Theory of Mesoscopic Systems

Boris Altshuler
Princeton University,
Columbia University &
NEC Laboratories America

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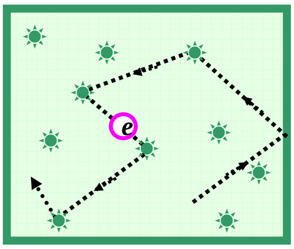
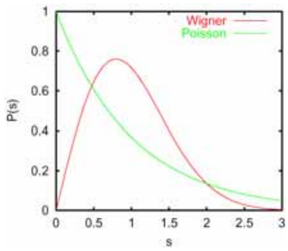
DE SUISSE OCCIDENTALE

Lecture 3 15 June 2006

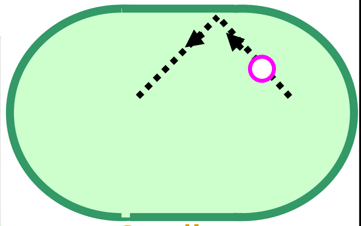
Previous Lecture

Quantum particle in random quenched potential





Quantum particle in a chaotic billiard



Stadium

Thouless Conductance and One-particle Spectral Statistics

0 — 1 — g

Localized states	*Extended states*
Insulator	*Metal*
Poisson spectral statistics	**Wigner-Dyson spectral statistics**

Q: *Does Anderson localization provide a generic scenario for the Wigner-Dyson to Poisson crossover* **?**

*Consider an **integrable** system. Each state is characterized by a **set of quantum numbers.***

*It can be viewed as a point in the **space of quantum numbers**. The whole set of the states forms a **lattice** in this space.*

*A **perturbation** that violates the integrability provides matrix elements of the **hopping** between different sites (**Anderson model !?**)*

Weak** enough **hopping** - Localization - **Poisson
Strong hopping** - transition to **Wigner-Dyson

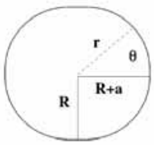
VOLUME 77, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1996

Diffusion and Localization in Chaotic Billiards

Fausto Borgonovi,[1,3,4] Giulio Casati,[2,3,5] and Baowen Li[6,7]
[1]Dipartimento di Matematica, Università Cattolica, via Trieste 17, 25121 Brescia, Italy
[2]Università di Milano, sede di Como, Via Lucini 3, Como, Italy
[3]Istituto Nazionale di Fisica della Materia, Unità di Milano, via Celoria 16, 22100, Milano, Italy
[4]Instituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy
[5]Instituto Nazionale di Fisica Nucleare, Sezione di Milano, Milano, Italy
[6]Department of Physics and Centre for Nonlinear and Complex Systems, Hong Kong Baptist University, Hong Kong
[*7]Center for Applied Mathematics and Theoretical Physics, University of Maribor, Krekova 2, 2000 Maribor, Slovenia
(Received 29 July 1996)

Localization and diffusion in the angular momentum space

$\varepsilon \equiv \frac{a}{R}$



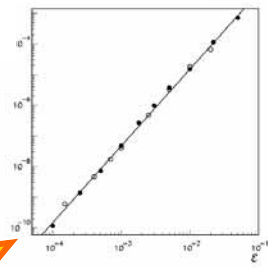
$\varepsilon > 0$ Chaotic stadium

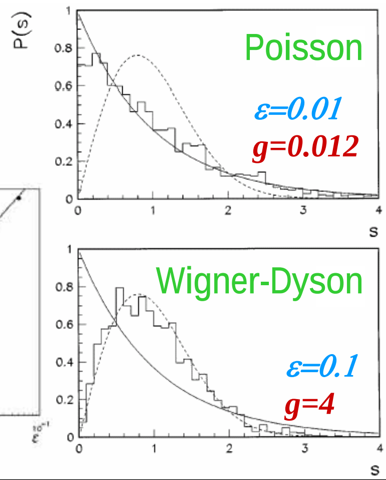
$\varepsilon \to 0$ Integrable circular billiard

Angular momentum is the integral of motion

$\hbar = 0; \quad \varepsilon << 1$

Diffusion in the angular momentum space $D \propto \varepsilon^{5/2}$





D.Poilblanc, T.Ziman, J.Bellisard, F.Mila & G.Montambaux
Europhysics Letters, v.22, p.537, 1993

1D Hubbard Model on a periodic chain

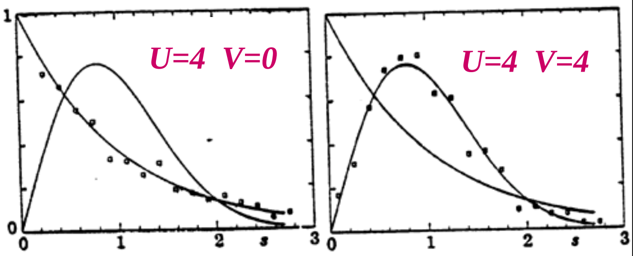
$$H = t\sum_{i,\sigma}\left(c^+_{i,\sigma}c_{i+1,\sigma} + c^+_{i+1,\sigma}c_{i,\sigma}\right) + U\sum_{i,\sigma} n_{i,\sigma}n_{i,-\sigma} + V\sum_{i,\sigma,\sigma'} n_{i,\sigma}n_{i+1,\sigma'}$$

Onsite interaction **n. neighbors interaction**

$V = 0$ Hubbard model integrable

$V \neq 0$ extended Hubbard model nonintegrable

12 sites
3 particles
Zero total spin
Total momentum $\pi/6$



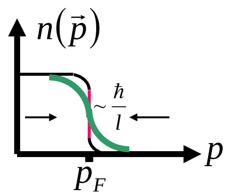
Wigner-Dyson random matrix statistics follows from the delocalization.

Q: ***Why the random matrix theory (RMT) works so well for nuclear spectra*** **?**

Spectra of Many-Body excitations !

Does it make sense to speak about the Fermi – liquid state in the presence of a quenched disorder

1. *Momentum is not a good quantum number – the momentum uncertainty is inverse proportional to the elastic mean free path, **l**. The step in the momentum distribution function is broadened by this uncertainty*



2. *Neither resistivity nor its temperature dependence is determined by the umklapp processes and thus does not behave as* T^2
3. *Sometimes (e.g., for random quenched magnetic field) the disorder averaged one-particle Green function even without interactions does not have a pole as a function of the energy,* ε*. The residue ,* Z*, makes no sense.*

Nevertheless even in the presence of the disorder

I. Excitations are similar to the excitations in a disordered Fermi-gas.
II. Small decay rate
III. Substantial renormalizations

Beforehand

Anderson Localization,
Mesoscopic Fluctuations,
Random Matrices, and
Quantum Chaos
Fermi liquid

Today

Continue.
Universal Hamiltonian, 0D
Inelastic relaxation rates.

Zero-dimensional Fermi Liquid

Zero Dimensional Fermi Liquid

Finite System → Thouless energy E_T

$$\varepsilon << E_T \xrightarrow{def} 0D$$

At the same time, we want the typical energies, ε, to exceed the mean level spacing, δ_1:

$$\delta_1 << \varepsilon << E_T$$

$$g \equiv \frac{E_T}{\delta_1} >> 1$$

Thouless Conductance and One-particle Quantum Mechanics

0 — 1 — g

Localized states	Extended states
Insulator	*Metal*
Poisson spectral statistics	**Wigner-Dyson spectral statistics**

$N \times N$ Random Matrices ($N \to \infty$)

Quantum Dots with dimensionless conductance g ($g \to \infty$)

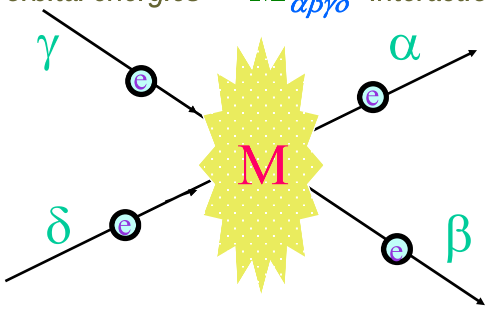
The same statistics of the random spectra and one-particle wave functions (eigenvectors)

Two-Body Interactions

$|\alpha,\sigma>$ Set of one particle states. σ and α label correspondingly spin and orbit.

$$\hat{H}_0 = \sum_{\alpha} \varepsilon_\alpha a^+_{\alpha,\sigma} a_{\alpha,\sigma} \qquad \hat{H}_{\text{int}} = \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \sigma,\sigma'}} M_{\alpha\beta\gamma\delta} a^+_{\alpha,\sigma} a^+_{\beta,\sigma'} a_{\gamma,\sigma} a_{\delta,\sigma'}$$

ε_α -one-particle orbital energies $\quad$ $M_{\alpha\beta\gamma\delta}$ -interaction matrix elements



Matrix Elements

$$\hat{H}_{\text{int}} = \sum_{\substack{\alpha,\beta,\gamma,\delta \\ \sigma,\sigma'}} M_{\alpha\beta\gamma\delta} a^+_{\alpha,\sigma} a^+_{\beta,\sigma'} a_{\gamma,\sigma} a_{\delta,\sigma'}$$

$$M_{\alpha\beta\gamma\delta}$$

Diagonal - $\alpha,\beta,\gamma,\delta$ are equal pairwise

$\alpha=\gamma$ and $\beta=\delta$ or $\alpha=\delta$ and $\beta=\gamma$ or $\alpha=\beta$ and $\gamma=\delta$

Offdiagonal - otherwise

It turns out that in the limit $g \to \infty$

- Diagonal matrix elements are much bigger than the offdiagonal ones

$$M_{\text{diagonal}} >> M_{\text{offdiagonal}}$$

- Diagonal matrix elements in a particular sample do not fluctuate - selfaveraging

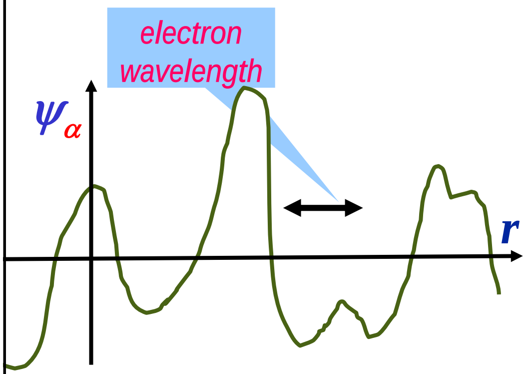
Toy model:

Short range ***e-e*** *interactions*

$$U(\vec{r}) = \frac{\lambda}{\nu}\delta(\vec{r})$$

$$M_{\alpha\beta\gamma\delta} = \frac{\lambda}{\nu}\int d\vec{r}\, \psi^*_\alpha(\vec{r})\psi^*_\beta(\vec{r})\psi_\gamma(\vec{r})\psi_\delta(\vec{r})$$

