# Theory of Mesoscopic Systems

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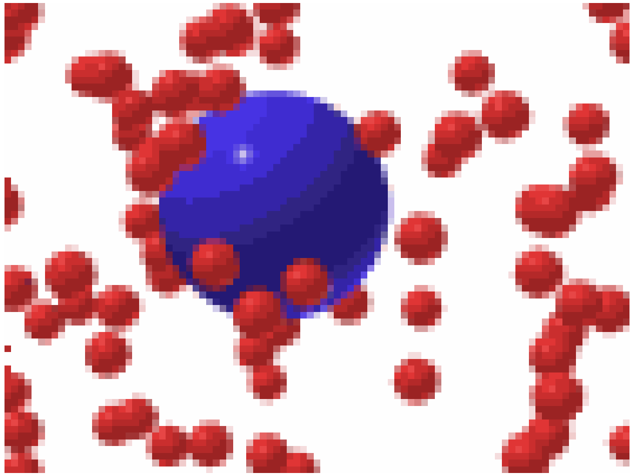
CUSO

CONFÉRENCE UNIVERSITAIRE
DE SUISSE OCCIDENTALE

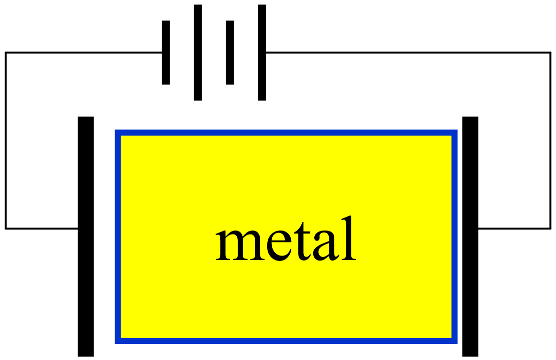
**Lecture 2   08 June 2006**

# Previous Lecture

# Brownian Motion - Diffusion



# Einstein-Sutherland Relation for electric conductivity $\sigma$




$$n = n(\mu)$$

Density of electrons

Chemical potential

$$\frac{dn}{dx} = \frac{dn}{d\mu}\frac{d\mu}{dx} = eE\frac{dn}{d\mu}$$

Electric field

No current

$$eD\frac{dn}{dx} = \sigma E$$

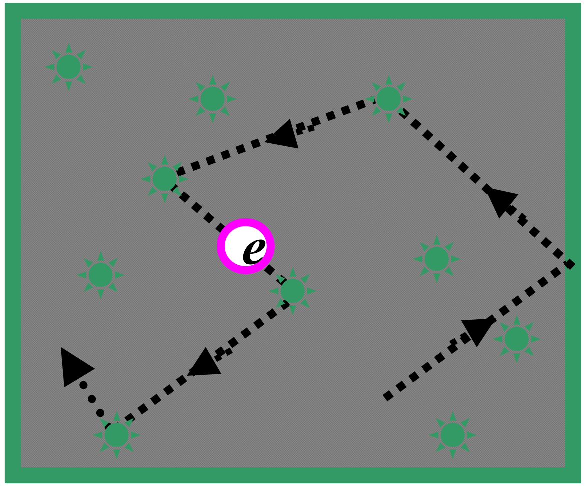
Conductivity

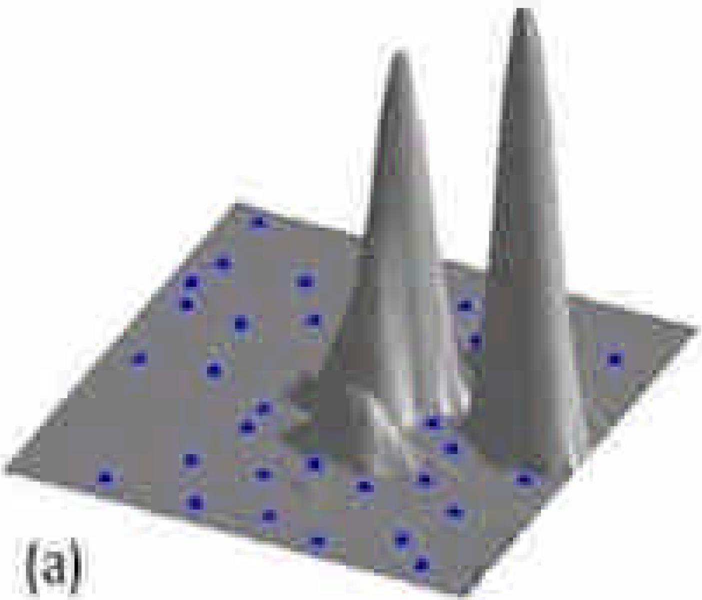
$$\boxed{\sigma = e^2 D\nu \qquad \nu \equiv \frac{dn}{d\mu}}$$

Density of states

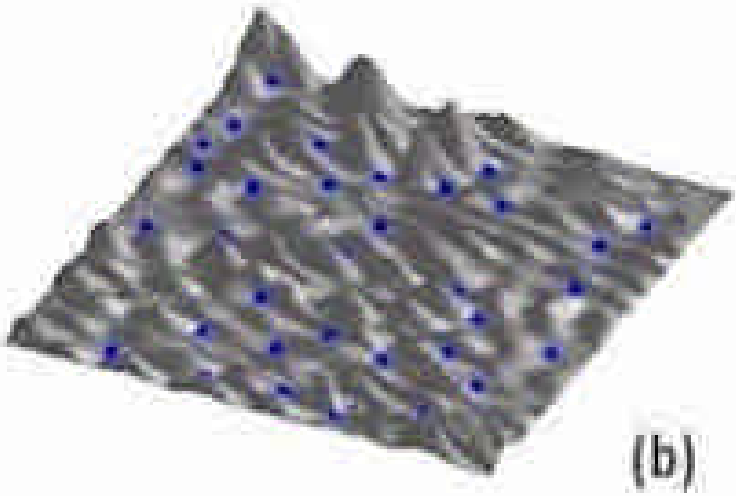
## Anderson localization

## Quantum particle in random quenched potential



(a)

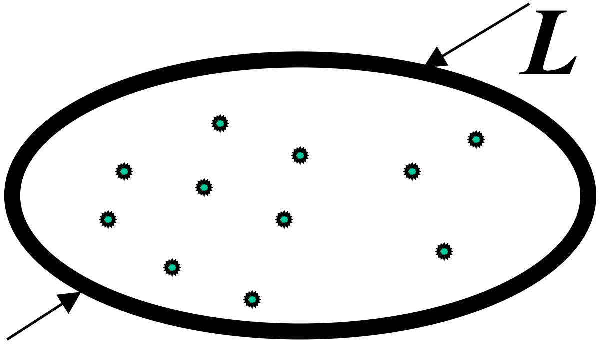


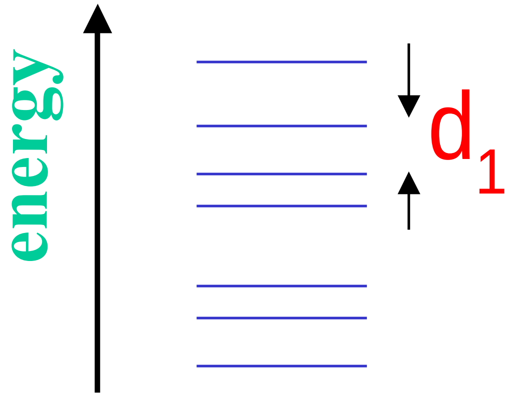
(b)

# Energy scales (*Thouless, 1972*)

## 1. Mean level spacing

$$d_1 = 1/n´ \, L^d$$




***L*** *is the system size;*

***d*** *is the number of dimensions*

## 2. Thouless energy

$$E_T = hD/L^2$$

***D*** *is the diffusion const*

$E_T$ *has a meaning of the **inverse diffusion time** of the traveling through the system or the **escape rate** (for open systems)*

