Theory of Mesoscopic Systems

Boris Altshuler

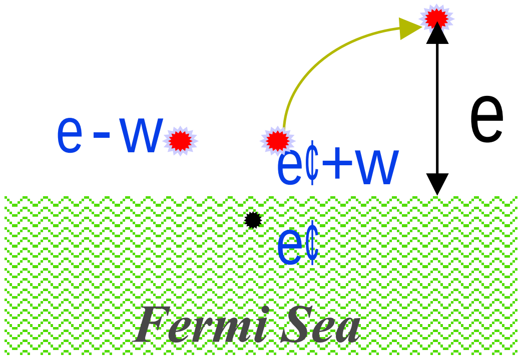
Princeton University,
Columbia University &
NEC Laboratories America

CONFÉRENCE UNIVERSITAIRE
DE SUISSE OCCIDENTALE

Lecture 4 20 June 2006

Previous Lecture

Quasiparticle decay rate at $T = 0$ in a clean Fermi Liquid.



$$\frac{\hbar}{t_{e-e}(e)} \propto \begin{cases} e^2/e_F & d = 3 \\ \left(e^2/e_F\right)\log\left(e_F/e\right) & d = 2 \\ e & d = 1 \end{cases}$$

Conclusions:

1. For d=3,2 from $e << e_F$ it follows that $et_{e\text{-}e} >> h$, i.e., that the qusiparticles are well determined and the Fermi-liquid approach is applicable.

2. For d=1 $et_{e\text{-}e}$ is of the order of h, i.e., that the Fermi-liquid approach is not valid for $1d$ systems of interacting fermions.

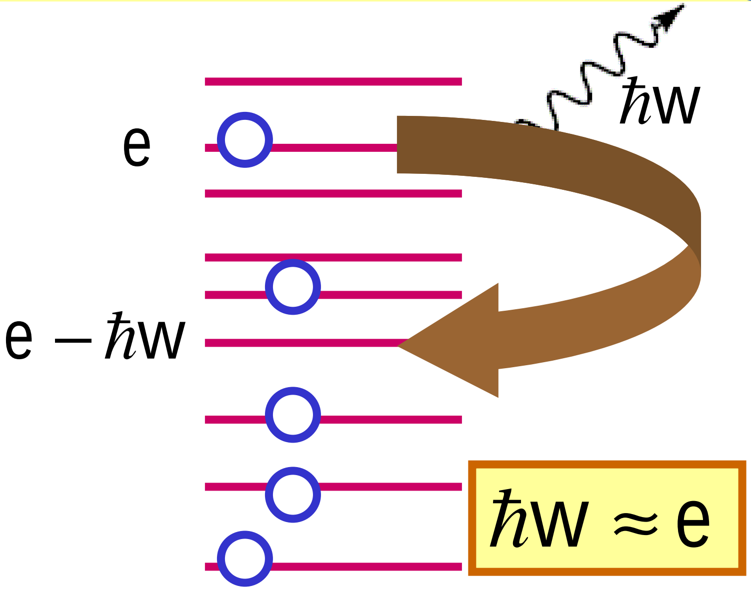
Luttinger liquids

Applicability of the FL approach is determined by the phase relaxation time

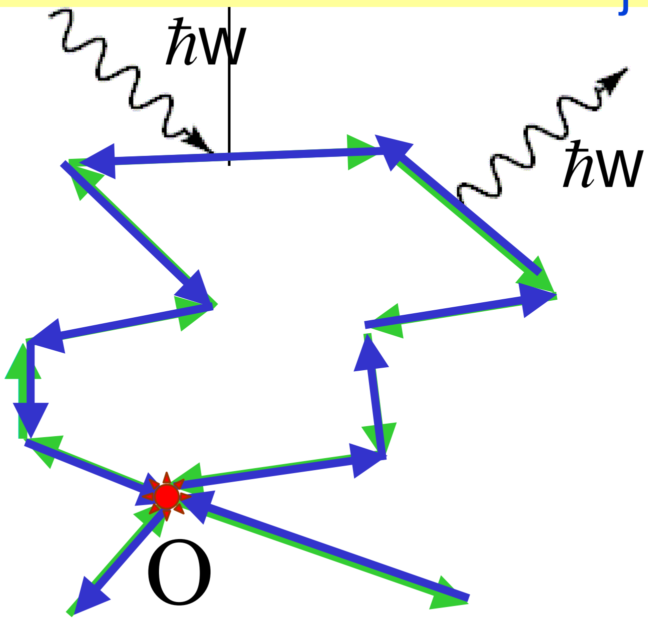
$$\delta\varepsilon(t^*) \approx \frac{\hbar}{t^*} \Rightarrow \delta\varphi(t^*) \approx 1 \Rightarrow \begin{array}{c} t^* \approx \tau_\varphi \\ \Delta\varepsilon_{\min} \approx \hbar/\tau_\varphi \end{array}$$

It is dephasing rate that determines the accuracy at which the energy of the quantum state can be measured in principle.