$\psi_\alpha(\vec{r})$ *one-particle eigenfunctions*



$\psi_\alpha(\vec{r})$ *is a random function that rapidly oscillates*

$$|\psi_\alpha(\vec{r})|^2 \geq 0$$

$$\psi_\alpha(\vec{r})^2 \geq 0$$

as long as ***T****-invariance is preserved*

In the limit

$$g \to \infty$$

- *Diagonal matrix elements are much bigger than the offdiagonal ones*

$$M_{\text{diagonal}} >> M_{\text{offdiagonal}}$$

- *Diagonal matrix elements in a particular sample do not fluctuate - selfaveraging*

$$M_{\alpha\beta\alpha\beta} = \frac{\lambda}{\nu}\int d\vec{r}\,|\psi_\alpha(\vec{r})|^2 |\psi_\beta(\vec{r})|^2$$

$$|\psi_\alpha(\vec{r})|^2 \Rightarrow \frac{1}{\text{volume}}$$

$$\Rightarrow \quad M_{\alpha\beta\alpha\beta} = \lambda\delta_1$$

More general: *finite range interaction potential* $U(\vec{r})$

$$M_{\alpha\beta\alpha\beta} = \frac{\lambda}{\nu}\int |\psi_\alpha(\vec{r}_1)|^2 |\psi_\beta(\vec{r}_2)|^2 U(\vec{r}_1 - \vec{r}_2)\, d\vec{r}_1 d\vec{r}_2$$

The same conclusion

Random Matrices:

E_α - spectrum

$\psi_\alpha(i)$ – ***i-th*** component of ***α-th*** eigenvector

$$\left\langle \psi_\alpha^*(i)\psi_\gamma(j)\right\rangle = \frac{1}{N}\delta_{\alpha\gamma}\delta_{ij}$$

$$\left\langle \psi_\alpha(i)\psi_\gamma(j)\right\rangle = \frac{2-\beta}{N}\delta_{\alpha\gamma}\delta_{ij}$$

in the limit $N \to \infty$

Components of the different eigenvectors as well as different components of the same eigenvector are not correlated

Berry Conjecture:

Exact wavefunctions at energy $\approx \varepsilon_F$ in chaotic systems behave as sums of plane waves with $\left|\vec{k}\right| \approx k_F$ and random coefficients:

$$\left\langle \psi_\alpha^*(\vec{r}_1)\psi_\gamma(\vec{r}_2)\right\rangle = \frac{\delta_{\alpha\gamma}}{V} f\left(\frac{2\pi\left|\vec{r}_1-\vec{r}_2\right|}{\lambda}\right)$$

$$\left\langle \psi_\alpha(\vec{r}_1)\psi_\gamma(\vec{r}_2)\right\rangle = (2-\beta)\frac{\delta_{\alpha\gamma}}{V} f\left(\frac{2\pi\left|\vec{r}_1-\vec{r}_2\right|}{\lambda}\right)$$

$$f(x) = \Gamma\left(\frac{d}{2}\right)x^{1-d/2}J_{d/2-1}(x)$$

d is # of dimensions,
$J_\mu(x)$ is Bessel function

Important:

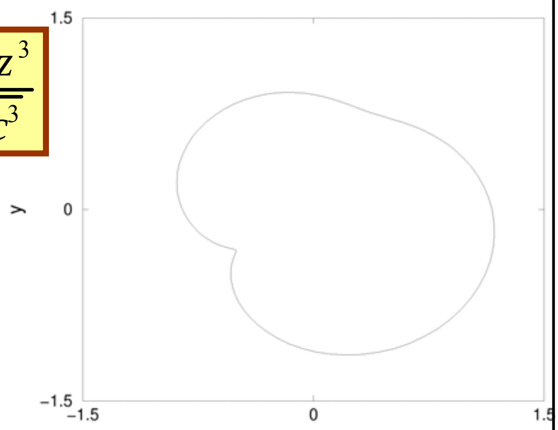
!

when x increases $f(x)$ decays quickly enough for the integral $\int_0^\infty f(x)x^{d-1}dx$ *to converge*

Only local correlations

AFRICA BILLIARD - *a conformal image of a unit circle*

$$\omega(z) = R\frac{z + bz^2 + ce^{i\delta}z^3}{\sqrt{1+2b^2+3c^3}}$$

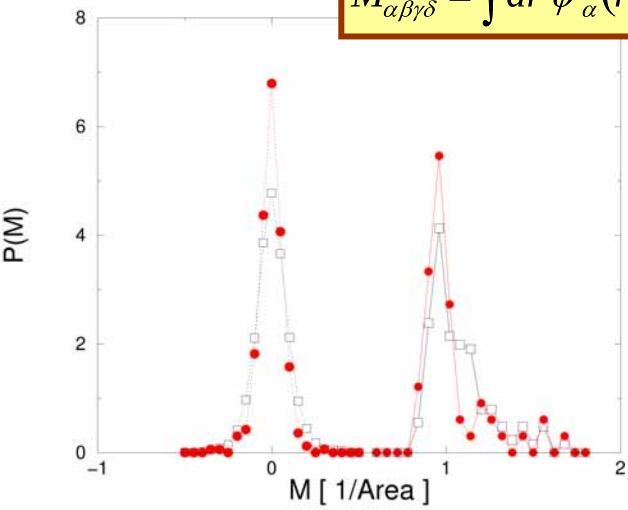


$$b = c = 0.2;$$

$$\delta = 1.5; R = 1$$

Distribution of the matrix elements

$$M_{\alpha\beta\gamma\delta} = \int d\vec{r}\, \psi_\alpha(\vec{r})\psi_\beta(\vec{r})\psi^*_\alpha(\vec{r})\psi^*_\beta(\vec{r})$$



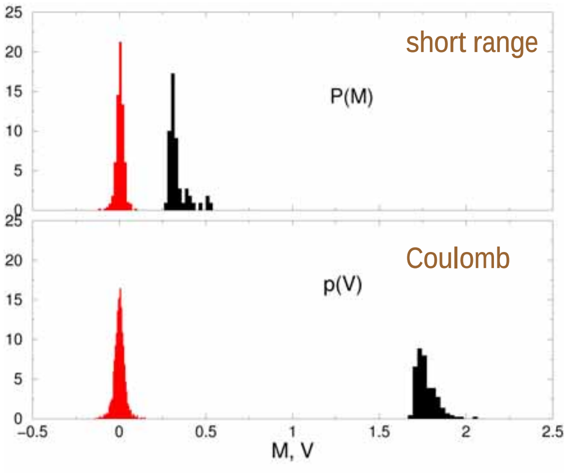
Open boxes -
levels from 20 to 30

Closed circuits –
levels from 30 to 40

$$M_{\alpha\beta\gamma\delta} = \frac{1}{\pi}\int d\vec{r}\,\psi_\alpha(\vec{r})\psi_\beta(\vec{r})\psi^*_\alpha(\vec{r})\psi^*_\beta(\vec{r})$$

$$V_{\alpha\beta\gamma\delta} \propto \int \frac{d\vec{r}_1 d\vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}\psi_\alpha(\vec{r}_1)\psi_\beta(\vec{r}_2)\psi^*_\alpha(\vec{r}_1)\psi^*_\beta(\vec{r}_2)$$

Distribution function of diagonal and offdiagonal matrix elements



Universal (Random Matrix) limit - Random Matrix symmetry of the correlation functions:

All correlation functions are invariant under arbitrary orthogonal transformation:

$$\tilde{\psi}_\mu(\vec{r}) = \sum_\nu \int d\vec{r}_1 O^\nu_\mu(\vec{r},\vec{r}_1)\psi_\nu(\vec{r}_1)$$

$$\int d\vec{r}_1 O^\nu_\mu(\vec{r},\vec{r}_1) O^\eta_\nu(\vec{r}_1,\vec{r}\,') = \delta_{\mu\eta}\delta(\vec{r}-\vec{r}\,')$$

There are only three operators, which are quadratic in the fermion operators a^+, a , and invariant under RM transformations:

$$\hat{n} = \sum_{\alpha,\sigma} a^+_{\alpha,\sigma} a_{\alpha,\sigma}$$

total number of particles

$$\hat{\vec{S}} = \sum_{\alpha,\sigma_1,\sigma_2} a^+_{\alpha,\sigma_1} \vec{\sigma}_{\sigma_1,\sigma_2} a_{\alpha,\sigma_2}$$

total spin

$$\hat{T}^+ = \sum_{\alpha} a^+_{\alpha,\uparrow} a^+_{\alpha,\downarrow}$$

????

Charge conservation (gauge invariance) -no $\hat{T}$ or $\hat{T}^+$ only $\hat{T}\hat{T}^+$

Invariance under rotations in spin space - no $\hat{\vec{S}}$ only $\hat{\vec{S}}^2$

Therefore, in a very general case

$$\hat{H}_{\text{int}} = eV\hat{n} + E_c\hat{n}^2 + J\hat{\vec{S}}^2 + \lambda_{BCS}\hat{T}^+\hat{T}.$$

Only three coupling constants describe all of the effects of e-e interactions

In a very general case only three coupling constants describe all effects of electron-electron interactions:

$$\hat{H} = \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha} + \hat{H}_{\text{int}}$$

$$\hat{H}_{\text{int}} = eV\hat{n} + E_c \hat{n}^2 + J\hat{S}^2 + \lambda_{BCS} \hat{T}^{+} \hat{T}.$$

I.L. Kurland, I.L.Aleiner & B.A., 2000

See also

P.W.Brouwer, Y.Oreg & B.I.Halperin, 1999

H.Baranger & L.I.Glazman, 1999

H-Y Kee, I.L.Aleiner & B.A., 1998

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For a short range interaction with a coupling constant λ

$$E_c = \frac{\lambda \delta_1}{2} \qquad J = -2\lambda \delta_1 \qquad \lambda_{BCS} = \lambda \delta_1 (2 - \beta)$$

where δ_1 is the one-particle mean level spacing

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

$$\hat{H}_0 = \sum_\alpha \varepsilon_\alpha n_\alpha$$

$$\hat{H}_{\text{int}} = eV\hat{n} + E_c\hat{n}^2 + J\hat{S}^2 + \lambda_{BCS}\hat{T}^+\hat{T}.$$

E_c determines the charging energy (Coulomb blockade)

J describes the spin exchange interaction

λ_{BCS} determines effect of superconducting-like pairing

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

$$\hat{H}_0 = \sum_\alpha \varepsilon_\alpha n_\alpha$$

$$\hat{H}_{\text{int}} = eV\hat{n} + E_c\hat{n}^2 + J\hat{S}^2 + \lambda_{BCS}\hat{T}^+\hat{T}.$$