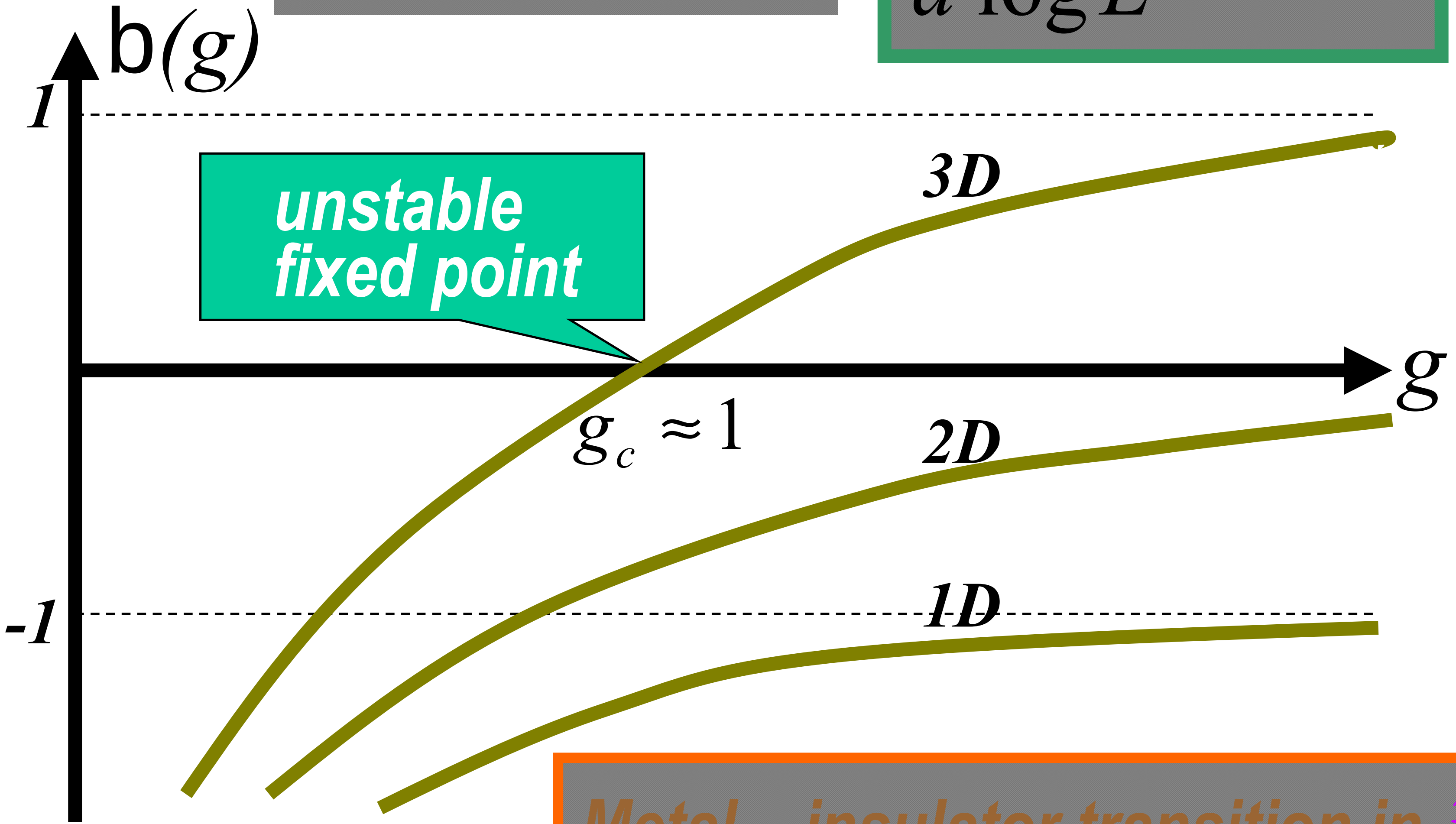
$$g = E_T / d_1$$

*dimensionless* ***Thouless*** *conductance*

$$g = Gh/e^2$$

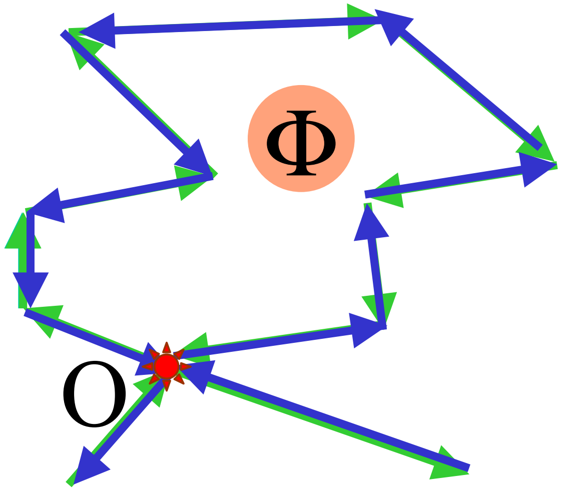
# b - function

$$\frac{d\log g}{d\log L} = \mathrm{b}(g)$$



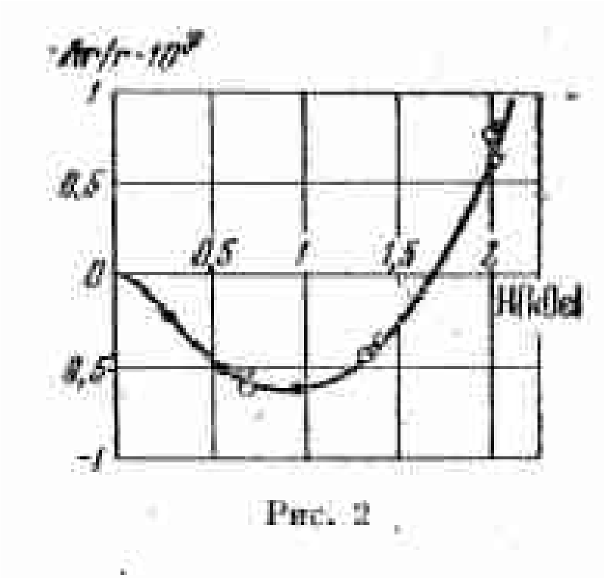

*Metal – insulator transition in* ***3D***

*All states are localized for* ***d=1,2***

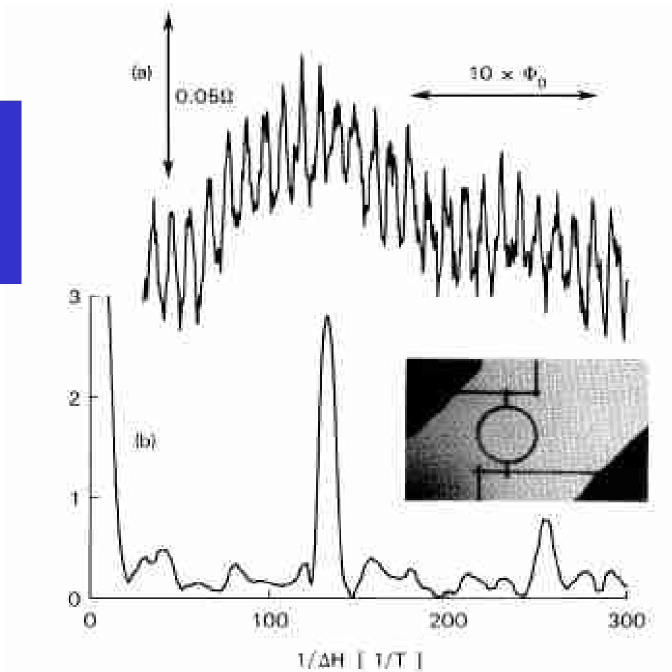


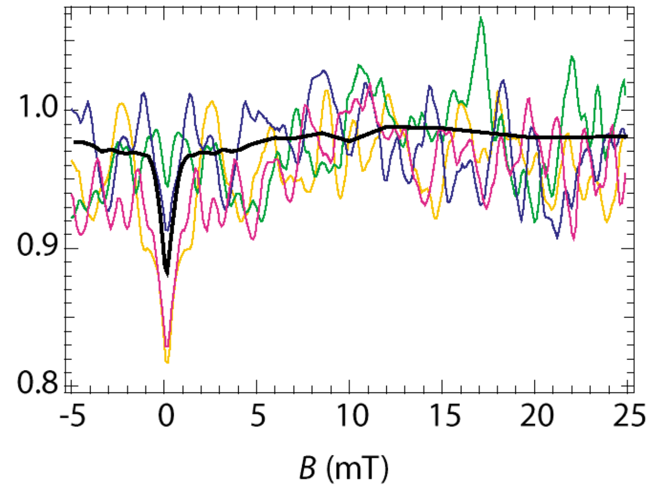

$$\varphi_1 - \varphi_2 = 2\times 2\pi\,\frac{\Phi}{\Phi_0};\quad \Phi_0 = \frac{hc}{e}$$

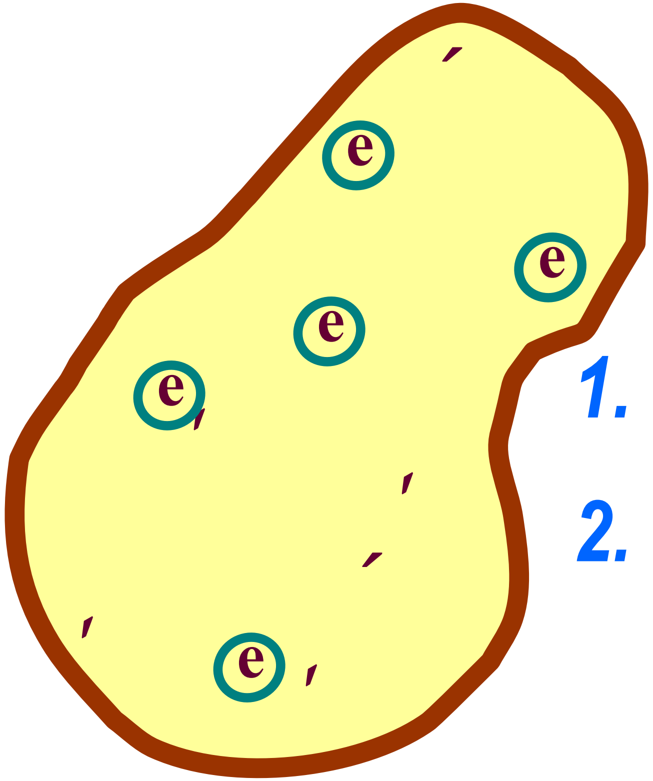
# Weak localization




# Mesoscopic fluctuations






1. *Disorder (´ - impurities)*
2. *Complex geometry*

**How to deal with disorder?**

- ~~Solve the Shrodinger equation exactly~~
- Make statistical analysis

**What if there in no disorder?**

| | |
|---|---|
| **Beforehand** | **Weak Localization and Mesoscopic Fluctuations** |
| **Today** | **Random Matrices, Anderson Localization, and Quantum Chaos** |
| **Later** | **Interaction between electrons in mesoscopic systems** |

# ORIGINS

**E.P. Wigner, Conference on Neutron Physics by Time of Flight, November 1956**

**P.W. Anderson, "*Absence of Diffusion in Certain Random Lattices*"; Phys.Rev., 1958, v.109, p.1492**

**L.D. Landau, "*Fermi-Liquid Theory*" Zh. Exp. Teor. Fiz.,1956, v.30, p.1058**

**J. Bardeen, L.N. Cooper & J. Schriffer, "*Theory of Superconductivity*"; Phys.Rev., 1957, v.108, p.1175.**

# Random Matrices

# Quantum Chaos

# Localization

# RANDOM MATRIX THEORY

**Spectral statstics**

$N$ ´ $N$ ***ensemble of Hermitian matrices with random matrix element*** $N$ ® $\infty$

$E_a$ - **spectrum (set of eigenvalues)**

$d_1 \equiv \langle E_{a+1} - E_a \rangle$ - **mean level spacing**

$\langle \ldots\ldots \rangle$ - **ensemble averaging**

$s \equiv \dfrac{E_{a+1} - E_a}{d_1}$ - **spacing between nearest neighbors**

$P(s)$ - **distribution function of nearest neighbors spacing between**

**Spectral Rigidity**

**Level repulsion**

$$P(s = 0) = 0$$

$$P(s << 1) \propto s^b \quad b=1,2,4$$

# Noncrossing rule (theorem) $P(s=0)=0$

Suggested by Hund (*Hund F. 1927 Phys. v.40, p.742*)

Justified by von Neumann & Wigner (*v. Neumann J. & Wigner E. 1929 Phys. Zeit. v.30, p.467*) . . . .

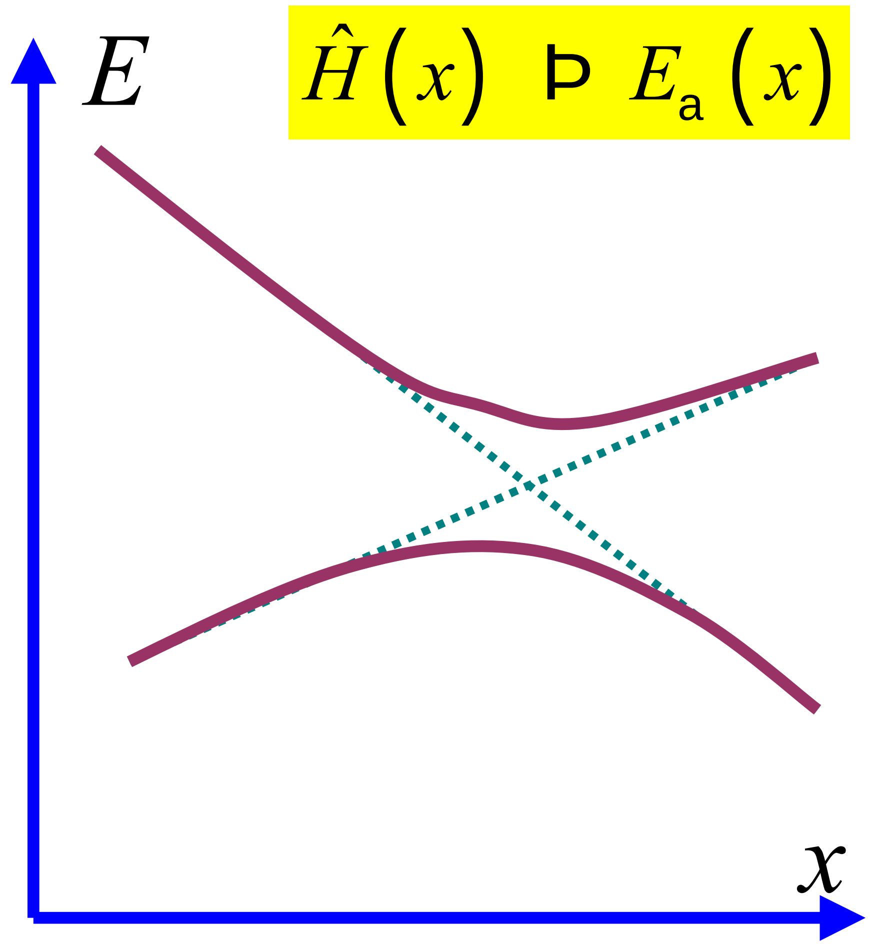
Usually textbooks present a simplified version of the justification due to Teller (*Teller E., 1937 J. Phys. Chem 41 109).*

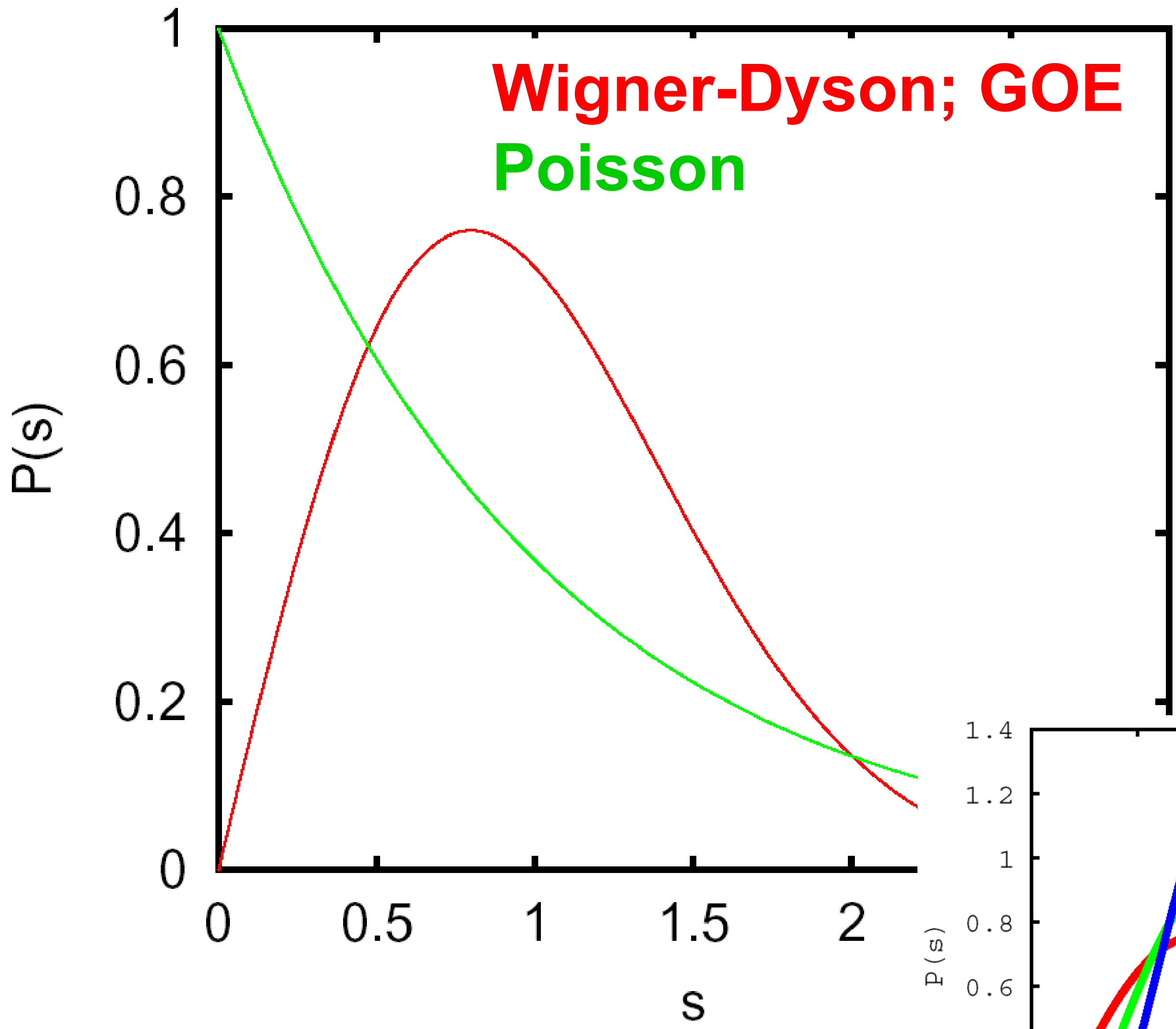
*Arnold V. I., 1972 Funct. Anal. Appl.v. 6, p.94*

*Mathematical Methods of Classical Mechanics (Springer-Verlag: New York), Appendix 10, 1989*

## **Arnold V.I.**, *Mathematical Methods of Classical Mechanics (Springer-Verlag: New York), Appendix 10, 1989*

**In general, a multiple spectrum in typical families of quadratic forms is observed only for two or more parameters, while in one-parameter families of general form the spectrum is simple for all values of the parameter. Under a change of parameter in the typical one-parameter family the eigenvalues can approach closely, but when they are sufficiently close, it is as if they begin to repel one another. The eigenvalues again diverge, disappointing the person who hoped, by changing the parameter to achieve a multiple spectrum.**