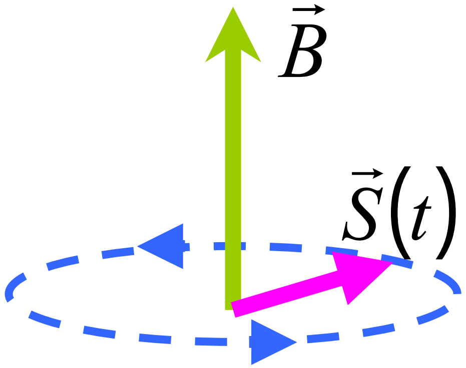
Energy relaxation rate $1/t_e$

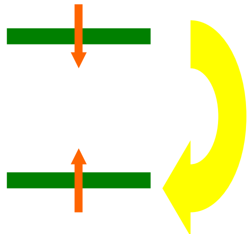


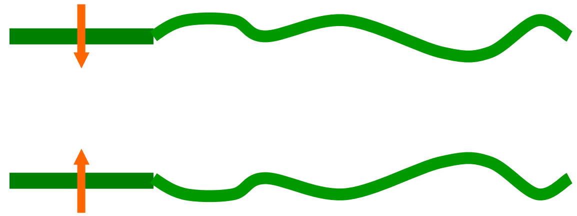
Inelastic dephasing rate $1/t_j$



Analogy: NMR relaxation rates T_1 and T_2



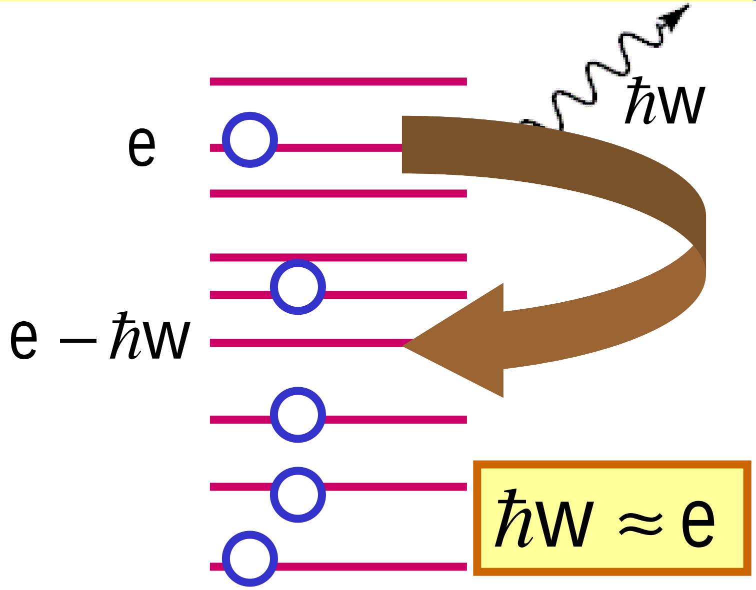




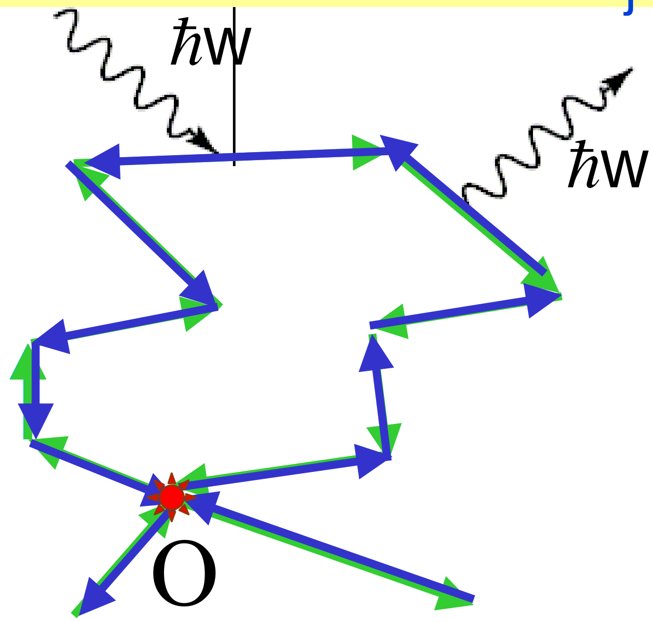
relaxation of S_z - T_1

relaxation of S_z - T_1