I. Excitations are similar to the excitations in a disordered Fermi-gas.

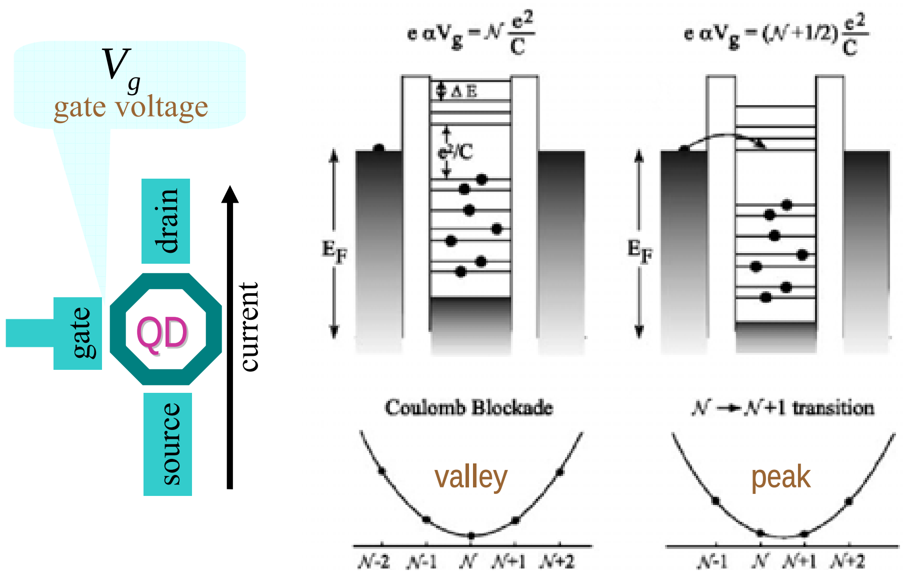
II. Small decay rate

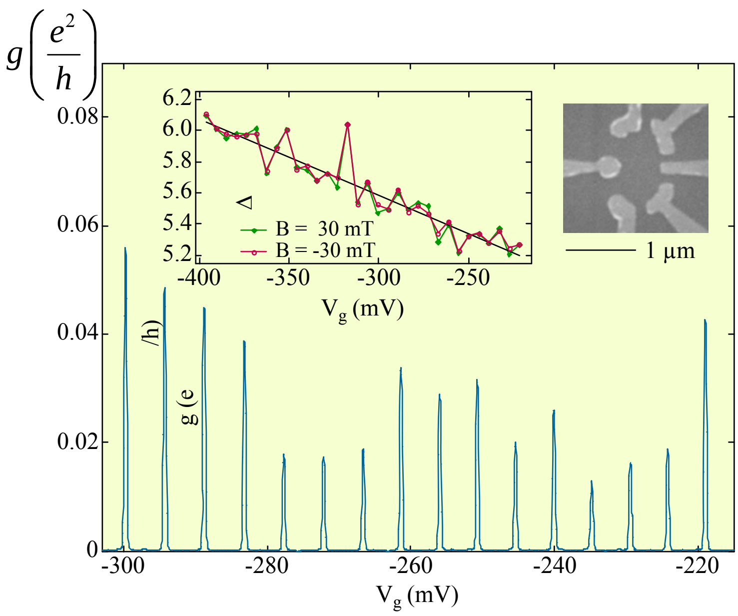
III. Substantial renormalizations

Isn't it a Fermi liquid ?

Fermi liquid behavior follows from the fact that different wave functions are almost uncorrelated

Example 1: Coulomb Blockade





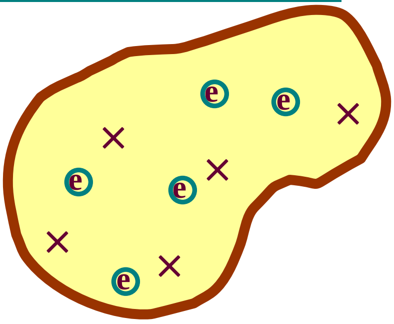
Coulomb Blockade Peak Spacing
Patel, et al. PRL 80 4522 (1998)
(Marcus Lab)

Example 2: Spontaneous Magnetization

1. Disorder (×impurities)
2. Complex geometry

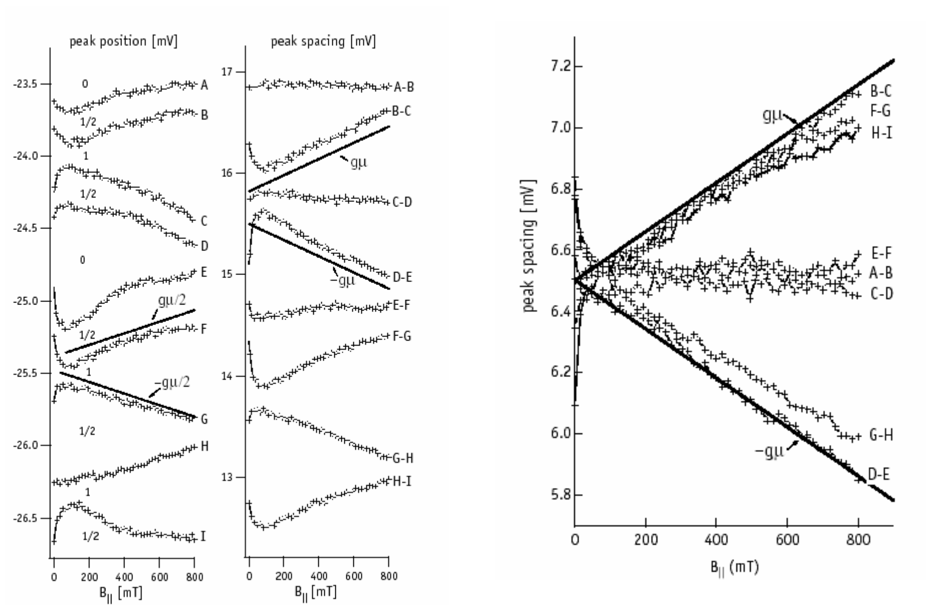
} *chaotic one-particle motion*

3. **e-e** interactions



Q: *What is the spin of the Quantum Dot in the ground state*?

How to measure the Magnetization – motion of the Coulomb blockade peaks in the parallel magnetic field



In the presence of magnetic field

$$\hat{H}_{\mathbf{int}} = \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha} + J\hat{\vec{S}}^{2} + \vec{B} \bullet \hat{\vec{S}}$$

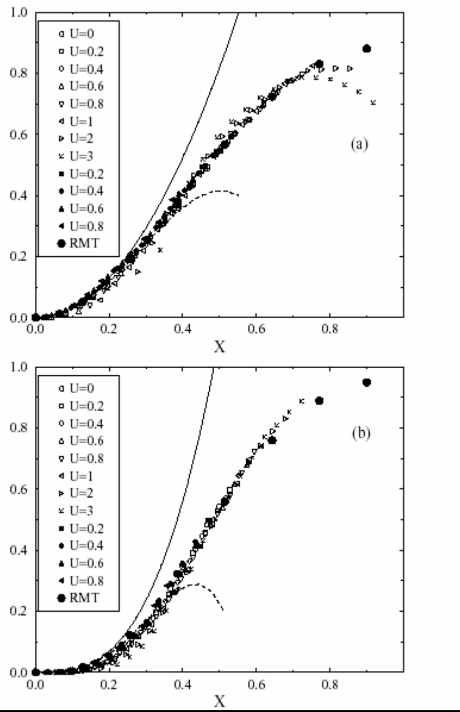
Scaling: the probability to find a ground state at a given magnetic field, ***B***, with a given spin, ***S***, depends on the combination rather than on ***B*** and ***J*** separately

$$X = J + g\mu_B \frac{B}{2S}$$

Probability to observe a triplet state as a function of the parameter X

● - results of the calculation based on the universal Hamiltonian with the RM one-particle states

The rest – exact diagonalization for Hubbard clusters with disorder. No adjustable parameters



$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

$$\hat{H}_0 = \sum_{\alpha} \varepsilon_\alpha n_\alpha$$

$$\hat{H}_{\text{int}} = eV\hat{n} + E_c\hat{n}^2 + J\hat{S}^2 + \lambda_{BCS}\hat{T}^{+}\hat{T}.$$

I. Excitations are similar to the excitations in a disordered Fermi-gas.

II. Small decay rate

III. Substantial renormalizations

Inelastic processes

Decoherence

Small decay rate

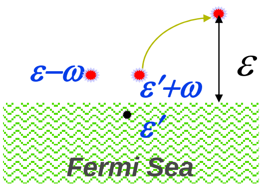
Q:

- Why is it small
- What is it equal to
- What is the connection between the decay rate of the quasiparticles and the dephasing rate

?

Quasiparticle decay rate at $T = 0$ in a *clean* Fermi Liquid.

I. d=3



$$\frac{\hbar}{\tau_{e-e}(\varepsilon)} \propto \left(\begin{matrix}\text{coupling}\\ \text{constant}\end{matrix}\right)^2 \frac{\varepsilon^2}{\varepsilon_F} \qquad d = 3$$

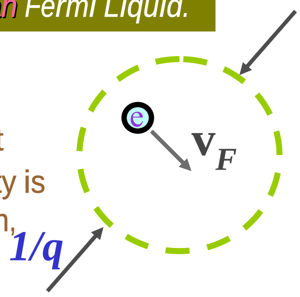
Reasons:

- At small ε the energy transfer, ω, is small and the integration over ε' and ω gives the factor ε^2.

- The momentum transfer, q , is large and thus the scattering probability at given ε' and ω does not depend on ε', ω or ε

Quasiparticle decay rate at $T = 0$ in a clean Fermi Liquid.

II. Low dimensions



Small momenta transfer, q , become important at low dimensions because the scattering probability is proportional to the squared time of the interaction, $(q v_F)^{-2}$

$$\frac{\hbar}{\tau_{e-e}(\varepsilon)} \propto \begin{cases} \varepsilon^2/\varepsilon_F & d = 3 \\ \left(\varepsilon^2/\varepsilon_F\right)\log\left(\varepsilon_F/\varepsilon\right) & d = 2 \\ \varepsilon & d = 1 \end{cases}$$

Quasiparticle decay rate at $T = 0$ in a clean Fermi Liquid.

III. Applicability

$$\frac{\hbar}{\tau_{e-e}(\varepsilon)} \propto \begin{cases} \varepsilon^2/\varepsilon_F & d = 3 \\ \left(\varepsilon^2/\varepsilon_F\right)\log\left(\varepsilon_F/\varepsilon\right) & d = 2 \\ \varepsilon & d = 1 \end{cases}$$

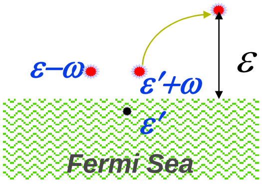
Conclusions:

1. For $d=3,2$ from $\varepsilon \ll \varepsilon_F$ it follows that $\varepsilon\tau_{e-e} \ll h$, i.e., that the qusiparticles are well determined and the Fermi-liquid approach is applicable.

2. For $d=1$ $\varepsilon\tau_{e-e}$ is of the order of h, i.e., that the Fermi-liquid approach is not valid for $1d$ systems of interacting fermions.

Luttinger liquids

Quasiparticle decay rate at $T = 0$ in a 0D Fermi Liquid.



Electronic spectrum is discrete

Need offdiagonal matrix elements

Quasiparticle decay is beyond the "universal Hamiltonian"

Quasiparticle decay rate is small as g^{-1}

$$\tau_{ee}(\varepsilon) \geq g \frac{\hbar}{\varepsilon}$$

CONCLUSIONS

One-particle chaos + moderate interaction of the electrons $\mapsto$ to a rather simple Hamiltonian of the system, which can be called Zero-dimensional Fermi liquid.