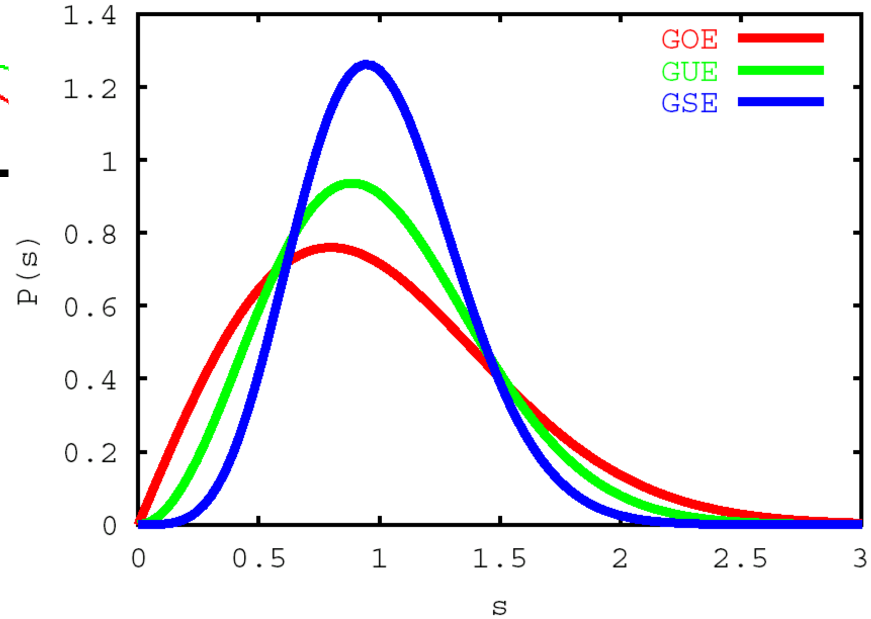



**Gaussian**
**Orthogonal**
**Ensemble**

**Orthogonal**
b=1

**Unitary**
b=2

**Simplectic**
b=4




**Poisson – completely uncorrelated levels**

# RANDOM MATRICES

*N* ´ *N* matrices with random matrix elements. *N* ® ∞

## Dyson Ensembles

| Matrix elements | Ensemble | b | realization |
|---|---|---|---|
| real | orthogonal | 1 | T-inv potential |
| complex | unitary | 2 | broken T-invariance (e.g., by magnetic field) |
| *2* ´ *2* matrices | simplectic | 4 | T-inv, but with spin-orbital coupling |

## Reason for $P(s) \rightarrow 0$ when $s \rightarrow 0$:

$$\hat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{12}^{*} & H_{22} \end{pmatrix}$$

$$E_2 - E_1 = \sqrt{\left(H_{22} - H_{11}\right)^2 + \left|H_{12}\right|^2}$$

small small small

1. The assumption is that the matrix elements are statistically independent. Therefore probability of two levels to be degenerate vanishes.

2. If $H_{12}$ is real (orthogonal ensemble), then for $s$ to be small two statistically independent variables ($(H_{22} - H_{11})$ and $H_{12}$) should be small and thus $P(s) \propto s \qquad \beta = 1$

3. Complex $H_{12}$ (unitary ensemble) $\Rightarrow$ both $Re(H_{12})$ and $Im(H_{12})$ are statistically independent $\Rightarrow$ three independent random variables should be small $\Rightarrow$ $P(s) \propto s^2 \qquad \beta = 2$

# Finite size quantum physical systems

Atoms

Nuclei

Molecules

.

.

.

Quantum Dots

**ATOMS** Main goal is to classify the eigenstates in terms of the quantum numbers

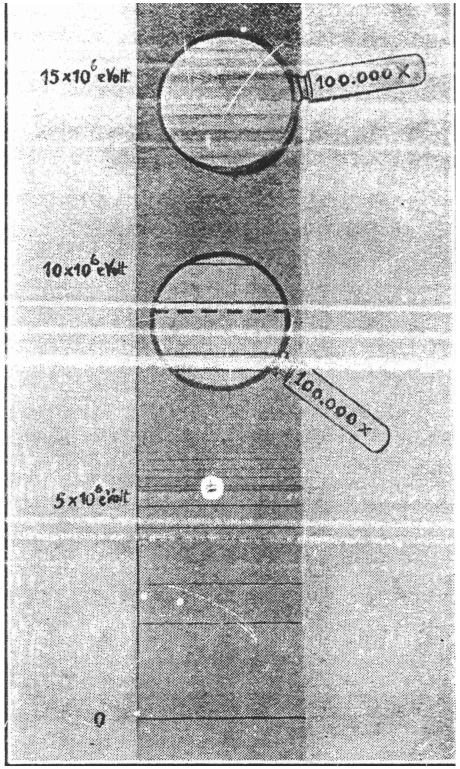
**NUCLEI** For the nuclear excitations this program does not work

***E.P. Wigner:*** Study spectral statistics of a particular quantum system – a given nucleus

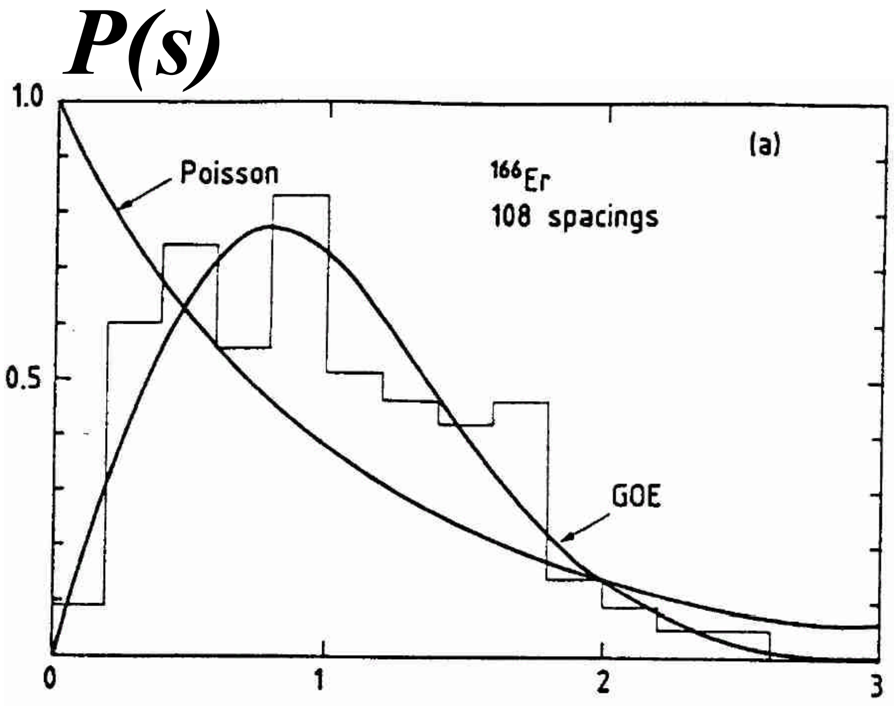
**Spectra: $\{E_a\}$**

| Random Matrices | Atomic Nuclei |
|---|---|
| • ***Ensemble*** | • ***Particular quantum system*** |
| • ***Ensemble averaging*** | • ***Spectral averaging (over a)*** |

**Nevertheless** Statistics of the nuclear spectra are almost exactly the same as the Random Matrix Statistics

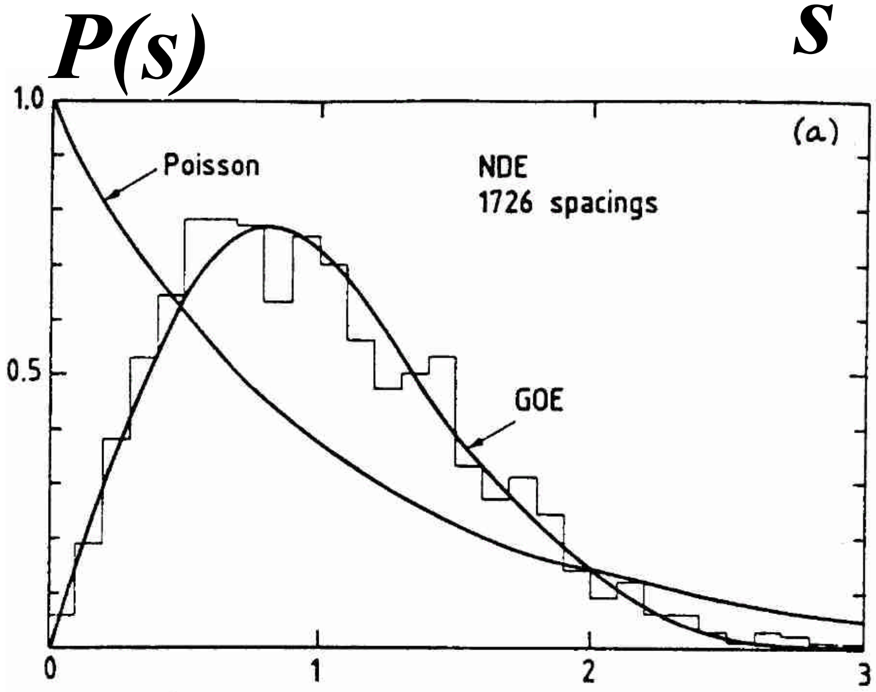



N. Bohr, Nature 137 (1936) 344.




Particular nucleus

$^{166}Er$




Spectra of several nuclei combined (after spacing) rescaling by the mean level

**Q:** *Why the random matrix theory (RMT) works so well for nuclear spectra* **?**

**Original answer:**

*These are systems with a large number of degrees of freedom, and therefore the "complexity" is high*

**Later it became clear that**

*there exist very "simple" systems with as many as 2 degrees of freedom (d=2), which demonstrate RMT - like spectral statistics*

# Classical ($h=0$) Dynamical Systems with $d$ degrees of freedom

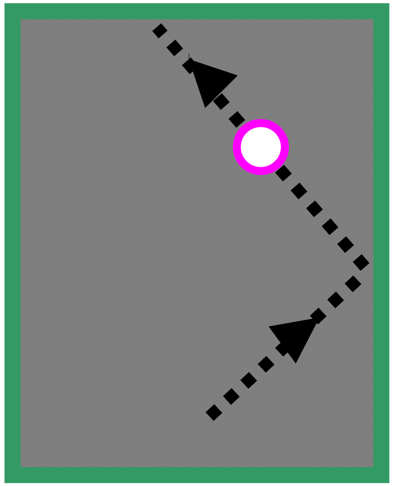
**Integrable Systems**

The variables can be separated and the problem reduces to $d$ one-dimensional problems ⇒ $d$ integrals of motion

## Examples

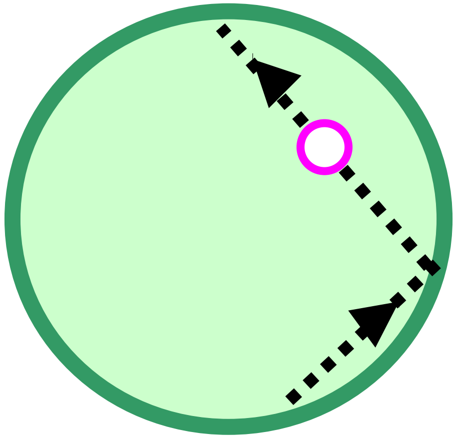
### 1. A ball inside rectangular billiard; $d=2$

- Vertical motion can be separated from the horizontal one
- Vertical and horizontal components of the momentum, are both integrals of motion



### 2. Circular billiard; $d=2$

- Radial motion can be separated from the angular one
- Angular momentum and energy are the integrals of motion



# Classical Dynamical Systems with *d* degrees of freedom

**Integrable Systems**

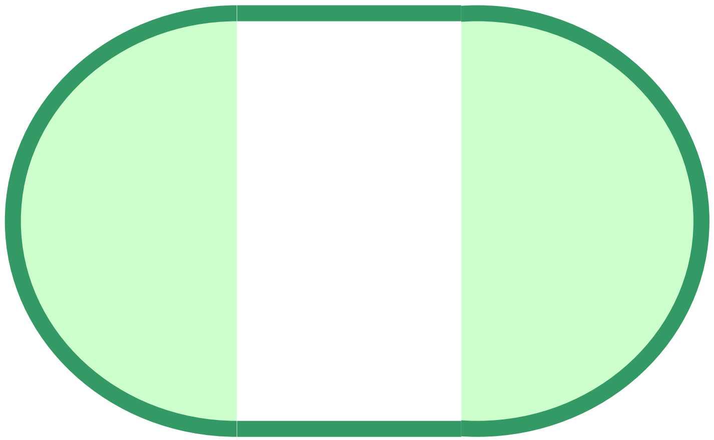
The variables can be separated ⇨ *d* one-dimensional problems ⇨ *d* integrals of motion

Rectangular and circular billiard, Kepler problem, . . . , 1d Hubbard model and other exactly solvable models, . .