Energy relaxation rate $1/t_e$



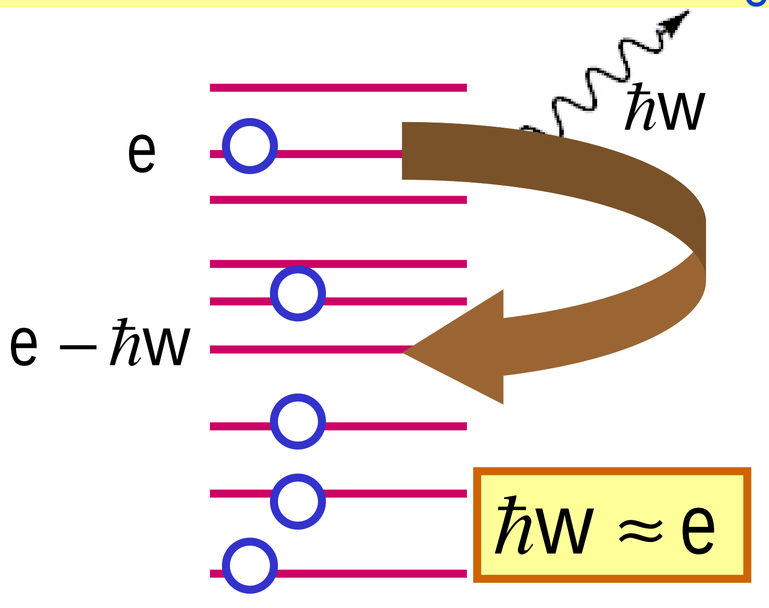
Inelastic dephasing rate $1/t_j$



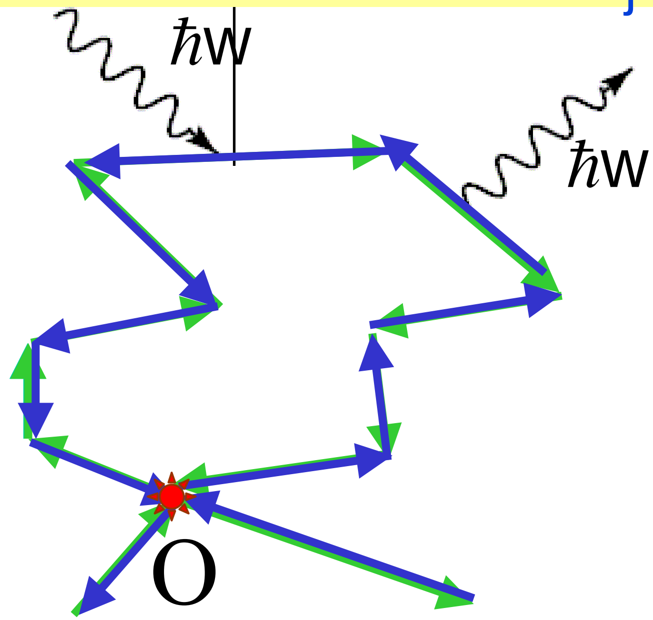
Problem: quasielastic scattering

Given the energy transfer in each scattering act w and the inelastic rate $1/t_{in}$ determine the rates $1/t_j$ and $1/t_e$. Consider both cases $wt_{in} << 1$ and $wt_{in} >> 1$.