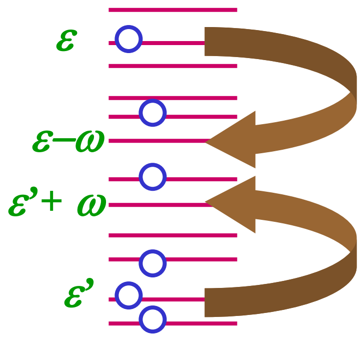
The main parameter that justifies this description is the Thouless conductance, which is supposed to be large

Excitations are characterized by their one-particle energy, charge and spin, but not by their momentum.

These excitations have the lifetime, which is proportional to the Thouless conductance, i.e., is long.

This approach allows to describe Coulomb blockade (renormalization of the compressibility), as well as the substantial renormalization of the magnetic susceptibility and effects of superconducting pairing

Quasiparticle relaxation rate in 0D case ***T=0***



Offdiagonal matrix element

$$M(\omega,\varepsilon,\varepsilon') \propto \frac{\delta_1}{g} << \delta_1$$

Quasiparticle relaxation rate in 0D case ***T=0***

Fermi Golden Rule

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon}\ \sum_{-\omega<\varepsilon'<0} \frac{\left|M(\omega,\varepsilon,\varepsilon')\right|^2}{\delta_1}$$

0D **case:** $L < L_\varepsilon$, i.e., $\varepsilon < E_T$

$$L_\varepsilon = \sqrt{\frac{hD}{\varepsilon}}$$

- $M \propto \delta_1(L)/g$
- **Each** $\sum$ **gives** $\approx \frac{\varepsilon}{\delta_1}$

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \delta_1 \left(\frac{\varepsilon}{E_T}\right)^2$$

(**U.Sivan, Y.Imry & A.Aronov,1994**)

Quasiparticle relaxation rate in disordered conductors ***T=0***

Fermi Golden Rule

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon} \sum_{-\omega<\varepsilon'<0} \frac{|M(\omega,\varepsilon,\varepsilon')|^2}{\delta_1}$$

0D **case:** $L < L_\varepsilon$, i.e., $\varepsilon < E_T$

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon)} \propto \delta_1 \left(\frac{\varepsilon}{E_T}\right)^2$$

$$L_\varepsilon = \sqrt{\frac{hD}{\varepsilon}}$$

d>0 **case:** $L > L_\varepsilon$, *i.e.,* $\varepsilon > E_T$

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon)} \propto \delta_1 (L_\varepsilon)$$

At $L \approx L_\varepsilon$ the rate is of the order of the mean level spacing δ_1. This relation should not change, when we keep increasing the system size, i.e. decreasing the Thouless energy E_T.

Quasiparticle relaxation rate in disordered conductors ***T=0***

Fermi Golden Rule

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon} \sum_{-\omega<\varepsilon'<0} \frac{|M(\omega,\varepsilon,\varepsilon')|^2}{\delta_1}$$

0D **case:** $L < L_\varepsilon$, i.e., $\varepsilon < E_T$

$$L_\varepsilon = \sqrt{\frac{hD}{\varepsilon}}$$

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon)} \propto \delta_1 \left(\frac{\varepsilon}{E_T}\right)^2$$

d>0 **case:** $L > L_\varepsilon$, *i.e.,* $\varepsilon > E_T$

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon)} \propto \delta_1 (L_\varepsilon)$$

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon)} \propto \frac{\varepsilon}{g(L_\varepsilon)} << \varepsilon$$

A. Schmid 1973
B.A. & A.Aronov 1979

Matrix elements at large, $\varepsilon,\omega \gg E_T$, energies

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon} \sum_{-\omega<\varepsilon'<0} \frac{|M(\omega,\varepsilon,\varepsilon')|^2}{\delta_1}$$

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \frac{\varepsilon}{g(L_\varepsilon)} \ll \varepsilon$$

$$|M(\omega,\varepsilon,\varepsilon')|^2 \propto \frac{\delta_1(L)^3\,\delta_1(L_\omega)}{\omega^2} \propto \frac{\omega^{-2+d/2}}{D^{d/2}}$$

~~$$\frac{|M(\omega,\varepsilon,\varepsilon')|^2}{\delta_1(L)} \xrightarrow[\omega\to 0]{} \infty$$~~

Quasiparticle relaxation rate in disordered conductors

$T>0$

$T=0$

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon} \sum_{-\omega<\varepsilon'<0} \frac{|M(\omega,\varepsilon,\varepsilon')|^2}{\delta_1}$$

$T>0$

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon,T)} \propto \sum_{\omega}(1-n_{\varepsilon-\omega}) \sum_{\varepsilon'} n_{\varepsilon'+\omega}(1-n_{\varepsilon'+\omega}) \frac{|M(\omega,\varepsilon,\varepsilon')|^2}{\delta_1}$$

$$n_\varepsilon = \left[\exp\frac{\varepsilon}{T}-1\right]^{-1}$$

Fermi distribution function

$$|M(\omega,\varepsilon,\varepsilon')|^2 \propto \frac{\omega^{-2+d/2}}{D^{d/2}}$$

Quasiparticle relaxation rate in disordered conductors **T>0**

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon} \sum_{-\omega<\varepsilon'<0} \frac{|M(\omega,\varepsilon,\varepsilon')|^2}{\delta_1}$$ **T=0**

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon,T)} \propto \sum_{\omega}(1-n_{\varepsilon-\omega})\sum_{\varepsilon'} n_{\varepsilon'+\omega}(1-n_{\varepsilon'+\omega})\frac{|M(\omega,\varepsilon,\varepsilon')|^2}{\delta_1}$$ **T>0**

$$n_\varepsilon = \left[\exp\frac{\varepsilon}{T} - 1\right]^{-1}$$ **Fermi distribution function**

$$|M(\omega,\varepsilon,\varepsilon')|^2 \propto \frac{\omega^{-2+d/2}}{D^{d/2}}$$

a) $T=0$ *-no problems:* $\sum_{\varepsilon'} \propto \omega$ *and* $\sum_{\omega}$ *converges*

b) $T>0$ *-a problem:* $\sum_{\varepsilon'} \propto T$ *and* $\sum_{\omega}$ *diverges !*

Abrahams, Anderson, Lee & Ramakrishnan 1981

T>0 *-a problem:* $1/\tau_{e\text{-}e}$ *diverges*

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon,T)} \propto T\sum_{\omega}\frac{(1-n_{\varepsilon-\omega})}{\omega^{2-d/2}D^{d/2}}$$

B.A., A.Aronov & D.E. Khmelnitskii (1983):

- Divergence of is not a catastrophe: $1/\tau_{e\text{-}e}$ has no physical meaning
- E.g., for energy relaxation of hot electrons processes with small energy transfer ω are irrelevant.

$$\frac{h}{\tau_\varepsilon} \propto \frac{\varepsilon}{g(L_\varepsilon)}$$

Q: Is it the energy relaxation rate that determines the applicability of the Fermi liquid approach **?**

$T>0$ - a problem: $1/\tau_{e\text{-}e}$ diverges

$$\frac{h}{\tau_{e\text{-}e}(\varepsilon,T)} \propto T\sum_{\omega}\frac{(1-n_{\varepsilon-\omega})}{\omega^{2-d/2}D^{d/2}}$$

B.A., A.Aronov & D.E. Khmelnitskii (1983):

- Divergence of is not a catastrophe: $1/\tau_{e\text{-}e}$ has no physical meaning
- E.g., for energy relaxation of hot electrons processes with small energy transfer ω are irrelevant. ⟹ $\frac{h}{\tau_\varepsilon} \propto \frac{\varepsilon}{g(L_\varepsilon)}$
- Phase relaxation: in a time t after a collision $\delta\varphi \approx (2\pi\,\omega\, t)/h \Rightarrow$ processes with energy transfer ω smaller than $1/\tau_\varphi$ are irrelevant.

$$\frac{h}{\tau_\varphi} \propto ?$$

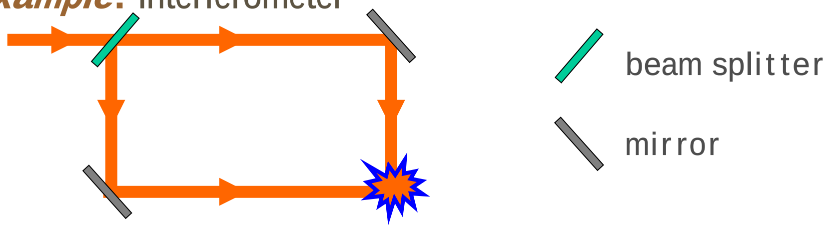
What is Dephasing?

1. Suppose that originally a system(an electron) was in a *pure* quantum state. It means that it could be described by a wave function with a given phase.

2. External perturbations can transfer the system to a different quantum state. Such a transition is characterized by its amplitude, which has a modulus and a phase.

3. The phase of the amplitude can be measured by comparing it with the phase of another amplitude of the same transition.

Example: interferometer



4. Usually we can not control all of the perturbations. As a result, even for fixed initial and final states, the phase of the transition amplitude has a random component.

5. We call this contribution to the phase, $\delta\varphi$, random if it changes from measurement to measurement in an uncontrollable way.

6. It usually also depends on the duration of the experiment, t:

$$\delta\varphi = \delta\varphi(t)$$

7. When the time t is large enough, $\delta\varphi$ exceeds 2π, and interference gets averaged out.

8. Definitions:

$$\delta\varphi(\tau_\varphi) \approx 2\pi$$

τ_φ phase coherence time; $1/\tau_\varphi$ dephasing rate

Why is Dephasing rate important?

Imagine that we need to measure the energy of a quantum system, which interacts with an environment and can exchange energy with it.

Let the typical energy transferred between our system an the environment in time t be $\delta\varepsilon(t)$. The total uncertainty of an ideal measurement is

$$\Delta\varepsilon(t) \approx \delta\varepsilon(t) + \frac{\hbar}{t}$$

environment — quantum uncertainty

$$\left.\begin{array}{l} \delta\varepsilon(t) \xrightarrow[t\to\infty]{} \infty; \\ \frac{\hbar}{t} \xrightarrow[t\to 0]{} \infty \end{array}\right\}$$

There should be an optimal measurement time $t=t^*$, which minimizes $\Delta\varepsilon(t)$:

$$\Delta\varepsilon(t^*) = \Delta\varepsilon_{min}$$

!

$$\delta\varepsilon(t^*) \approx \frac{\hbar}{t^*} \Rightarrow \delta\varphi(t^*) \approx 1 \Rightarrow \begin{array}{l} t^* \approx \tau_\varphi \\ \Delta\varepsilon_{\min} \approx \hbar/\tau_\varphi \end{array}$$

!

Why is Dephasing rate important?

$$\delta\varepsilon(t^*) \approx \frac{\hbar}{t^*} \Rightarrow \delta\phi(t^*) \approx 1 \Rightarrow \begin{matrix} t^* \approx \tau_\varphi \\ \Delta\varepsilon_{\min} \approx \hbar/\tau_\varphi \end{matrix}$$

It is dephasing rate that determines the accuracy at which the energy of the quantum state can be measured in principle.