**Chaotic Systems**

The variables can not be separated ⇨ there is only one integral of motion - energy

## Examples



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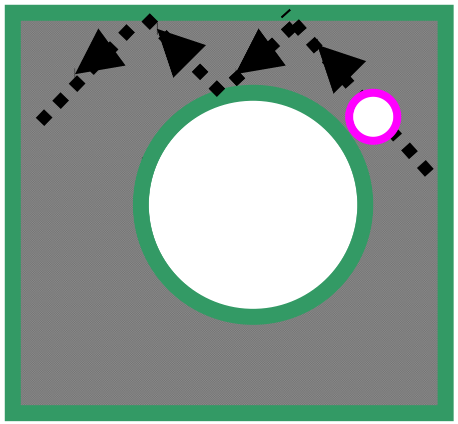
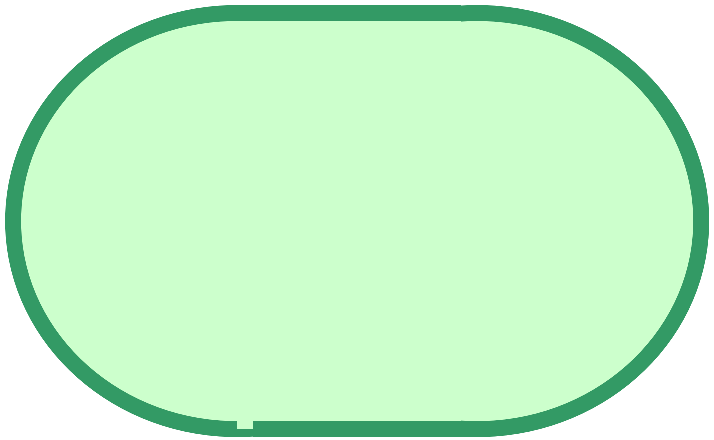
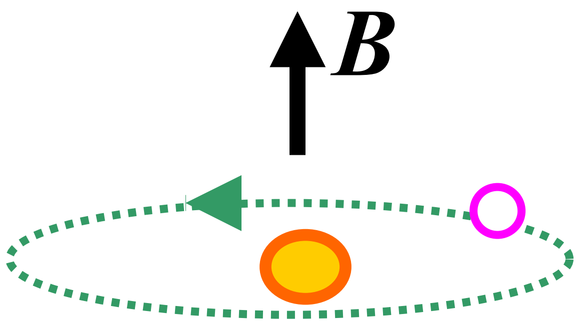


Sinai billiard



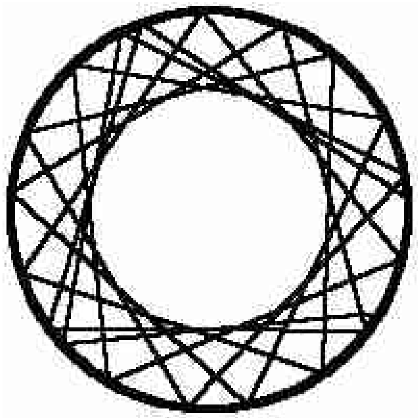
Stadium

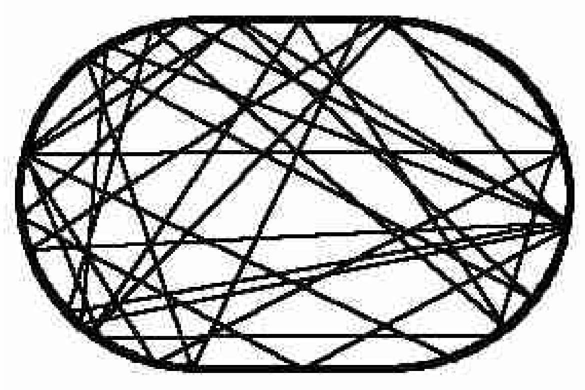



Kepler problem in magnetic field

## Classical Chaos
## h =0

- *Nonlinearities*
- *Exponential dependence on the original conditions (Lyapunov exponents)*
- *Ergodicity*





**Quantum description of any System with a finite number of the degrees of freedom is a linear problem – Shrodinger equation**

## Q: What does it mean Quantum Chaos ?

# $\hbar$ 1 0 Bohigas – Giannoni – Schmit conjecture

VOLUME 52 2 JANUARY 1984 NUMBER 1

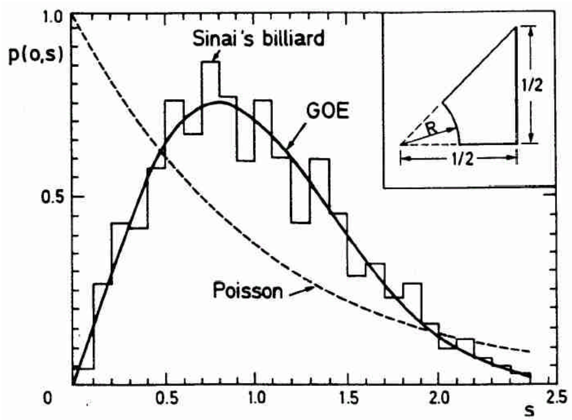
**Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws**

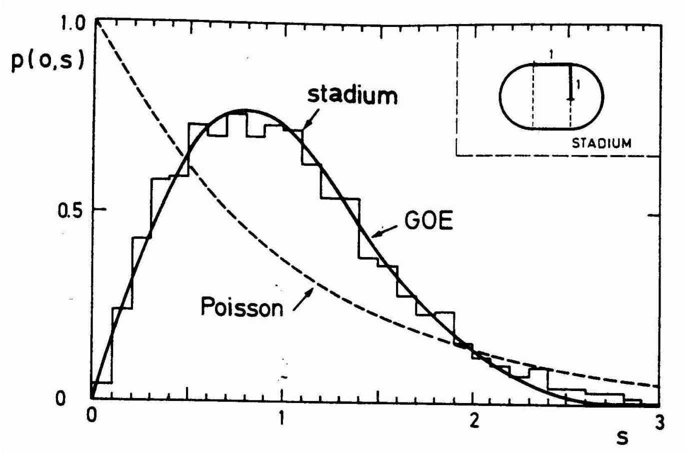
O. Bohigas, M. J. Giannoni, and C. Schmit
*Division de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France*

(Received 2 August 1983)

It is found that the level fluctuations of the quantum Sinai's billiard are consistent with the predictions of the Gaussian orthogonal ensemble of random matrices. This reinforces the belief that level fluctuation laws are universal.

In summary, the question at issue is to prove or disprove the following conjecture: Spectra of time-reversal-invariant systems whose classical analogs are K systems show the same fluctuation properties as predicted by GOE




**Chaotic classical analog**

**Wigner- Dyson spectral statistics**

**No quantum numbers except energy**

# Q: What does it mean Quantum Chaos ?

***Two possible definitions***

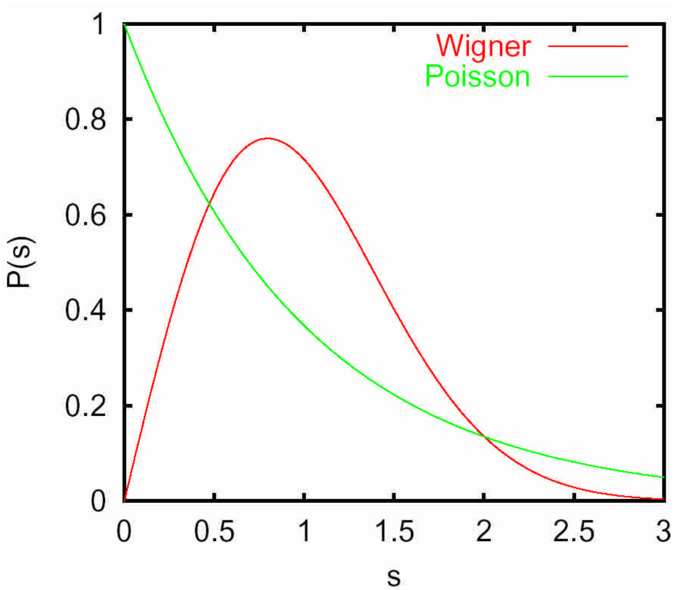
**Chaotic** classical analog

**Wigner - Dyson**-like spectrum

Classical Quantum

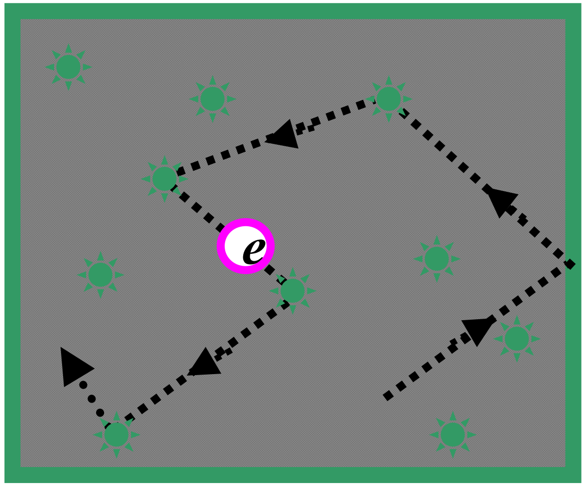
**Integrable** ? ⟷ **Poisson**

**Chaotic** ? ⟷ **Wigner-Dyson**



# Poisson to Wigner-Dyson crossover

Important example: quantum particle subject to a random potential – disordered conductor



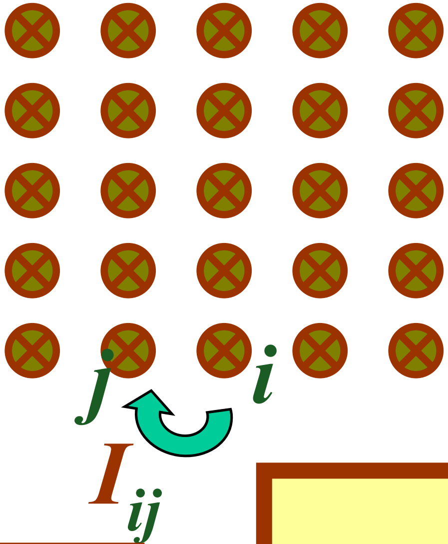

✷ *Scattering centers, e.g., impurities*

- As well as in the case of Random Matrices (RM) there is a luxury of ensemble averaging.
- The problem is much richer than RM theory
- There is still a lot of universality.

## Anderson localization (1958)

At strong enough disorder all eigenstates are localized in space

# Anderson Model




- *Lattice - tight binding model*
- *Onsite energies* $e_i$ *- random*
- *Hopping matrix elements* $I_{ij}$

$$-W < e_i < W$$

*uniformly distributed*

$$I_{ij} = \begin{cases} I & i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases}$$

## Anderson Transition

| $I < I_c$ | $I > I_c$ |
|---|---|
| ***Insulator*** | ***Metal*** |
| *All eigenstates are localized* | *There appear states extended all over the whole system* |
| *Localization length* X | |

# Anderson Transition

**Strong disorder**

$I < I_c$

## *Insulator*

***All eigenstates are localized***

***Localization length X***

***The eigenstates, which are localized at different places will not repel each other***

***Poisson spectral statistics***

**Weak disorder**

$I > I_c$

## *Metal*

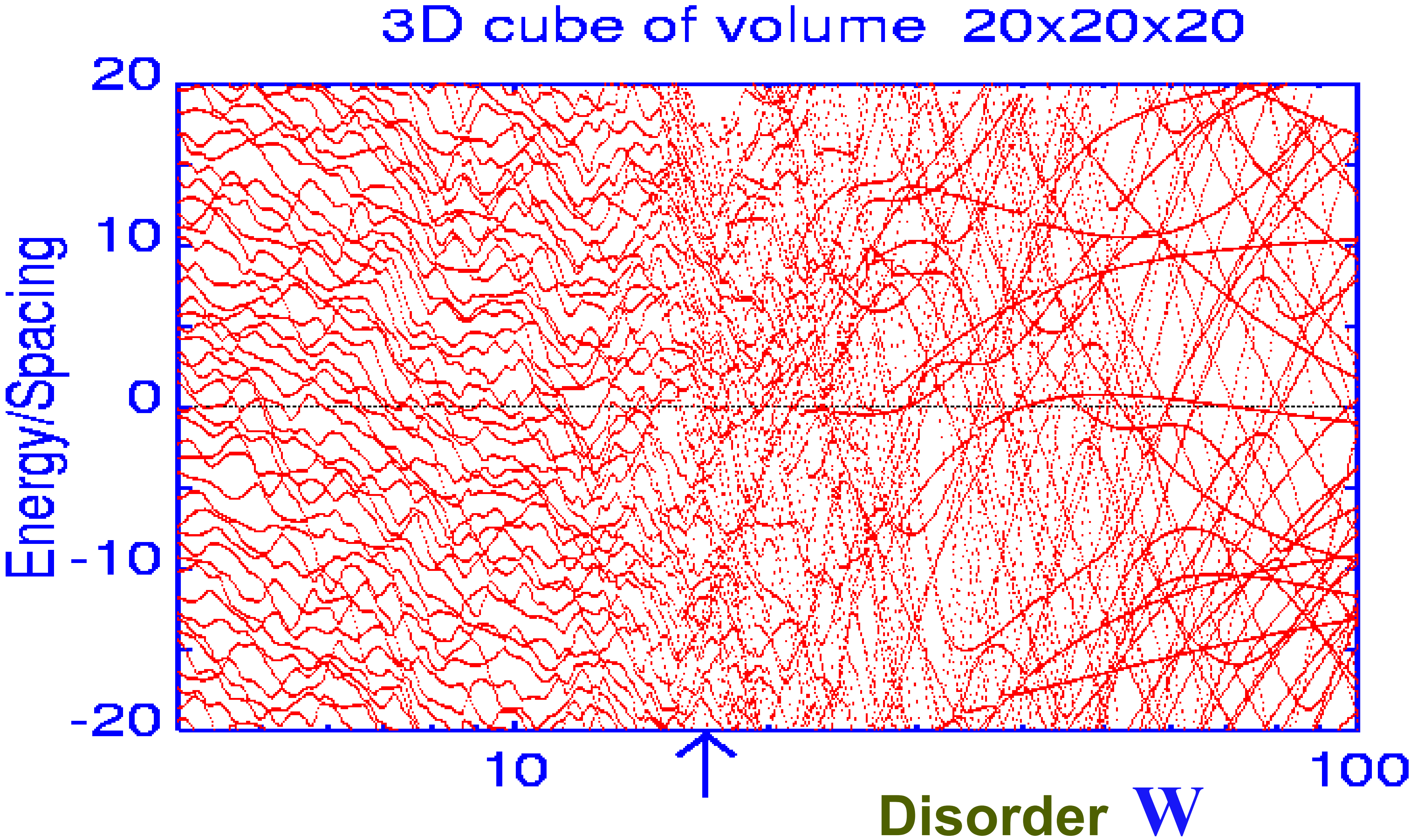
***There appear states extended all over the whole system***

***Any two extended eigenstates repel each other***

***Wigner – Dyson spectral statistics***

Zharekeschev & Kramer.