Energy relaxation rate $1/\tau_\varepsilon$



Inelastic dephasing rate $1/\tau_\varphi$



For electron-electron interaction in the presence of disorder

$$\frac{\hbar}{\tau_\varepsilon} \approx \frac{\varepsilon}{g(L_\varepsilon)}$$

$$L_\varepsilon = \sqrt{\hbar D/\varepsilon}$$

g – Thouless conductance

D – diffusion constant

ν – density of states

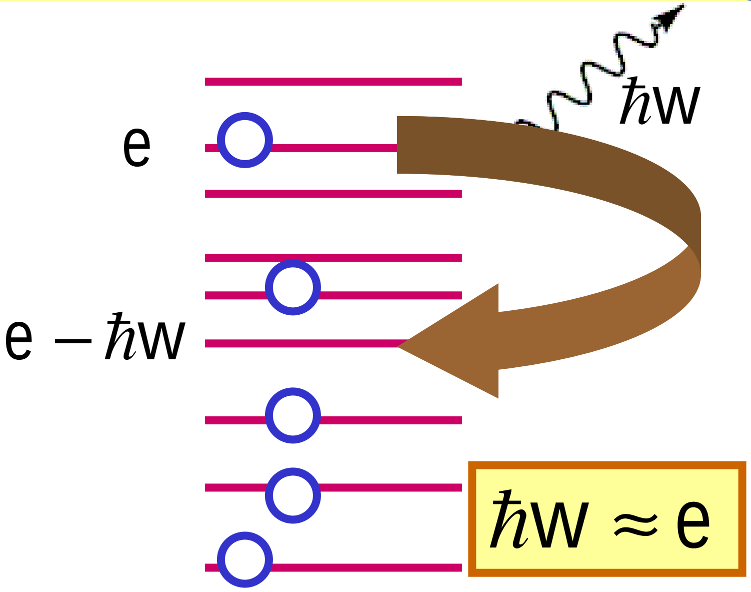
$$\frac{\hbar}{\tau_\varphi} \approx \frac{T}{g(L_\varphi)}$$

$$L_e = \sqrt{D\tau_\varphi}$$

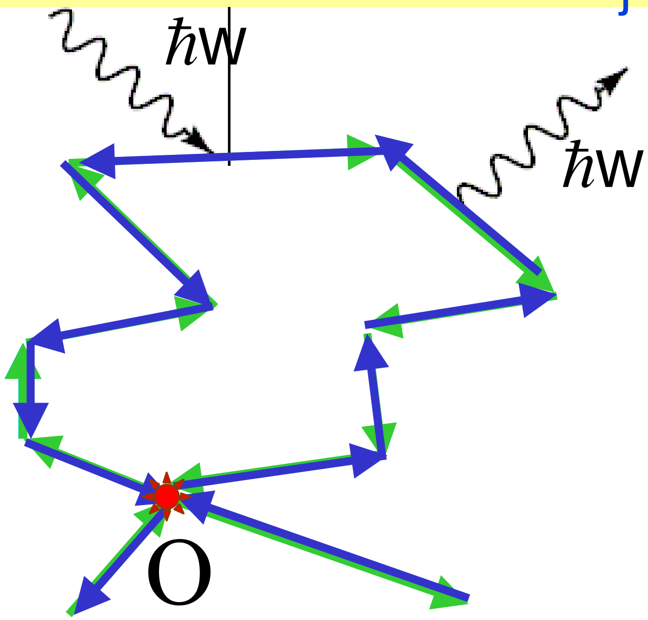
$$\frac{\hbar}{\tau_\varepsilon} \approx \frac{\varepsilon^{d/2}}{\nu D^{d/2}}$$

$$\frac{\hbar}{\tau_\varphi} \approx \frac{\hbar^{d-6/d-4}}{\nu^{2/d-4} D^{d/d-4}} T^{2/d-4}$$

Energy relaxation rate $1/t_e$



Inelastic dephasing rate $1/t_j$



For electron-electron interaction in the presence of disorder

$$\frac{\hbar}{t_e} \approx \frac{e}{g(L_e)}$$

$$L_e = \sqrt{\hbar D/e}$$

$$t_e \xrightarrow[e, T \to 0]{} \infty$$

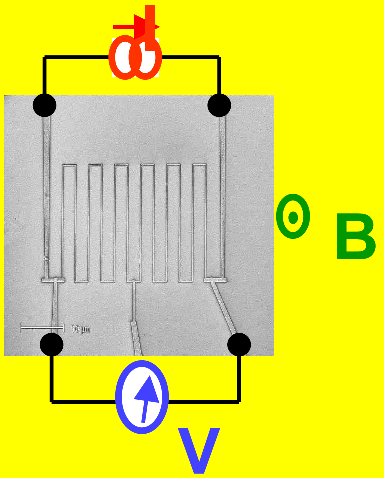
$$t_j \xrightarrow[T \to 0]{} \infty$$

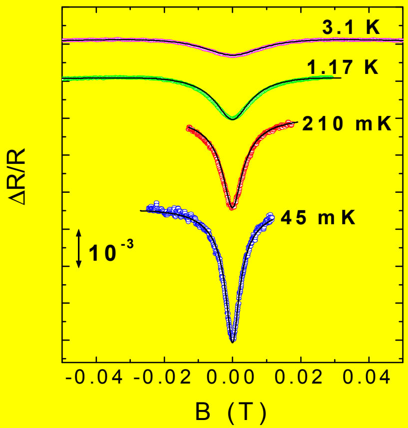
$$\frac{\hbar}{t_j} \approx \frac{T}{g(L_j)}$$