***T*>0 -a problem: $1/\tau_{e\text{-}e}$ diverges**

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon,T)} \propto T\sum_{\omega}\frac{(1-n_{\varepsilon-\omega})}{\omega^{2-d/2}D^{d/2}}$$

B.A., A.Aronov & D.E. Khmelnitskii (1983):

- Divergence of is not a catastrophe: $1/\tau_{e\text{-}e}$ has no physical meaning
- E.g., for energy relaxation of hot electrons processes with small energy transfer ω are irrelevant.

$$\frac{h}{\tau_\varepsilon} \propto \frac{\varepsilon}{g(L_\varepsilon)}$$

- Phase relaxation: in a time t after a collision $\delta\varphi \approx (2\pi\,\omega\, t)/h \Rightarrow$ processes with energy transfer ω smaller than $1/\tau_\varphi$ are irrelevant.

$$\frac{h}{\tau_\varphi(\varepsilon,T)} \propto T\sum_{\omega>h/\tau_\varphi}\frac{(1-n_{\varepsilon-\omega})}{\omega^{2-d/2}D^{d/2}}$$

!

e-e interaction – Electric noise

Fluctuation- dissipation theorem:

Electric noise - randomly time and space - dependent electric field $E^{\alpha}(\vec{r},t) \Leftrightarrow E^{\alpha}(\vec{k},\omega)$
Correlation function of this field is completely determined by the conductivity $\sigma(\vec{k},\omega)$:

$$\left\langle E^{\alpha}E^{\beta}\right\rangle_{\omega,\vec{k}} = \frac{\omega}{\sigma_{\alpha\beta}(\omega,\vec{k})}\coth\left(\frac{\omega}{2T}\right)\frac{k_{\alpha}k_{\beta}}{k^2} \propto \frac{T}{\sigma_{\alpha\beta}(\omega,\vec{k})}$$

Noise intensity increases with the temperature, T, and with resistance

g

$$\left\langle E^{\alpha}E^{\beta}\right\rangle_{\omega,\vec{k}} = \frac{\omega}{\sigma_{\alpha\beta}(\omega,\vec{k})}\coth\left(\frac{\omega}{2T}\right)\frac{k_{\alpha}k_{\beta}}{k^2} \propto \frac{T}{\sigma_{\alpha\beta}(\omega,\vec{k})}$$

$g(L) \equiv \frac{h}{e^2 R(L)}$ *- Thouless conductance – def.*

$R(L)$ *- resistance of the sample with* $\left\{\begin{array}{l}\text{length (1d)}\\ \text{area (2d)}\end{array}\right\}$ L

$$\frac{1}{\tau_{\varphi}} \propto \frac{T}{g(L_{\varphi})}$$

$L_{\varphi} \equiv \sqrt{D\tau_{\varphi}}$ *- dephasing length*

D *- diffusion constant of the electrons*

$$\frac{1}{\tau_\varphi} \propto \frac{T}{g(L_\varphi)}$$

This is an equation!

$$\boldsymbol{g(L) \propto L^{d-2}}$$

$$L_\varphi \propto \sqrt{\tau_\varphi}$$

where is the number of dimensions:
d=1 *for wires;* ***d=2*** *for films, ...*

$$L_\varphi \propto T^{-1/(4-d)}$$

$$\tau_\varphi \propto T^{-2/(4-d)} \propto \begin{matrix} T^{-1} & d=2 \\ T^{-2/3} & d=1 \end{matrix}$$

$$\frac{1}{\tau_\varphi} \propto \frac{T}{g(L_\varphi)} \qquad L_\varphi \equiv \sqrt{D\tau_\varphi}$$

Fermi liquid is valid (one particle excitations are well defined), provided that

$$T\tau_\varphi(T) > \hbar$$

1. *In a purely* ***1d*** *chain,* $g \leq 1$ *,and, therefore, Fermi liquid theory is never valid.*
2. *In a multichannel wire* $g(L_\varphi) > 1$ *,provided that* L_φ *is smaller than the localization length, and Fermi liquid approach is justified*

How to detect phase coherence / measure the dephasing/decoherence rate ?

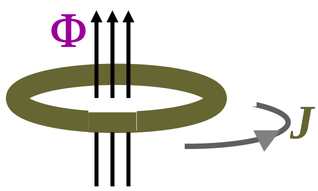
Quantum phenomena in electronic systems:

- Weak localization — yes
- Mesoscopic fluctuations — yes
- Persistent current — no
-
-

Persistent current at zero temperature is a property of the ground state!

$$J \propto \frac{\partial E}{\partial \Phi}$$

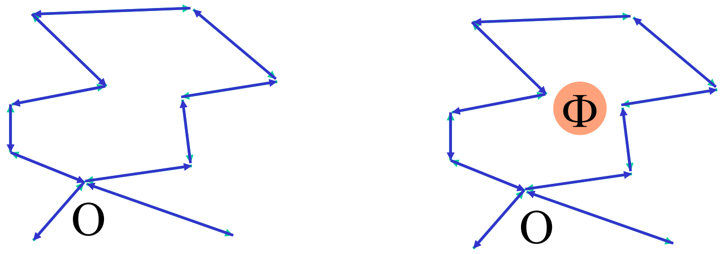
E is the ground state energy



Interaction between electrons can change both E and J, but this does not mean that there is dephasing of the ground state wave function.

Measurements of the persistent current as well as of other thermodynamic properties do not allow to extract the dephasing rate. Only transport measurements

Magnetoresistance



No magnetic field

$$\varphi_1 = \varphi_2$$

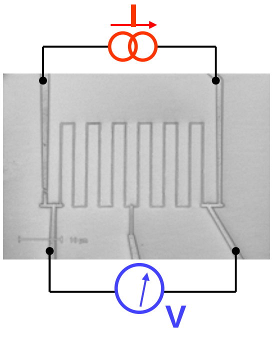
With magnetic field H

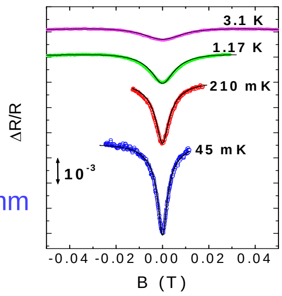
$$\varphi_1 - \varphi_2 = 2 * 2\pi\, \Phi/\Phi_0$$

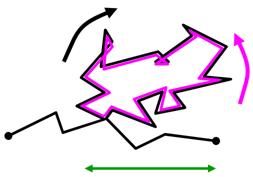
$\Phi = HS$ - *magnetic flux through the loop*

$\Phi_0 = hc/e$ - *flux quantum*

Weak Localization, Magnetoresistance in Metallic Wires







$$L_\phi = \sqrt{D\tau_\phi}$$

1d **case; strong spin-orbital coupling**

$$\frac{\Delta R}{R} = -\frac{h}{e^2 L} R \sqrt{\frac{1}{L_\varphi^{\ 2}} + \frac{1}{3}\left(\frac{A}{L_H}\right)^2}$$

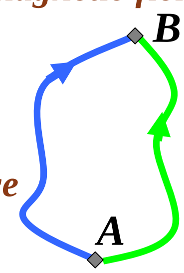
$$L_H = \sqrt{\frac{h}{eH}}$$

A – area of the wire cross-section

Can we always reliably extract the inelastic dephasing rate from the experiment ?

Weak localization: NO - *everything that violates T-invariance will destroy the constructive interference*

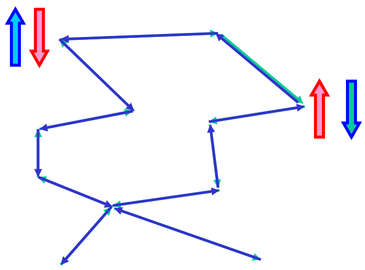
EXAMPLE: *random quenched magnetic field*



Mesoscopic fluctuations: YES - *Even strong magnetic field will not eliminate these fluctuations. It will only reduce their amplitude by factor 2.*

But *Slow diffusion of the impurities will look as dephasing in mesoscopic fluctuations measurements*

Magnetic Impurities



⇧ - before ⇧ - after

T-invariance is clearly violated, therefore we have dephasing

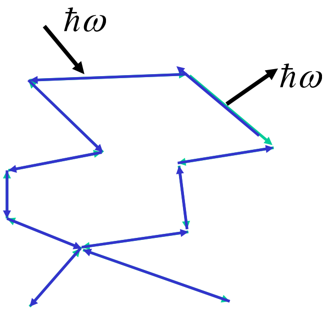
Mesoscopic fluctuations

Magnetic impurities cause dephasing only through effective interaction between the electrons.

$T \to 0$ *Either Kondo scattering or quenching due to the RKKY exchange.*

In both cases no "elastic dephasing"

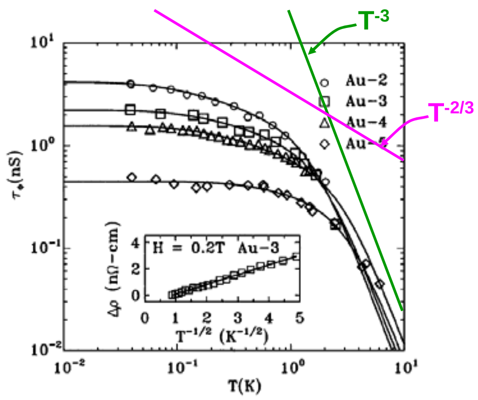
Inelastic dephasing rate $1/\tau_\varphi$ can be separated at least in principle