*Exact diagonalization of the Anderson model*



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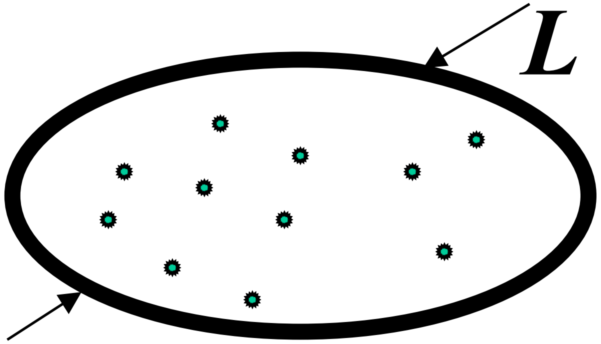


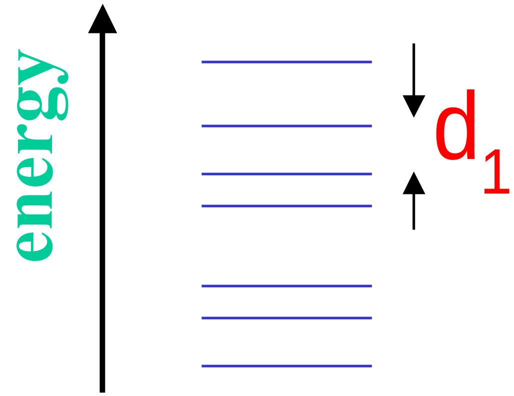


*$L$ is the system size;*

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*$D$ is the diffusion const*

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$$g = E_T / d_1$$

*dimensionless* ***Thouless*** *conductance*

$$g = Gh/e^2$$

# Thouless Conductance and One-particle Spectral Statistics

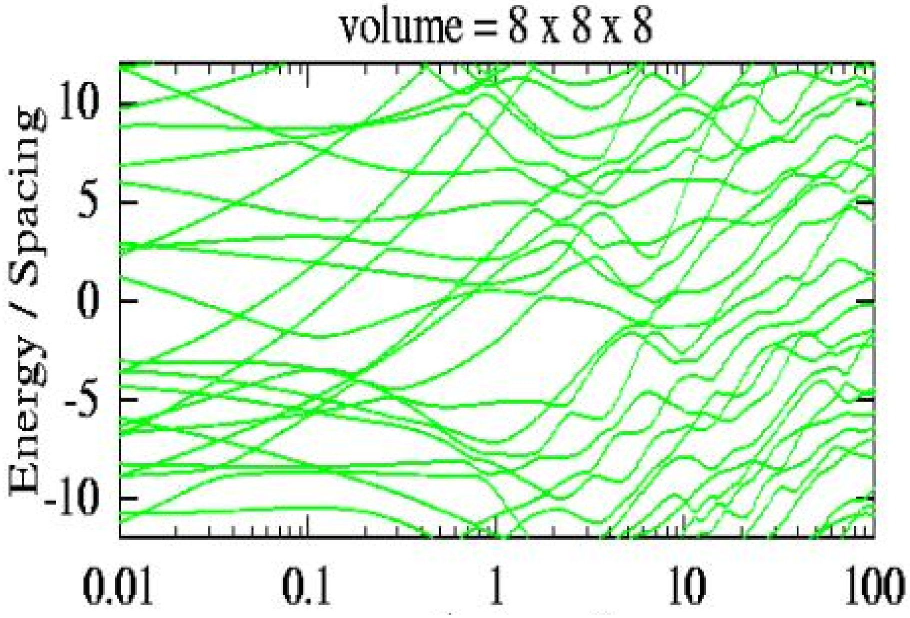

***Localized states***
***Insulator***

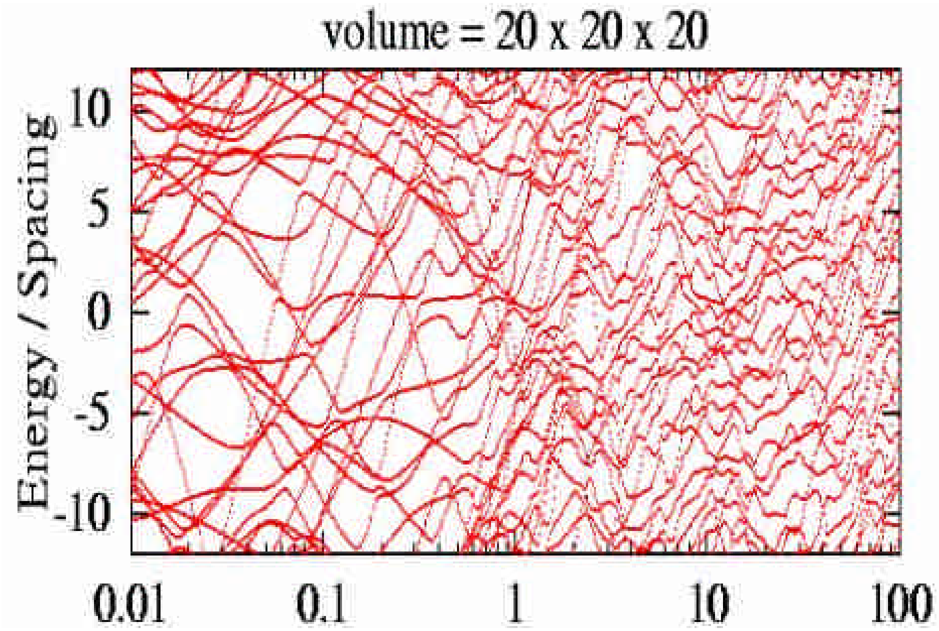
**Poisson spectral statistics**

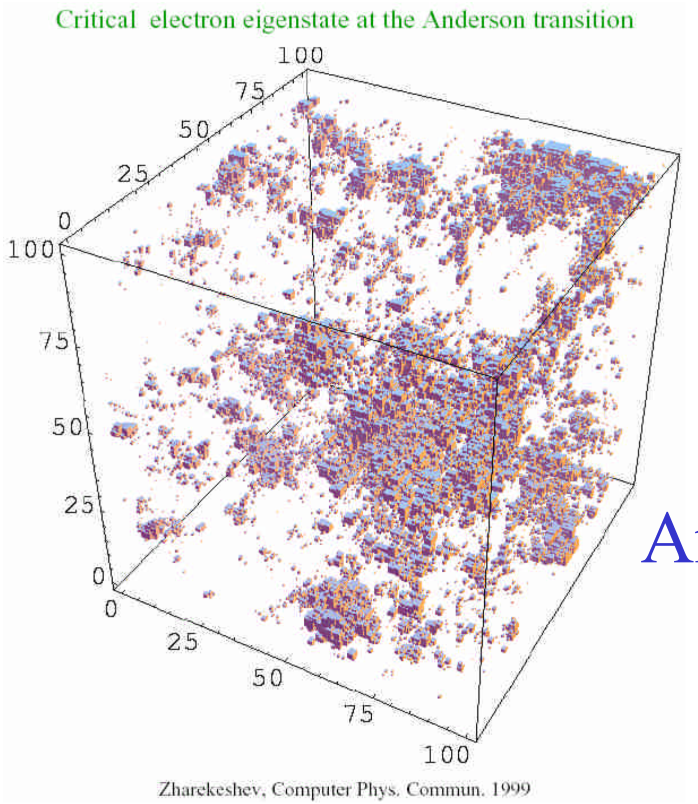
***Extended states***
***Metal***

**Wigner-Dyson spectral statistics**

Transition at $g\sim 1$.
Is it sharp?