- other electrons
- phonons
- magnons
- two level systems
-
-

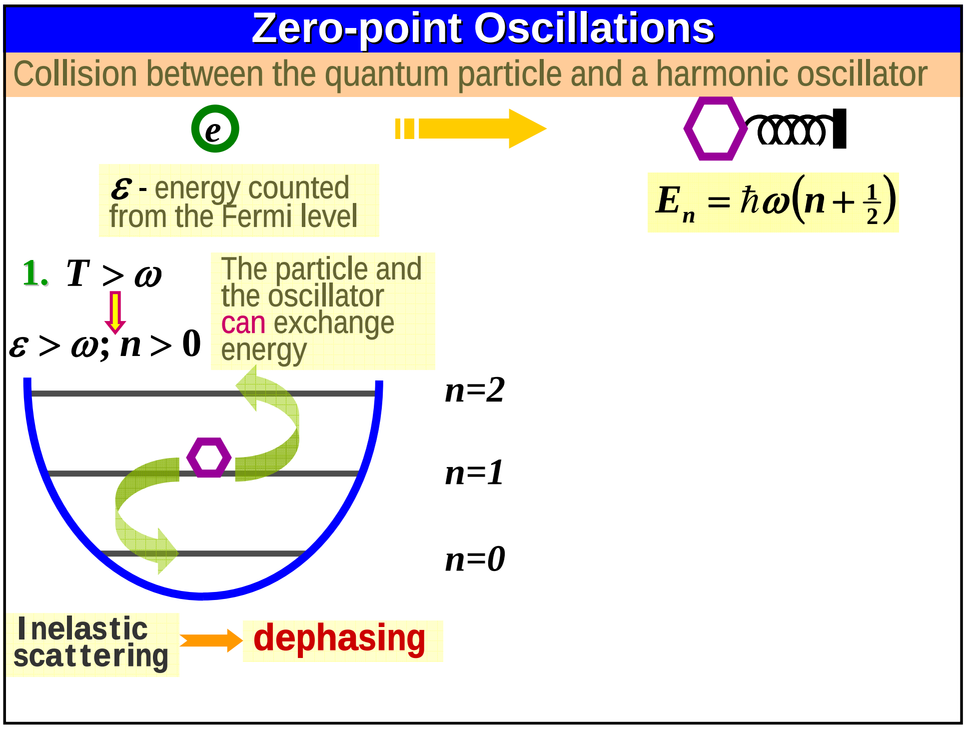
THE EXPERIMENTAL CONTROVERSY

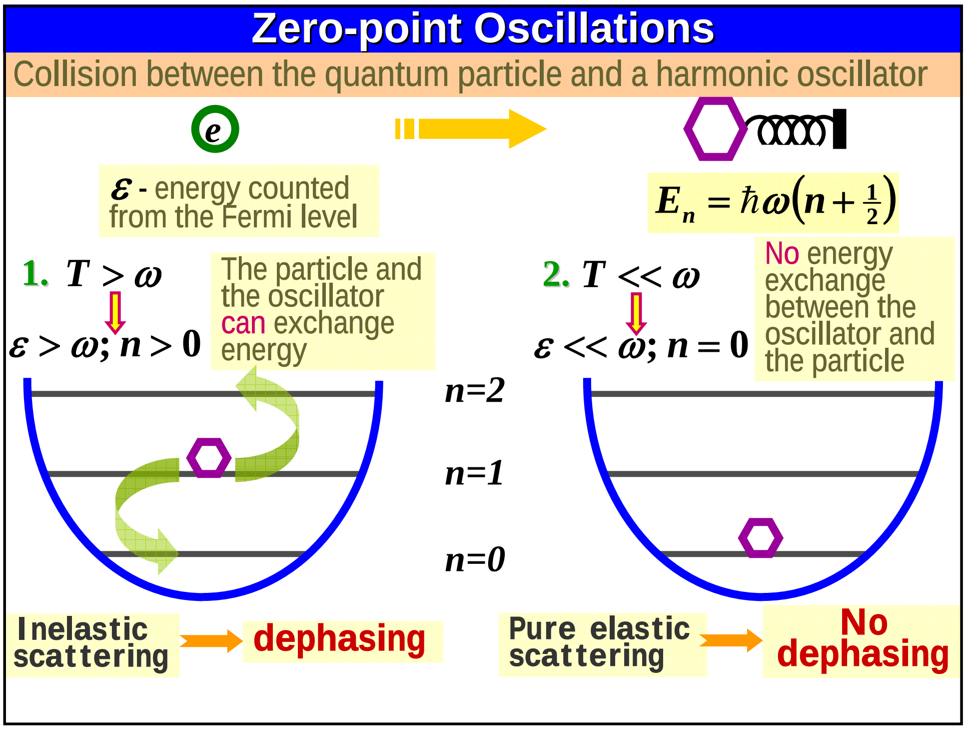
Mohanty, Jariwala and Webb, **PRL 78, 3366 (1997)**

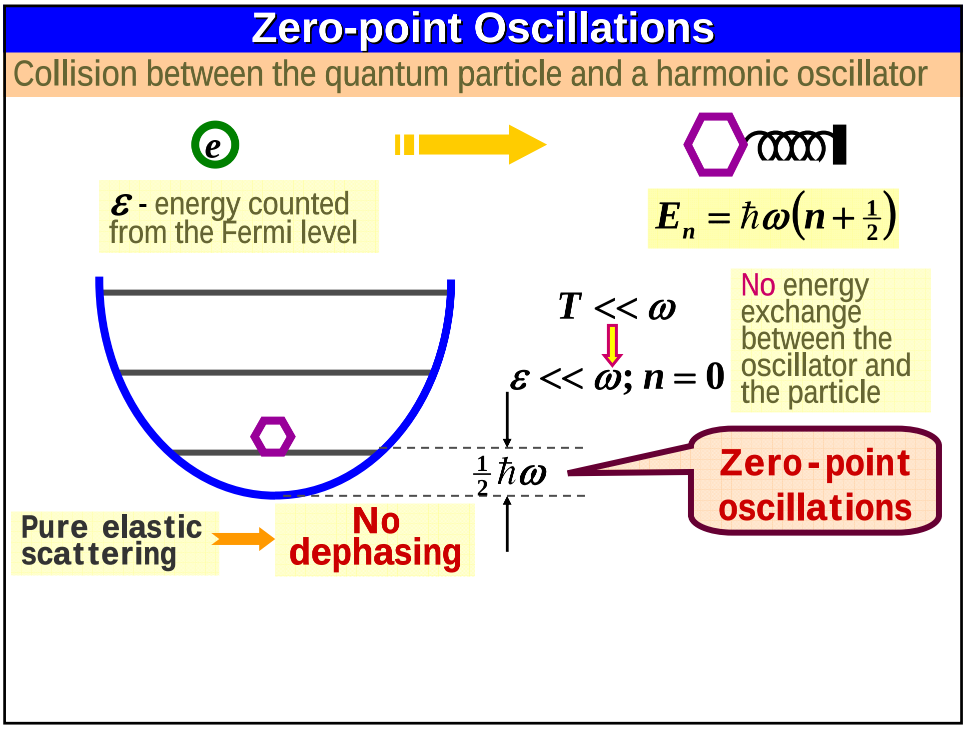


Saturation of τ_φ:

Artifact of measurement ?
Real effect in samples ?


Zero-point Oscillations
Collision between the quantum particle and a harmonic oscillator
e
ε - energy counted from the Fermi level
$E_n = \hbar\omega(n + \frac{1}{2})$
1. $T > \omega$
$\varepsilon > \omega; n > 0$
The particle and the oscillator can exchange energy
n=2
n=1
n=0
Inelastic scattering
dephasing


Zero-point Oscillations
Collision between the quantum particle and a harmonic oscillator
e
ε - energy counted from the Fermi level
$E_n = \hbar\omega(n + \frac{1}{2})$
1. $T > \omega$
$\varepsilon > \omega; n > 0$
The particle and the oscillator can exchange energy
2. $T << \omega$
$\varepsilon << \omega; n = 0$
No energy exchange between the oscillator and the particle
n=2
n=1
n=0
Inelastic scattering
dephasing
Pure elastic scattering
No dephasing



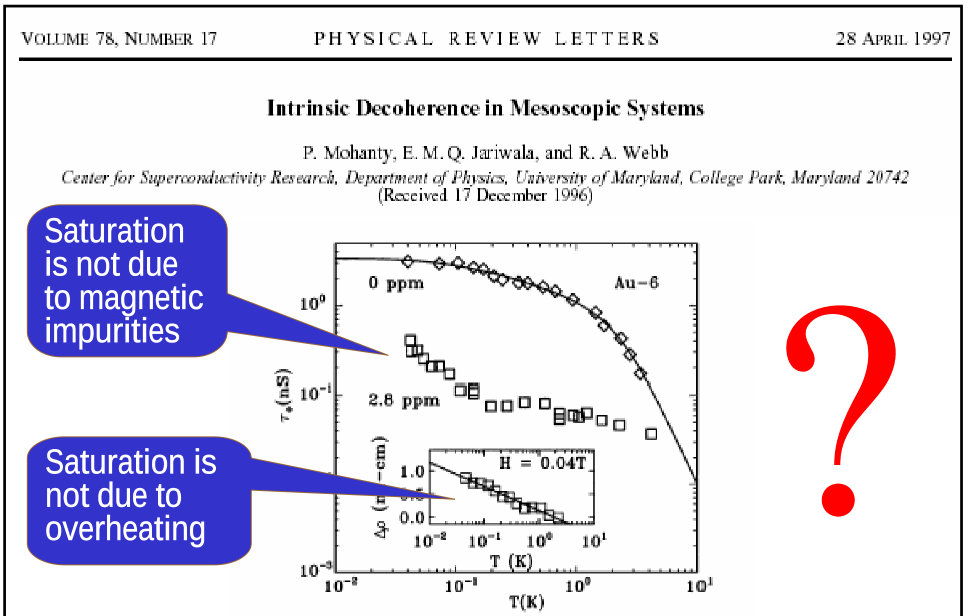
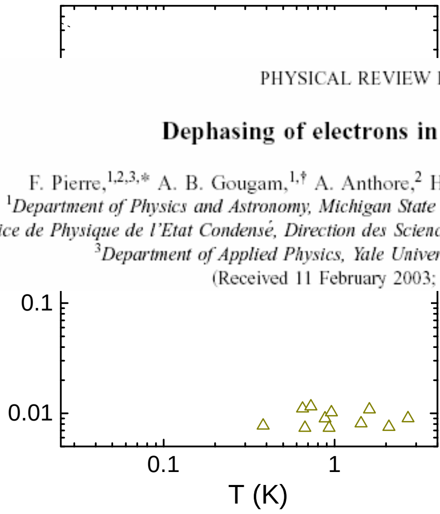


FIG. 3. Temperature dependence of τ_ϕ before (diamonds) and after (boxes) Fe implantation. The solid line is a fit to Eq. (1) with phonons. The inset shows the log(T) dependence of $\Delta\rho$ due to magnetic impurities with a theoretical fit.

$\tau_\phi(T)$ in Au wires



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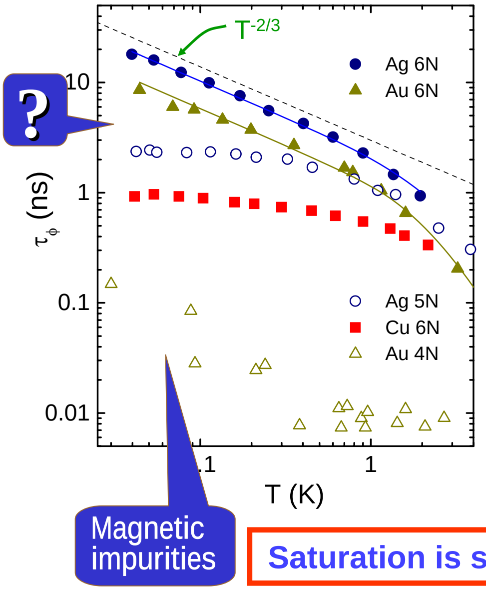
Dephasing of electrons in mesoscopic metal wires

F. Pierre,[1,2,3,*] A. B. Gougam,[1,†] A. Anthore,[2] H. Pothier,[2] D. Esteve,[2] and Norman O. Birge[1]
[1]*Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824-2320, USA*
[2]*Service de Physique de l'Etat Condensé, Direction des Sciences de la Matière, CEA-Saclay, 91191 Gif-sur-Yvette, France*
[3]*Department of Applied Physics, Yale University, New Haven, Connecticut 06520, USA*
(Received 11 February 2003; published 26 August 2003)

$\tau_\phi(T)$ in Ag, Au & Cu wires



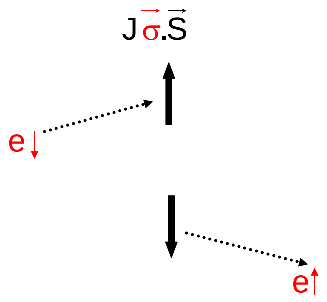
xN = 99.9…9 % source material purity (x nines)

Low T behavior vs. Purity:

- Ag 6N, Au 6N
  → agreement with AAK theory
- Ag 5N, Cu 6N
  → saturation of $\tau_\phi(T)$
- Au 4N
  → contains ~ 30 ppm of **Fe**

Saturation is sample dependent

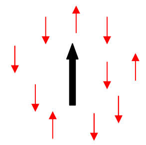
Magnetic Impurities: the Kondo Effect



Spin-flip scattering

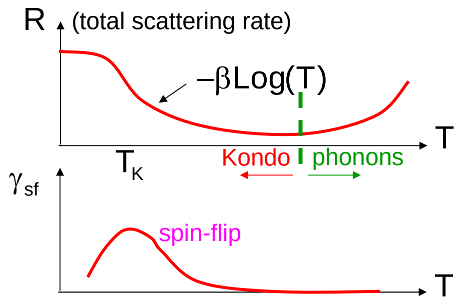
⇨ increased resistivity
⇨ reduction of τ_ϕ

Collective effect:

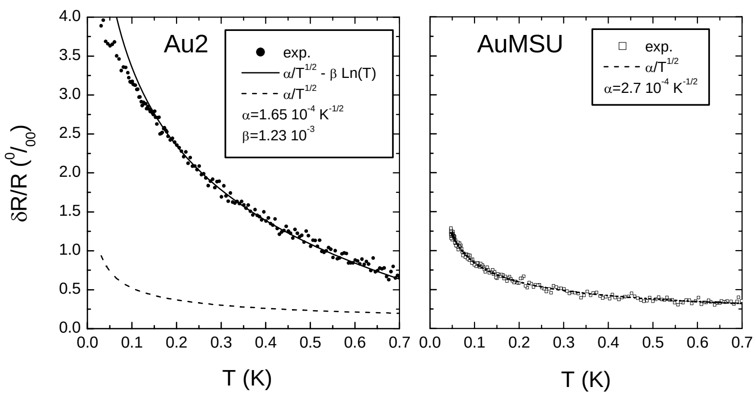


Formation of a singlet spin state

$k_B T_K \approx E_F e^{-1/\nu J}$

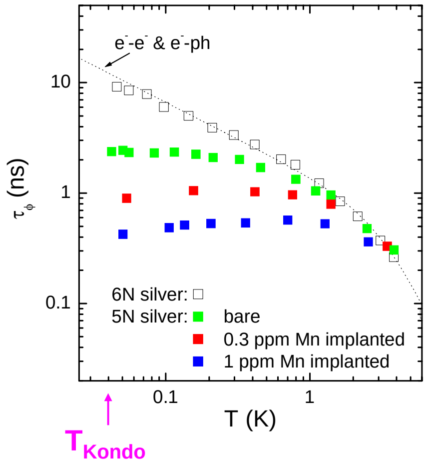


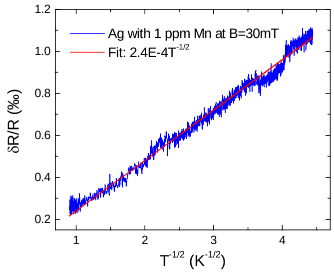
Au with and without Fe impurities



Kondo effect: R(T) = A - B log(T)

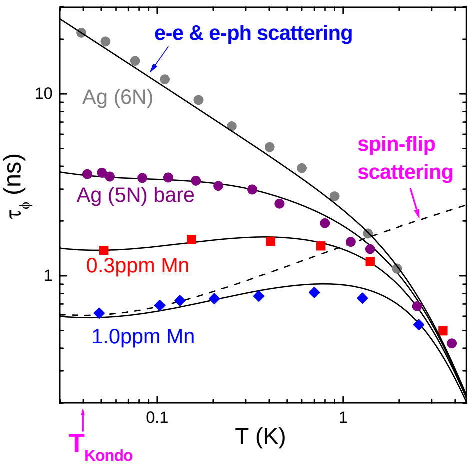
Dephasing and temperature dependence of the resistivity.





1 ppm of Mn is <u>invisible</u> in R(T)

Can magnetic impurities cause an apparent saturation of τ_ϕ ?



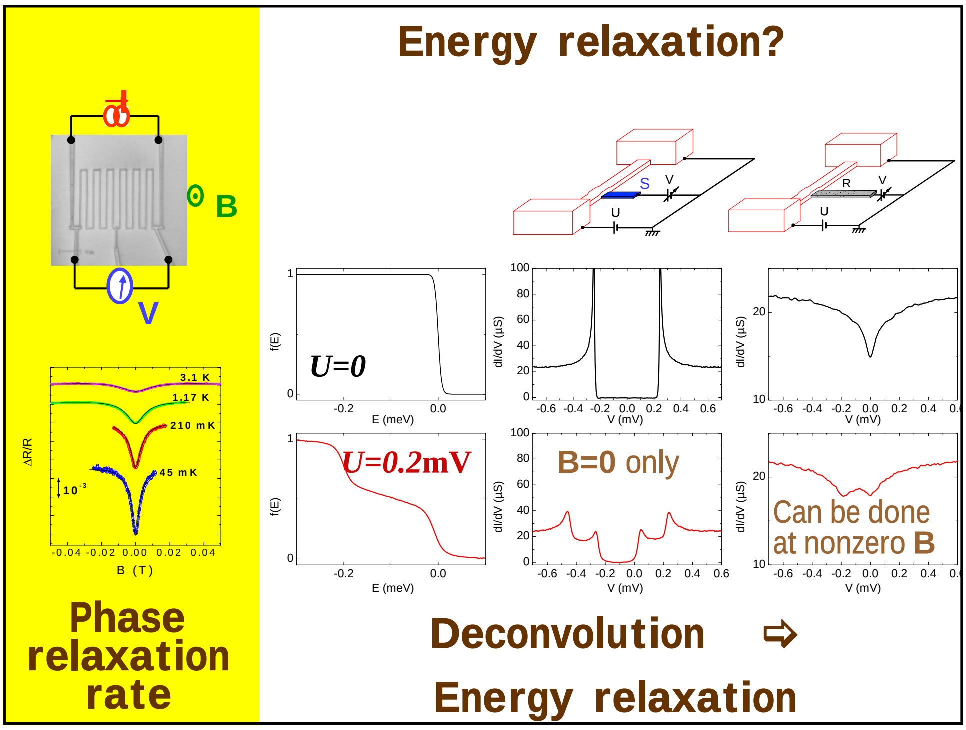
Spin-flip rate peaks at T_K:

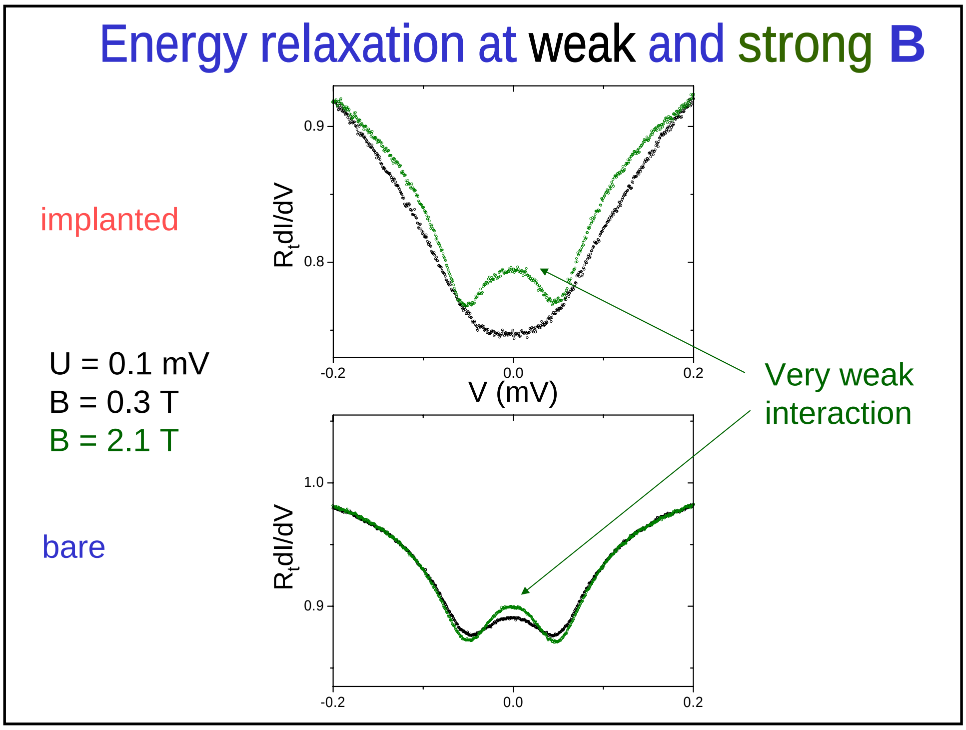
$$\tau_\phi(T_K) = \frac{h\,\nu_F}{4\,n_{imp}} \approx \frac{0.6\text{ ns}}{c_{imp}\text{ (ppm)}}$$

YES!
But only if $T \approx T_{Kondo}$

Is it a proof of the magnetic impurities domination in the decoherence ?