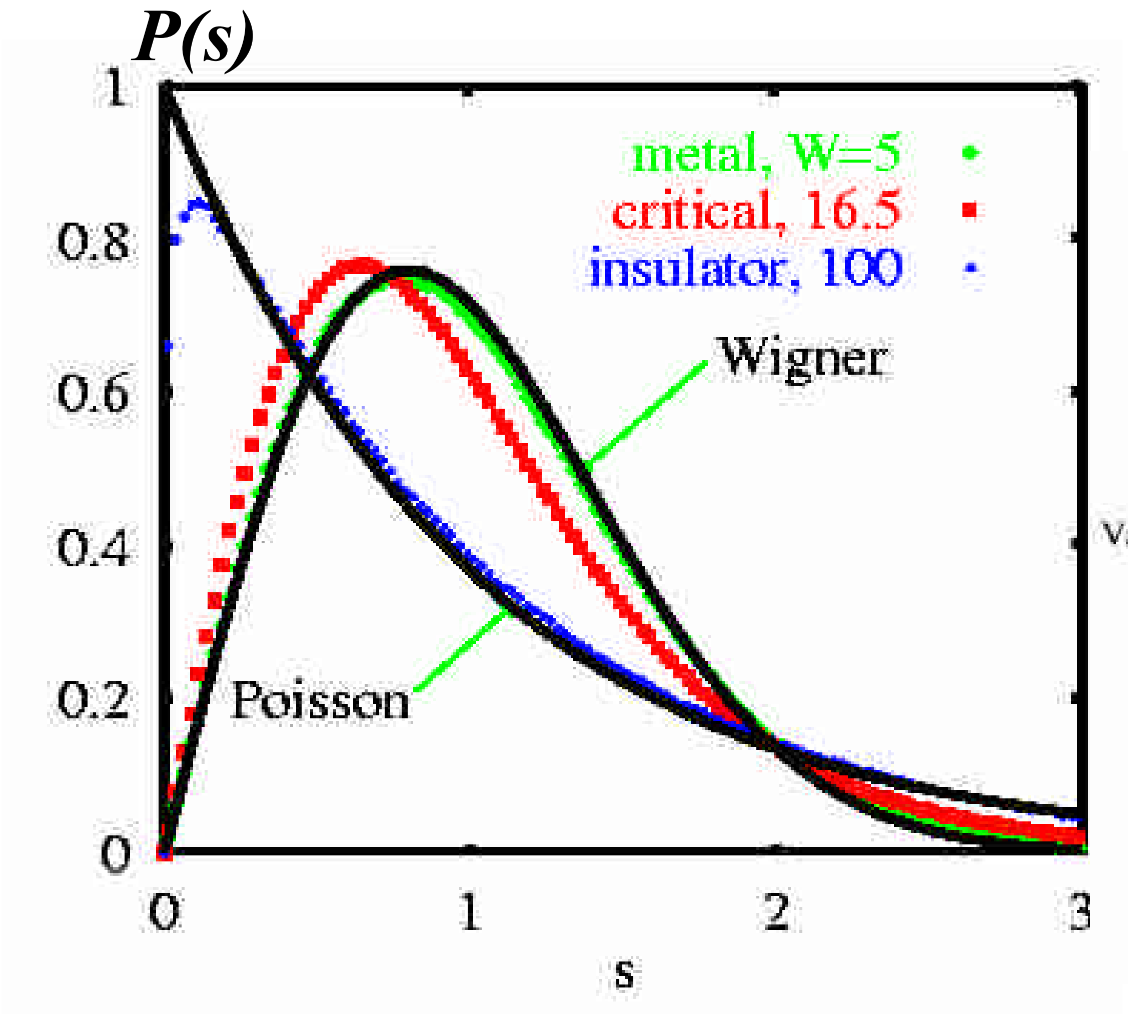



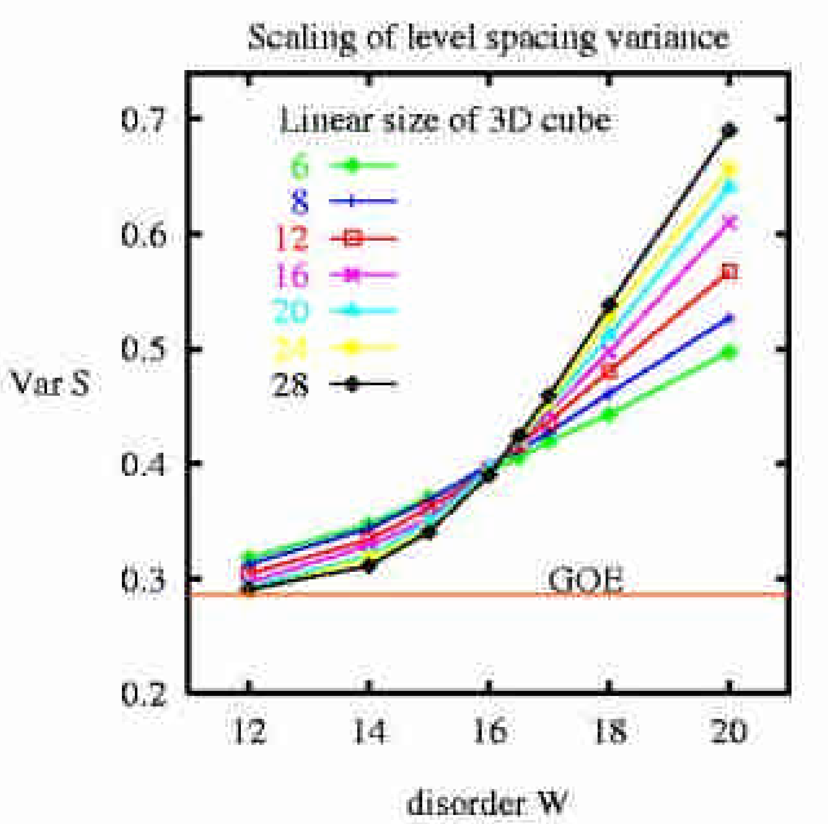

**Conductance *g***

$$100 \times 100 \times 100$$

Anderson model cube

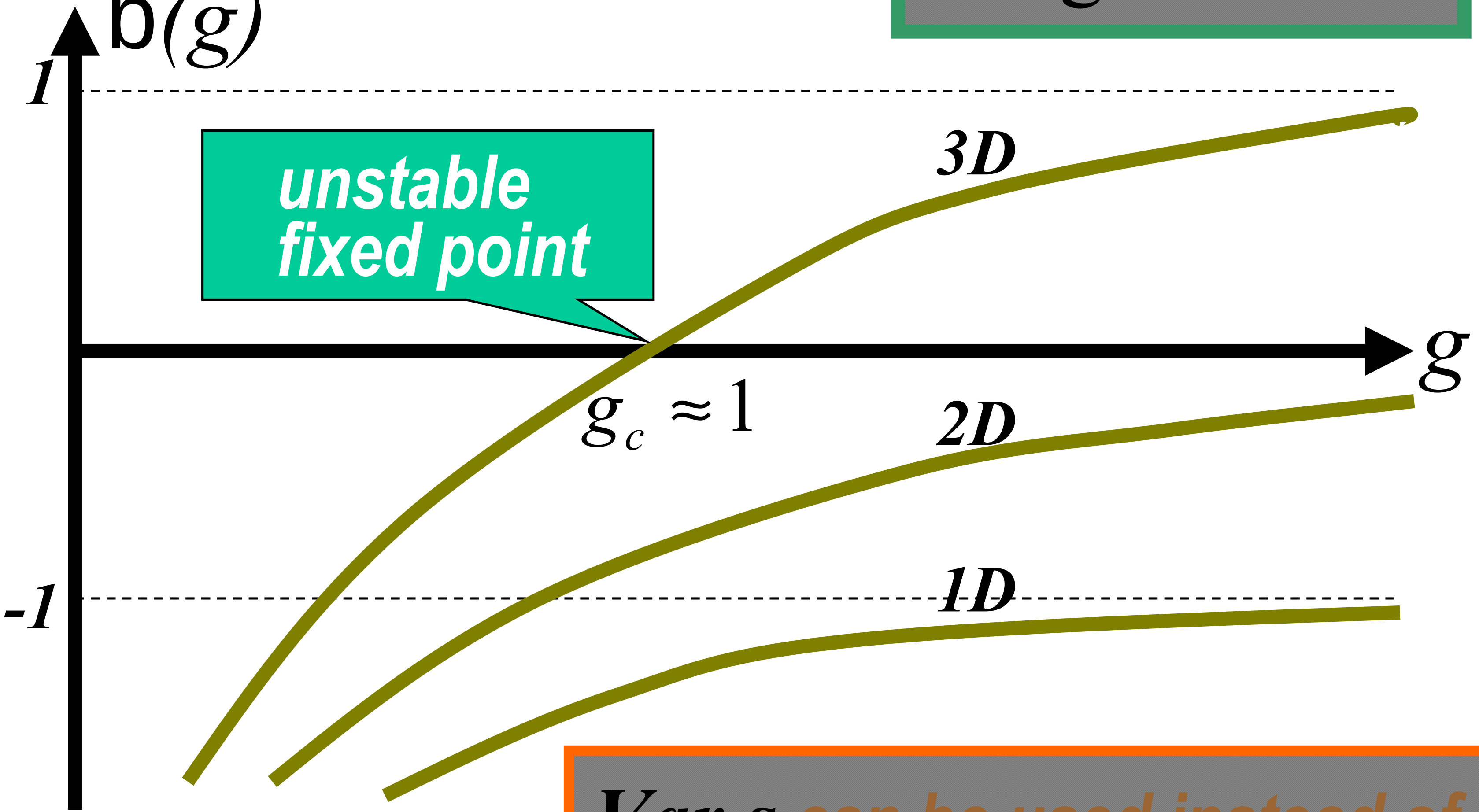
# Anderson transition in terms of pure level statistics



# b - function

$$\frac{d \log g}{d \log L} = \mathrm{b}(g)$$




***Var s** can be used instead of **g***

## Suggested problem:

Consider $$\tilde{b}(V) \equiv \frac{dV}{d(\log g)}$$

where $$V \equiv Var\, s \equiv \langle s^2 \rangle - \langle s \rangle^2$$

Is $\tilde{b}(V)$ universal function?

Sketch this function

# Thouless Conductance and One-particle Spectral Statistics

0 1 *g*

*Localized states*
*Insulator*

**Poisson spectral statistics**

*Extended states*
*Metal*

**Wigner-Dyson spectral statistics**

N ´ N
*Random Matrices*

*Quantum Dots with Thouless conductance* ***g***

N® ¥

*The same statistics of the random spectra and one-particle wave functions (eigenvectors)*

*g*® ¥

VOLUME 85, NUMBER 11 PHYSICAL REVIEW LETTERS 11 SEPTEMBER 2000

**Correlations due to Localization in Quantum Eigenfunctions of Disordered Microwave Cavities**

Prabhakar Pradhan and S. Sridhar
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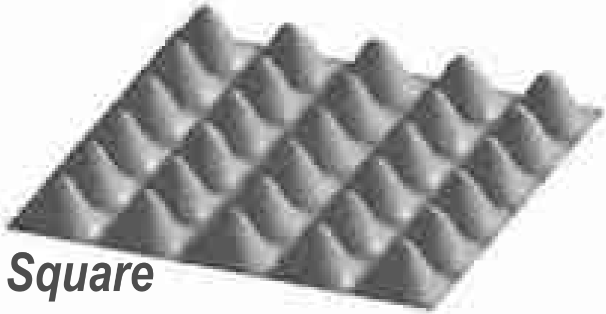
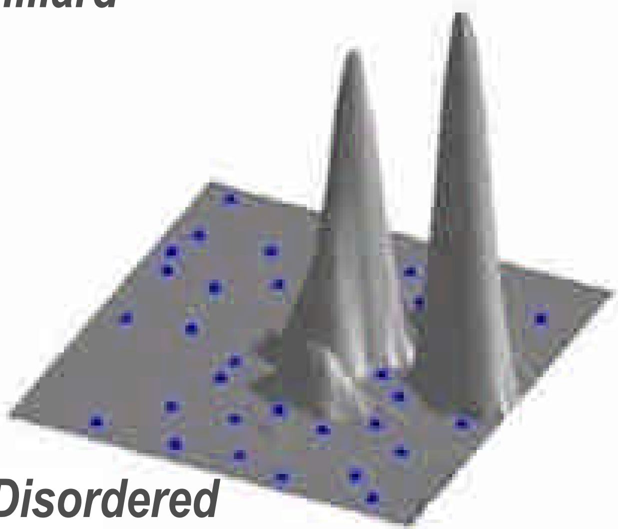
## Integrable



Square billiard

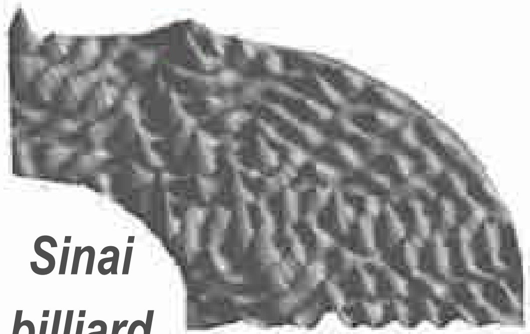


Disordered localized

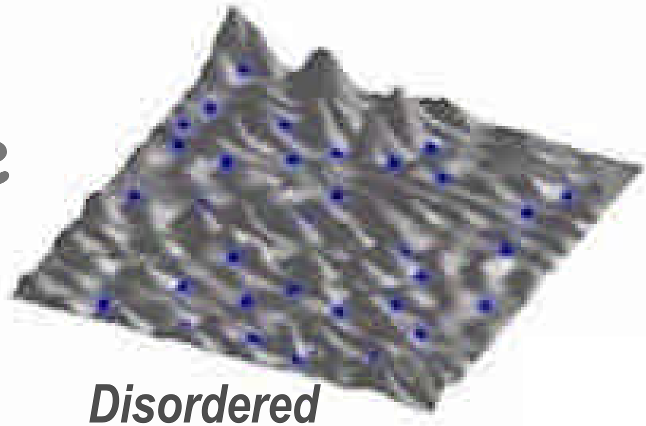
All chaotic systems resemble each other.

All integrable systems are integrable in their own way

## Chaotic



Sinai billiard



Disordered extended

## Disordered Systems:

*Anderson* ***metal;***
***Wigner-Dyson*** *spectral statistics*

*Anderson* ***insulator;***
***Poisson*** *spectral statistics*

**Q:** ***Is it a generic scenario for the Wigner-Dyson to Poisson crossover*** **?**

## Speculations

***Consider an integrable system. Each state is characterized by a set of quantum numbers.***

***It can be viewed as a point in the space of quantum numbers. The whole set of the states forms a lattice in this space.***

***A perturbation that violates the integrability provides matrix elements of the hopping between different sites (Anderson model !?)***

## Q: *Does Anderson localization provide a generic scenario for the Wigner-Dyson to Poisson crossover* ?

*Consider an **integrable** system. Each state is characterized by a **set of quantum numbers**.*

*It can be viewed as a point in the **space of quantum numbers**. The whole set of the states forms a **lattice** in this space.*

*A **perturbation** that violates the integrability provides matrix elements of the **hopping** between different sites (**Anderson model !?**)*

***Weak enough hopping** - Localization - **Poisson***
***Strong hopping** - transition to **Wigner-Dyson***

The very definition of the localization is **not invariant** – one should specify in which space the eigenstates are localized.

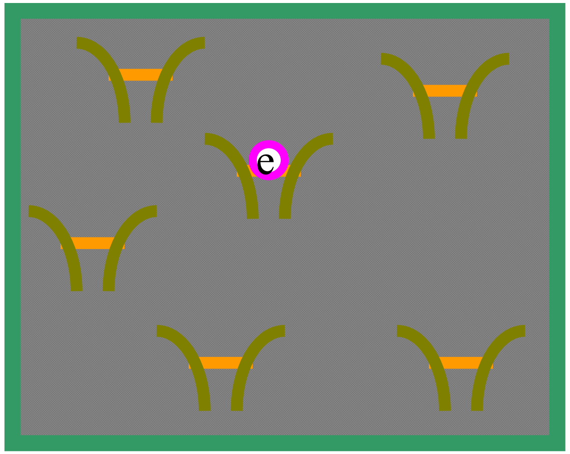
Level statistics **is invariant**:

**Poissonian statistics** $\exists$ basis where the eigenfunctions are localized

**Wigner -Dyson statistics** $\forall$ basis the eigenfunctions are extended

## Example 1

## Doped semiconductor




**Low concentration of donors** → Electrons are localized on donors ⇨ **Poisson**

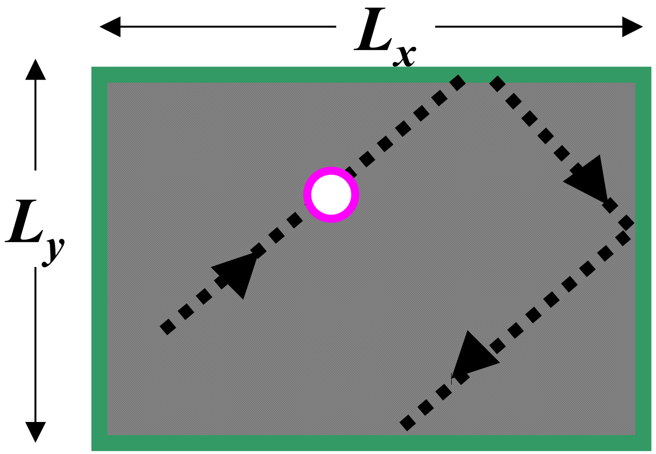
**Higher donor concentration** → Electronic states are extended ⇨ **Wigner-Dyson**

## Example 2

## Rectangular billiard

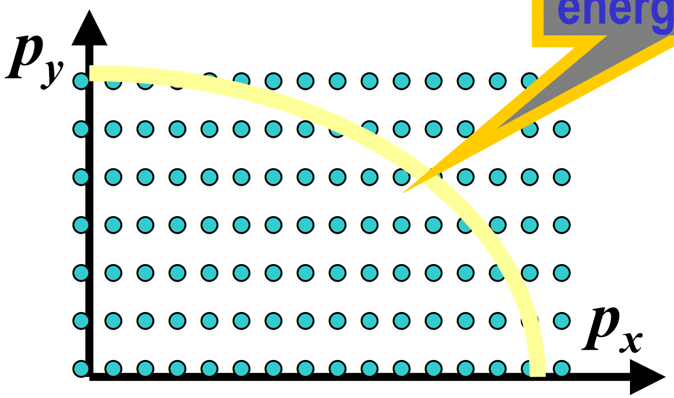
**Two integrals of motion**

$$p_x = \frac{\mathrm{p}n}{L_x}; \quad p_y = \frac{\mathrm{p}m}{L_x}$$




**Lattice in the momentum space**

Line (surface) of constant energy




**Ideal billiard** – **localization in the momentum space** ⇨ **Poisson**

**Deformation or smooth random potential** – **delocalization in the momentum space** ⇨ **Wigner-Dyson**

VOLUME 77, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1996

**Diffusion and Localization in Chaotic Billiards**

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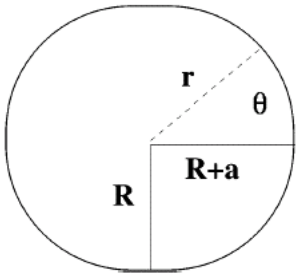
(Received 29 July 1996)