Not yet


Energy relaxation?
B
V
ΔR/R
3.1 K
1.17 K
210 mK
45 mK
B (T)
Phase relaxation rate
U=0
U=0.2mV
B=0 only
Can be done at nonzero B
f(E)
E (meV)
dI/dV (µS)
V (mV)
Deconvolution ⇨
Energy relaxation


Energy relaxation at weak and strong B
implanted
U = 0.1 mV
B = 0.3 T
B = 2.1 T
bare
R_tdI/dV
V (mV)
Very weak interaction

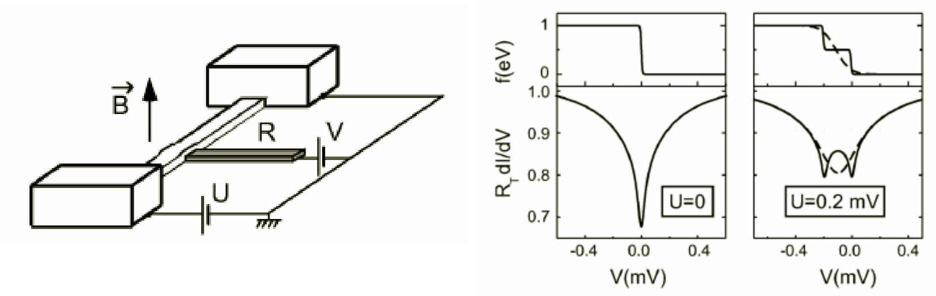
VOLUME 90, NUMBER 7 PHYSICAL REVIEW LETTERS week ending 21 FEBRUARY 2003

Magnetic-Field-Dependent Quasiparticle Energy Relaxation in Mesoscopic Wires

A. Anthore, F. Pierre, H. Pothier, and D. Esteve
Service de Physique de l'Etat Condensé, Direction des Sciences de la Matière, CEA-Saclay, 91191 Gif-sur-Yvette, France
(Received 5 August 2002; published 20 February 2003)

Effect of magnetic impurities on energy exchange between electrons

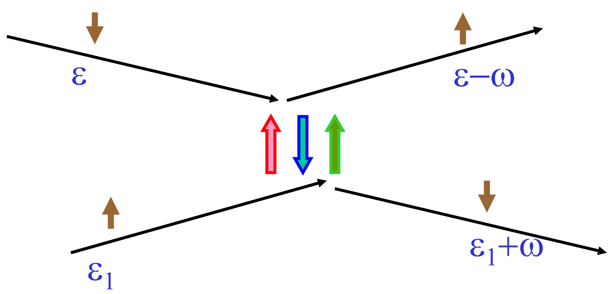
B. Huard, A. Anthore,* Norman O. Birge,† H. Pothier,‡ and D. Esteve
Quantronics Group, Service de Physique de l'État Condensé, DRECAM, CEA-Saclay, 91191 Gif-sur-Yvette, France
(Dated: January 17, 2005)



VOLUME 86, NUMBER 11 PHYSICAL REVIEW LETTERS 12 MARCH 2001

Electron Energy Relaxation in the Presence of Magnetic Impurities

A. Kaminski and L. I. Glazman
Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455
(Received 26 October 2000)

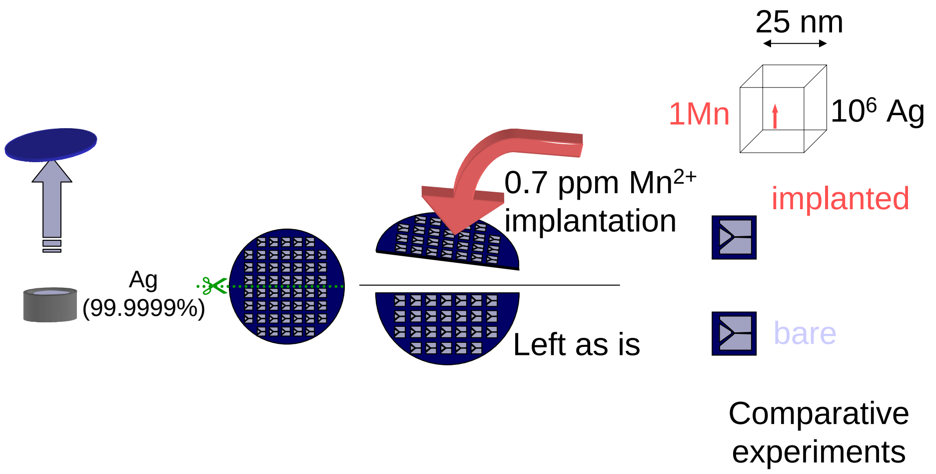


Second order in electron – impurity exchange

One electron can not change energy after scattering by a system of two degenerated states.

Two electrons can exchange energy in spite of the fact that the states of the impurity are degenerate and it can not be excited

Controlled experiment II: energy exchange



Effect of 1 ppm Mn on interactions ?

Effect of magnetic impurities on energy exchange between electrons

B. Huard, A. Anthore,* Norman O. Birge,† H. Pothier,‡ and D. Esteve
Quantronics Group, Service de Physique de l'État Condensé,
DRECAM, CEA-Saclay, 91191 Gif-sur-Yvette, France

(Dated: January 17, 2005)

Concentration of Mn impurities

	"Bare" wire c_b	Implanted wire c_{im}
Neutralization current + Monte Carlo simulations	?	$c_b + (0.7 \pm 0.1)\ ppm$
Dephasing time τ_ϕ		
Energy relaxation		

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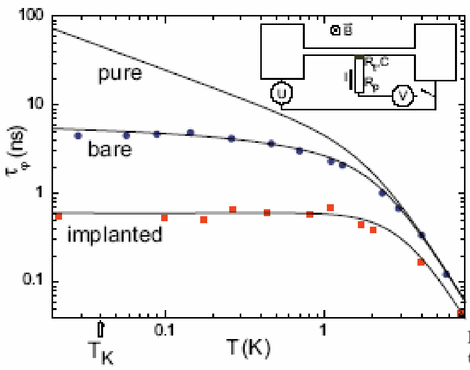

FIG. 1: (Color online) Symbols: measured phase coherence time in the two wires. Solid lines: best fits with Eq. (1), obtained with $c_b = 0.10 \pm 0.01$ ppm (bare wire) and $c_i = 0.95 \pm 0.1$ ppm (implanted wire). The upper line is the prediction without spin-flip scattering ($c = 0$). Inset: layout of the circuit. The switch is open for magnetoresistance measurements, closed for energy exchange measurements.

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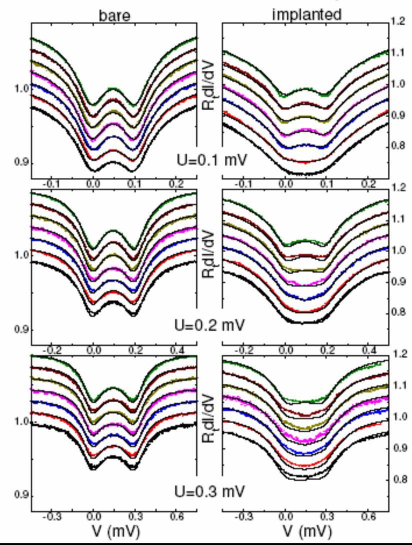


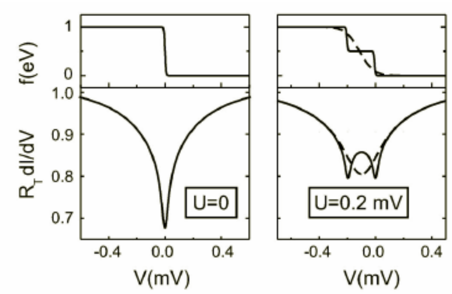
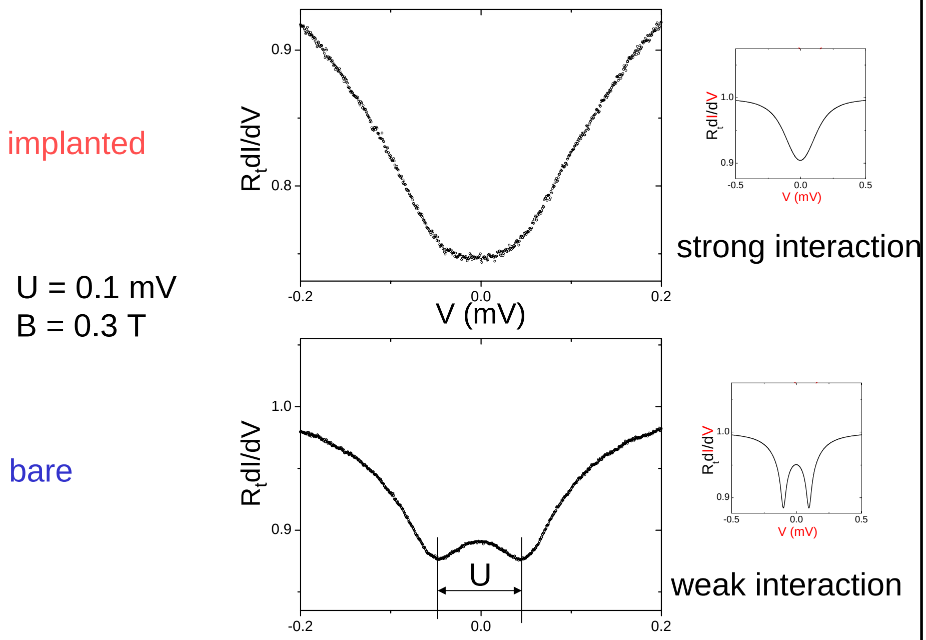


FIG. 2: (Color online) Differential conductance $dI/dV(V)$ of the tunnel junction (see inset of Fig. 1) for the bare (left) and implanted (right) wires, for $U = 0.1$ mV, 0.2 mV and 0.3 mV (top to bottom panels), and for $B = 0.3$ to 2.1 T by steps of 0.3 T (bottom to top in each panel). The curves were shifted vertically for clarity. Symbols: experiment. Solid lines: calculations using $c_b = 0.1$ ppm, $c_i = 0.95$ ppm and $\kappa_{ee} = 0.05$ ns^{-1}meV$^{-1/2}$.

Energy relaxation at weak **B**



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Magnetic-field effects in energy relaxation mediated by Kondo impurities in mesoscopic wires

G. Göppert,[1] Y. M. Galperin,[2] B. L. Altshuler,[1,3] and H. Grabert[4]
[1]*Physics Department, Princeton University, Princeton, New Jersey 08544*
[2]*Department of Physics, University of Oslo, P.O. Box 1048, N-0316 Oslo, Norway and Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, Illinois 6043*
[3]*NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540*
[4]*Fakultät für Physik, Albert-Ludwigs-Universität, Hermann-Herder-Straße 3, D 79104 Freiburg, Germany*
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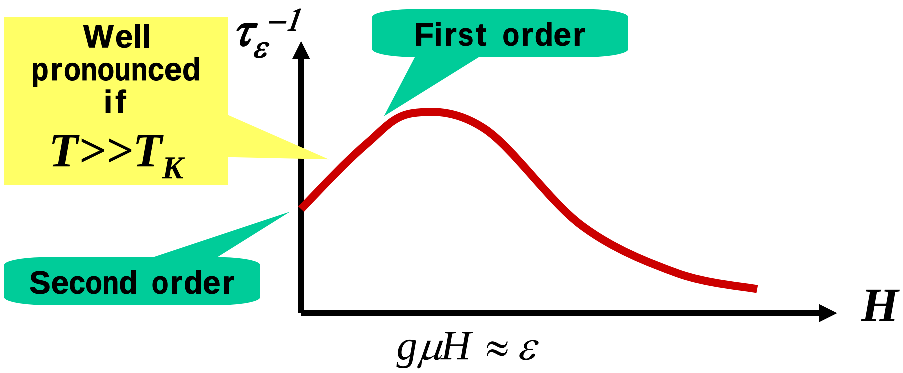

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Concentration of Mn impurities

	"Bare" wire c_b	Implanted wire c_{im}
Neutralization current + Monte Carlo simulations	?	$c_b + (0.7 \pm 0.1)\ ppm$
Dephasing time τ_ϕ	$(0.1 \pm 0.01)\ ppm$	$(0.95 \pm 0.1)\ ppm$
Energy relaxation		$(0.9 \pm 0.3)\ ppm$

1 Dephasing and energy relaxation give very close concentration of the magnetic impurities

2 Energy relaxation is suppressed by the magnetic field as was predicted

↓

Mn impurities dominate both energy and phase relaxation in this temperature interval. No room for mysteries like zero-point dephasing.

Both energy and phase relaxation times should increase dramatically when temperature is reduces further – Kondo effect

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