# Localization and diffusion in the angular momentum space

$$e \equiv \frac{a}{R}$$




$e > 0$ **Chaotic stadium**

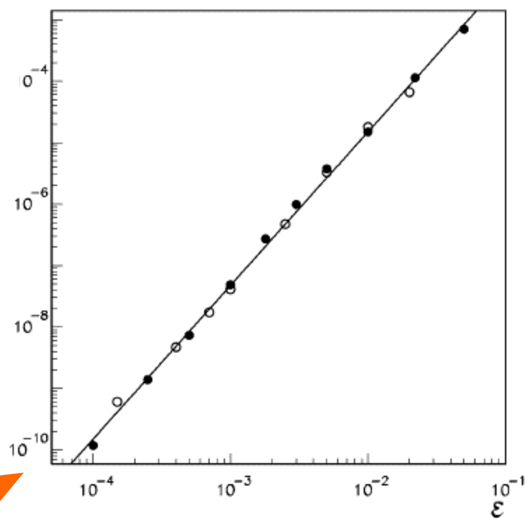
$e \to 0$ **Integrable circular billiard**

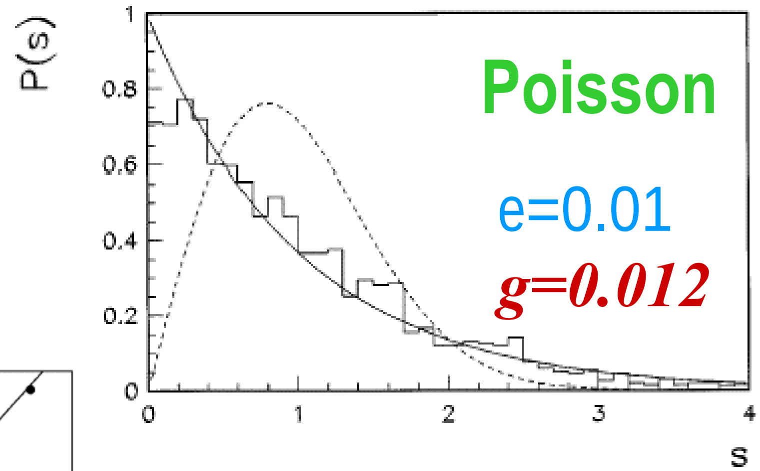
**Angular momentum is the integral of motion**

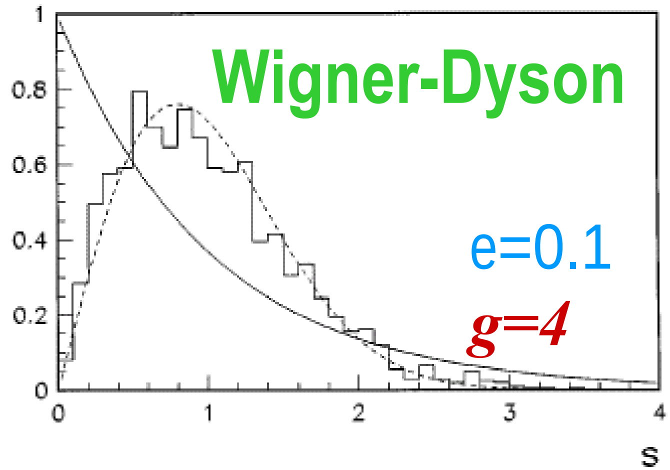
$$\hbar = 0; \quad e << 1$$

**Diffusion in the angular momentum space**

$$D \propto e^{5/2}$$



# Localization and diffusion in the angular momentum space

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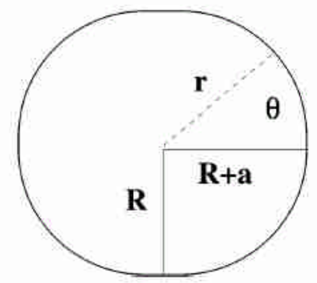


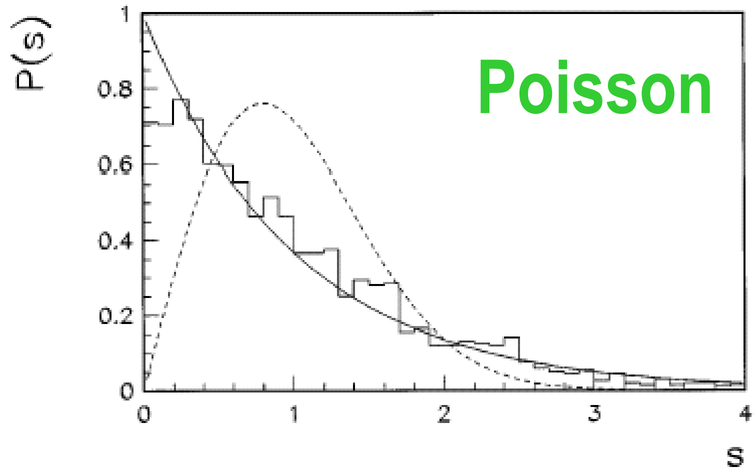

$\mathrm{e} > 0$ **Chaotic stadium**

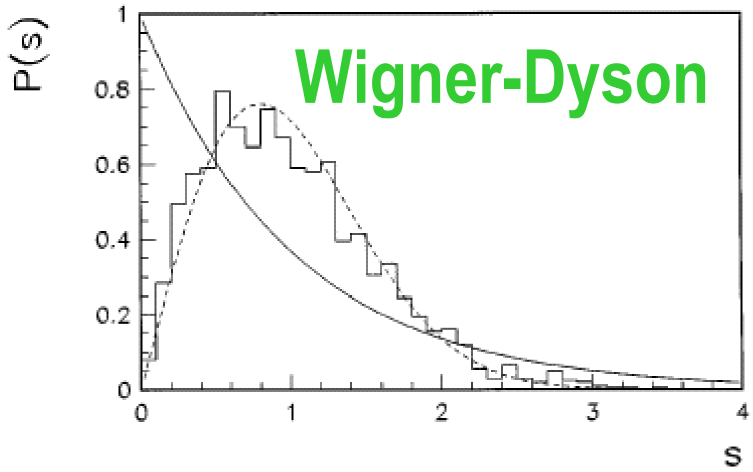
$\mathrm{e} \to 0$ **Integrable circular billiard**

**Angular momentum is the integral of motion**

$\hbar = 0; \quad \mathrm{e} << 1$

**Angular momentum is not conserved**



D.Poilblanc, T.Ziman, J.Bellisard, F.Mila & G.Montambaux

*Europhysics Letters, v.22, p.537, 1993*

## 1D Hubbard Model on a periodic chain

$$H = t\sum_{i,s}\left(c^{+}_{i,s}\,c_{i+1,s} + c^{+}_{i+1,s}\,c_{i,s}\right) + U\sum_{i,s} n_{i,s}\,n_{i,-s} + V\sum_{i,s,s'} n_{i,s}\,n_{i+1,s'}$$

Onsite interaction

n. neighbors interaction

$V = 0$ Hubbard model **integrable**

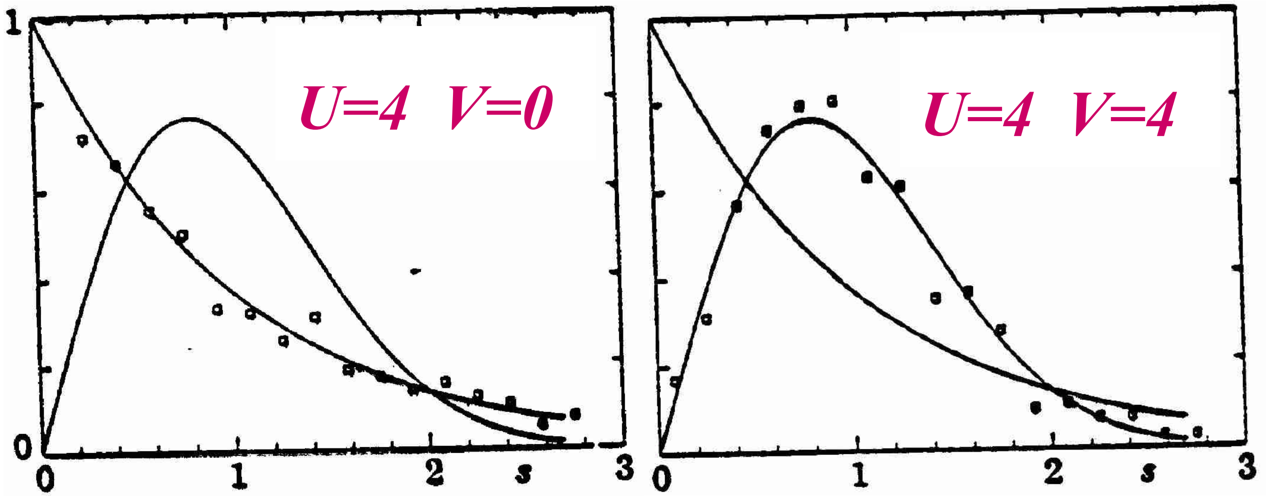
$V \neq 0$ extended Hubbard model **nonintegrable**

12 sites

3 particles

Zero total spin

Total momentum p/6



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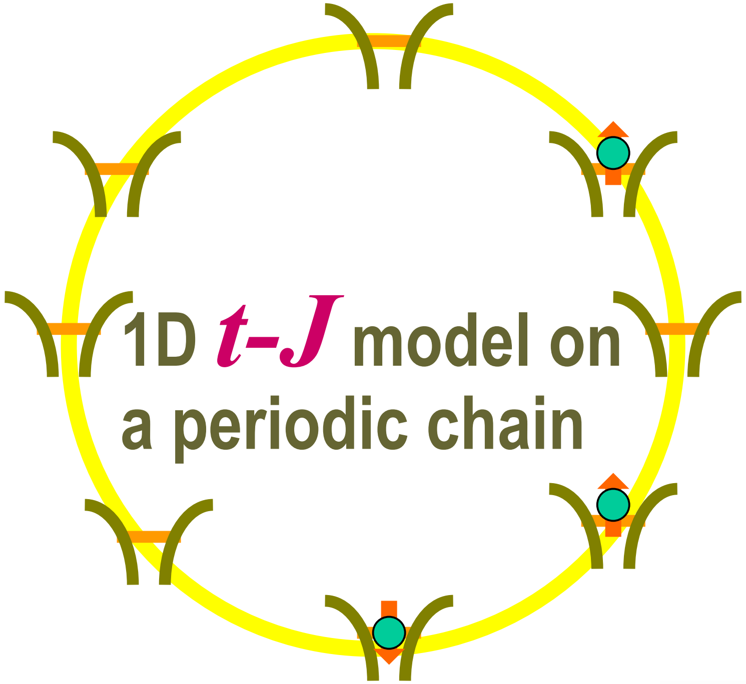
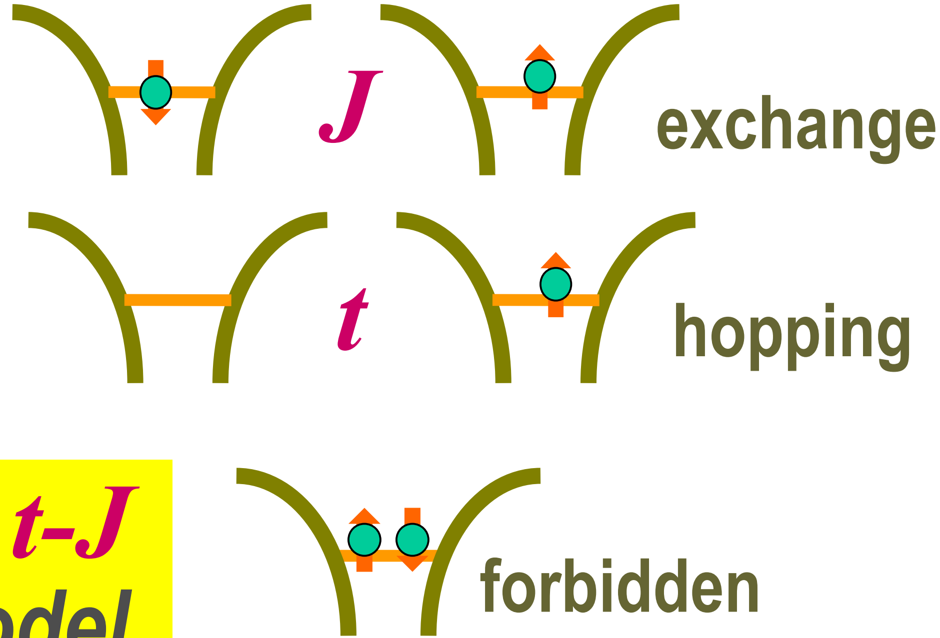
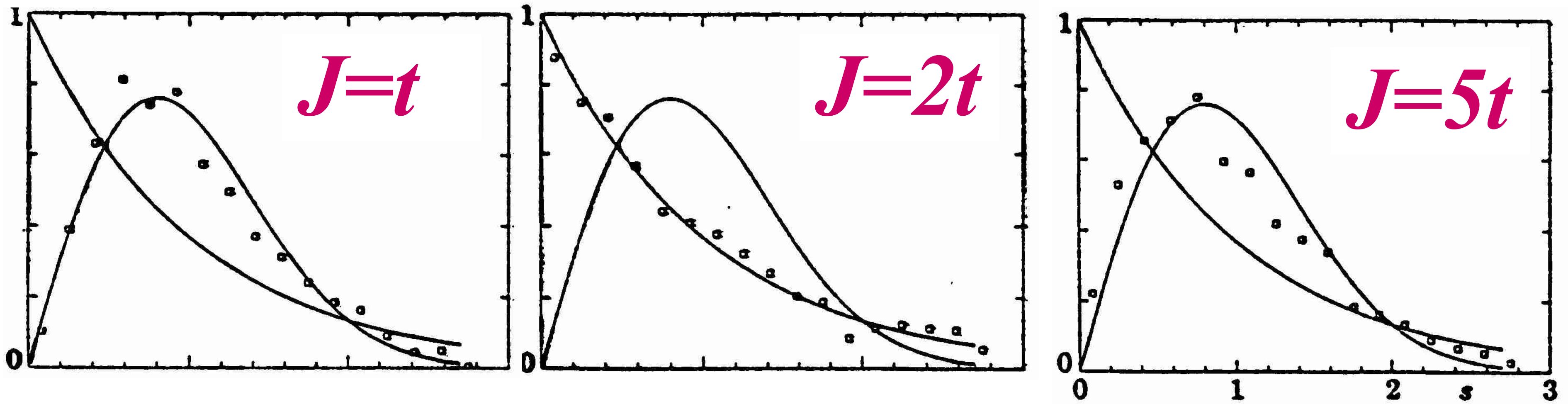



**1d *t-J***
***model***

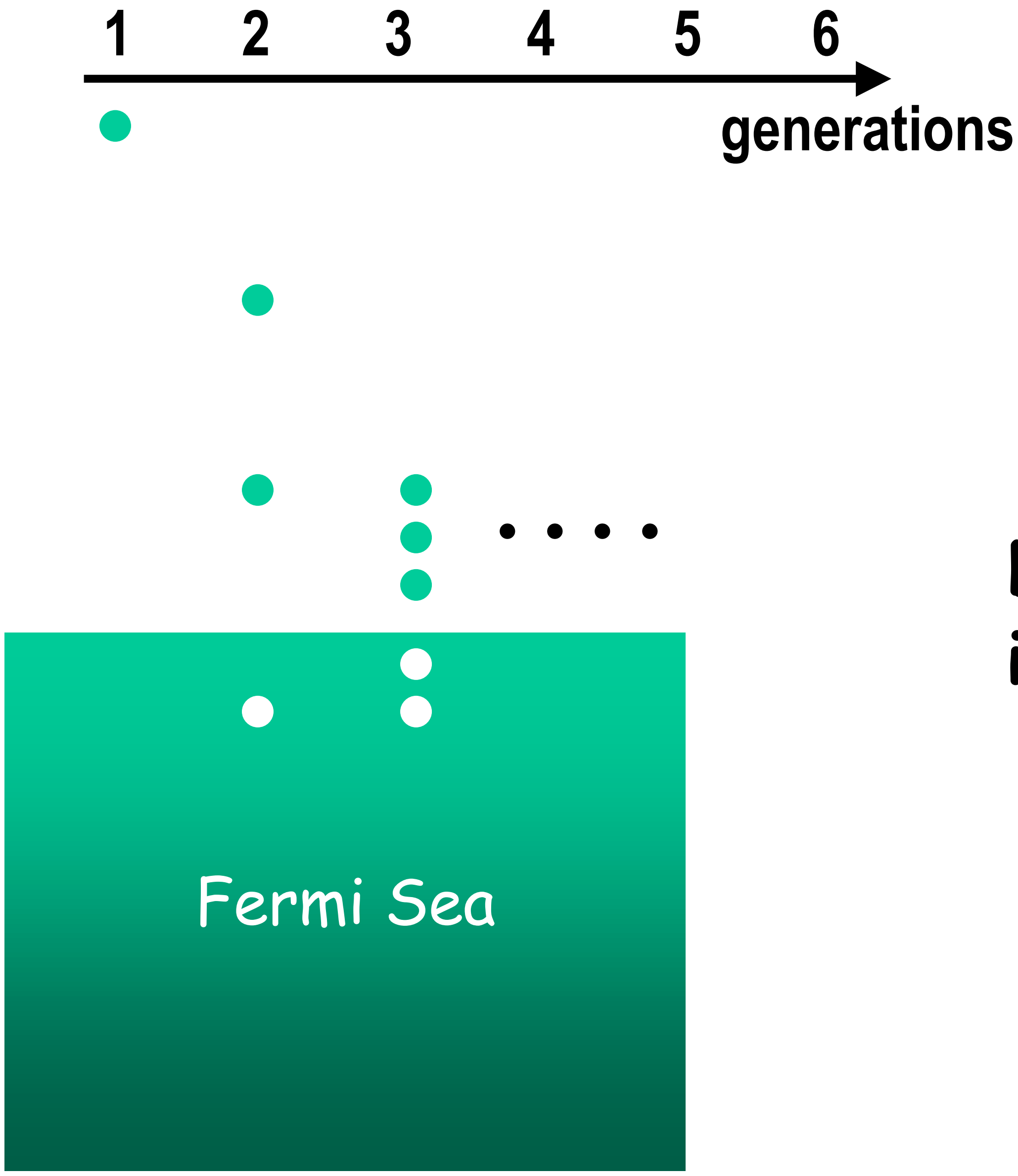



*N=16*; one hole

Q: ***Why the random matrix theory (RMT) works so well for nuclear spectra*** ?

Spectra of Many-Body excitations !

# Chaos in Nuclei – Delocalization?




**Delocalization in Fock space**

# Zero-dimensional

# Fermi Liquid

# ORIGINS

**E.P. Wigner, Conference on Neutron Physics by Time of Flight, November 1956**

**P.W. Anderson, "*Absence of Diffusion in Certain Random Lattices*"; Phys.Rev., 1958, v.109, p.1492**

**L.D. Landau, "*Fermi-Liquid Theory*" Zh. Exp. Teor. Fiz.,1956, v.30, p.1058**

**J. Bardeen, L.N. Cooper & J. Schriffer, "*Theory of Superconductivity*"; Phys.Rev., 1957, v.108, p.1175.**

~~What does it mean - non-Fermi liquid ?~~

Q: What is the difference between Fermi-liquid and non-Fermi liquid ?

A: The difference is the same as between bananas and non-bananas.

What does it mean Fermi liquid ?

- *Fermi statistics*
- *Low temperatures*
- *Not too strong interactions*
- ***Translation invariance***

$\Longrightarrow$ **Fermi Liquid**

***It means that***

1. ***Excitations are similar to the excitations in a Fermi-gas:***
   
   ***a) the same quantum numbers – momentum, spin ½ , charge $e$***
   
   ***b) decay rate is small as compared with the excitation energy***

2. ***Substantial renormalizations. For example, in a Fermi gas***

$$\partial n/\partial \mathrm{m}, \quad \mathrm{g} = c/T, \quad \mathrm{c}/g\mathrm{m}_B$$

***are all equal to the one-particle density of states n .***
***These quantities are different in a Fermi liquid***

# Signatures of the Fermi - Liquid state ?!

## 1. Resistivity is proportional to $T^2$ :

**L.D. Landau & I.Ya. Pomeranchuk** ***"To the properties of metals at very low temperatures"*****; Zh.Exp.Teor.Fiz., 1936, v.10, p.649**

**…The increase of the resistance caused by the interaction between the electrons is proportional to $T^2$ and at low temperatures exceeds the usual resistance, which is proportional to $T^5$.**

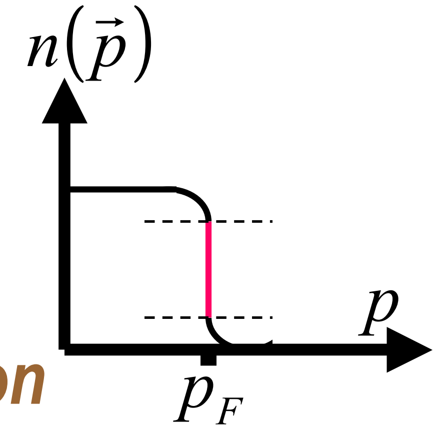
**… the sum of the momenta of the interaction electrons can change by an integer number of the periods of the reciprocal lattice. Therefore the momentum increase caused by the electric field can be destroyed by the interaction between the electrons, not only by the thermal oscillations of the lattice.**

# Signatures of the Fermi - Liquid state ?!

1. *Resistivity is proportional to* $T^2$ :

   **L.D. Landau & I.Ya. Pomeranchuk** ***"To the properties of metals at very low temperatures"*****; Zh.Exp.Teor.Fiz., 1936, v.10, p.649**

   *Umklapp electron – electron scattering dominates the charge transport (?!)*




2. *Jump in the momentum distribution function at* $T=0$.

2a. *Pole in the one-particle Green function*

$$G(\varepsilon,\vec{p}) = \frac{Z}{i\varepsilon_n - \xi(\vec{p})}$$

***Fermi liquid = 0<Z<1 (?!)***

# Landau Fermi - Liquid theory

| | |
|---|---|
| ***Momentum*** | $\vec{p}$ |
| ***Momentum distribution*** | $n(\vec{p})$ |
| ***Total energy*** | $E\{n(\vec{p})\}$ |
| ***Quasiparticle energy*** | $\mathrm{x}(\vec{p}) \equiv \mathrm{d}E/\mathrm{d}n(\vec{p})$ |
| ***Landau f-function*** | $f(\vec{p},\vec{p}\,') \equiv \mathrm{dx}(\vec{p})/\mathrm{d}n(\vec{p}\,')$ |

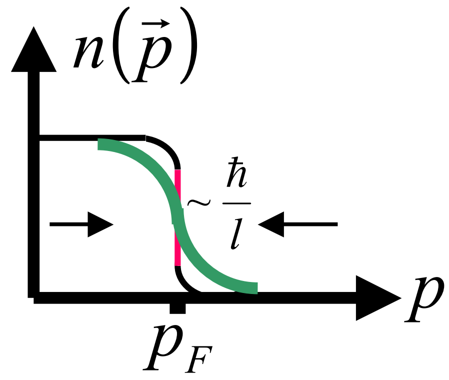
**Q:** ***Can Fermi – liquid survive without the momenta***

***Does it make sense to speak about the Fermi – liquid state in the presence of a quenched disorder***

**?**

## Does it make sense to speak about the Fermi – liquid state in the presence of a quenched disorder

1. Momentum is not a good quantum number – the momentum uncertainty is inverse proportional to the elastic mean free path, $l$. The step in the momentum distribution function is broadened by this uncertainty




2. Neither resistivity nor its temperature dependence is determined by the umklapp processes and thus does not behave as $T^2$

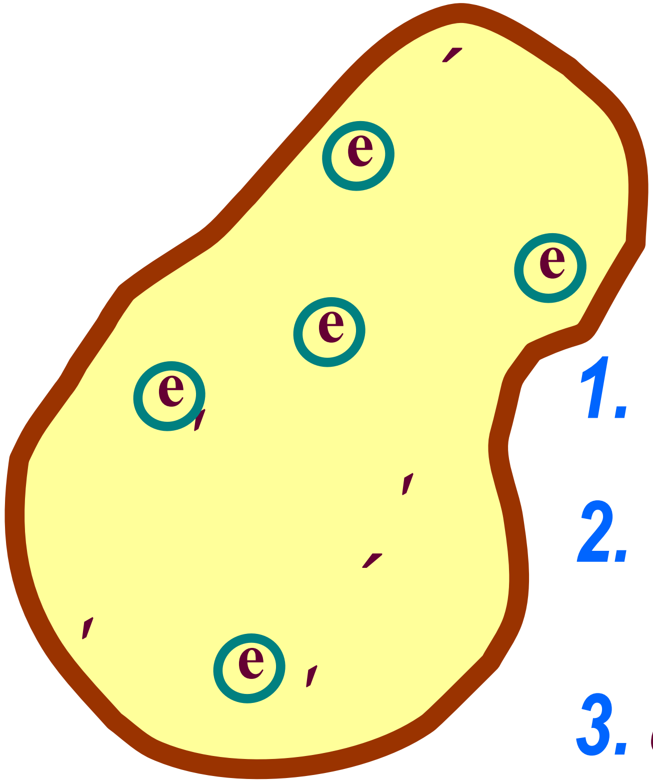
3. Sometimes (e.g., for random quenched magnetic field) the disorder averaged one-particle Green function even without interactions does not have a pole as a function of the energy, $\epsilon$. The residue , $Z$, makes no sense.

**Nevertheless even in the presence of the disorder**

I. Excitations are similar to the excitations in a disordered Fermi-gas.

II. Small decay rate

III. Substantial renormalizations




# Quantum Dot

1. *Disorder (´ impurities)*
2. *Complex geometry*
3. *$e$-$e$ interactions*

## Realizations:

- Metallic clusters
- Gate determined confinement in 2D gases (e.g. *GaAs/AlGaAs*)
- Carbon nanotubes
- 
- 

# Zero Dimensional Fermi Liquid

**Finite System** $\Longrightarrow$ **Thouless energy** $E_T$

$$e << E_T \xrightarrow{def} \mathbf{0D}$$

***At the same time, we want the typical energies, e , to exceed the mean level spacing, $d_1$ :***

$$d_1 << e << E_T$$

$$g \equiv \frac{E_T}{d_1} >> 1$$

# Plan:

- Try to describe the e-e interaction effects in Quantum Dots in the limit $g \to \infty$

- Calculate/estimate corrections when $1 << g < \infty$

**Interaction is not supposed to be weak !**

# Thouless Conductance and One-particle Quantum Mechanics

**0** — **1** — ***g***

***Localized states***
***Insulator***

**Poisson spectral statistics**

***Extended states***
***Metal***

**Wigner-Dyson spectral statistics**

N ´ N
***Random Matrices***

***Quantum Dots with dimensionless conductance g***

N® ¥

*g*® ¥

***The same statistics of the random spectra and one-particle wave functions (eigenvectors)***