$$L_e = \sqrt{D t_j}$$

$$\frac{\hbar}{t_e} \approx \frac{e^{d/2}}{n D^{d/2}}$$

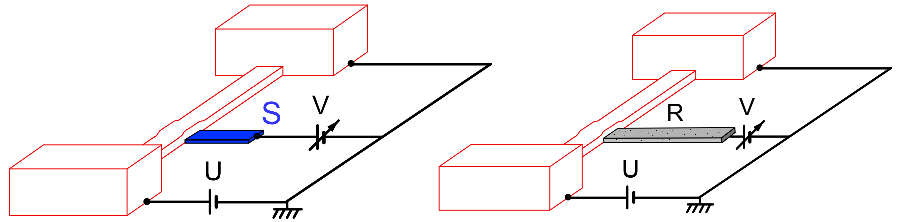
$$\frac{\hbar}{t_j} \approx \frac{\hbar^{d-6/d-4}}{n^{2/d-4} D^{d/d-4}} T^{2/d-4}$$

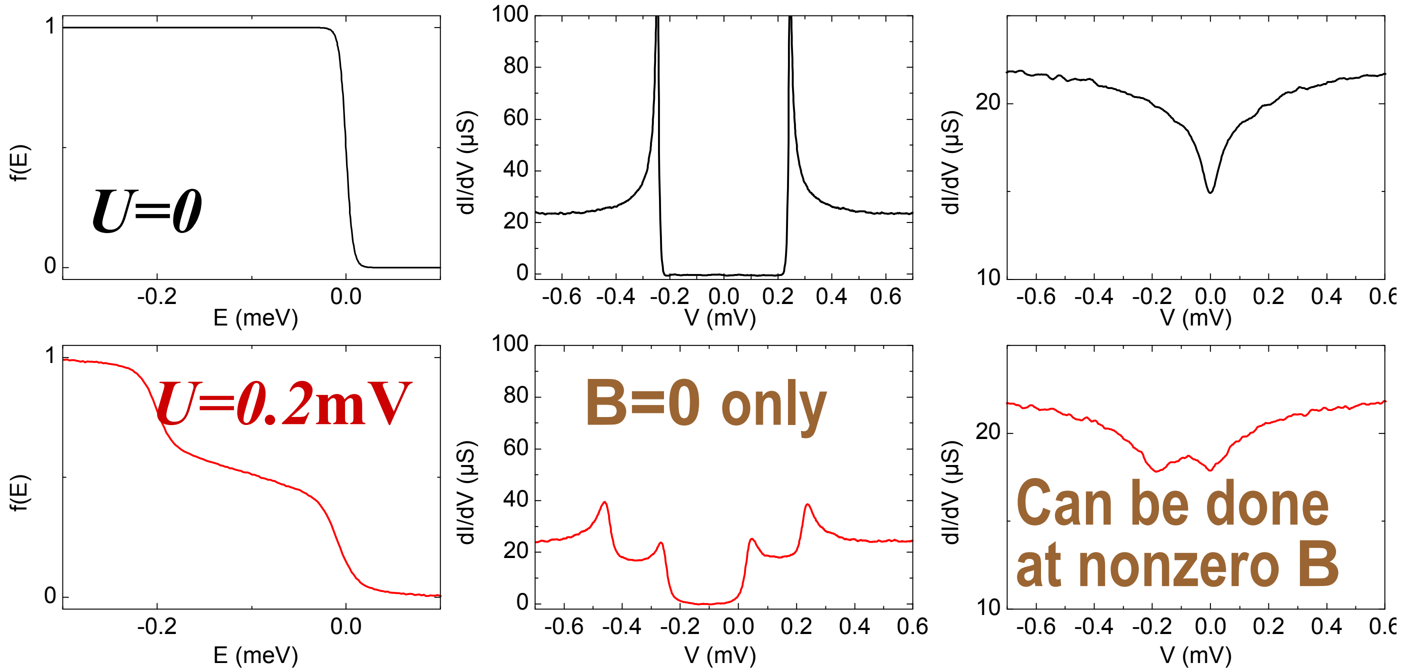




Phase relaxation rate

Energy relaxation?

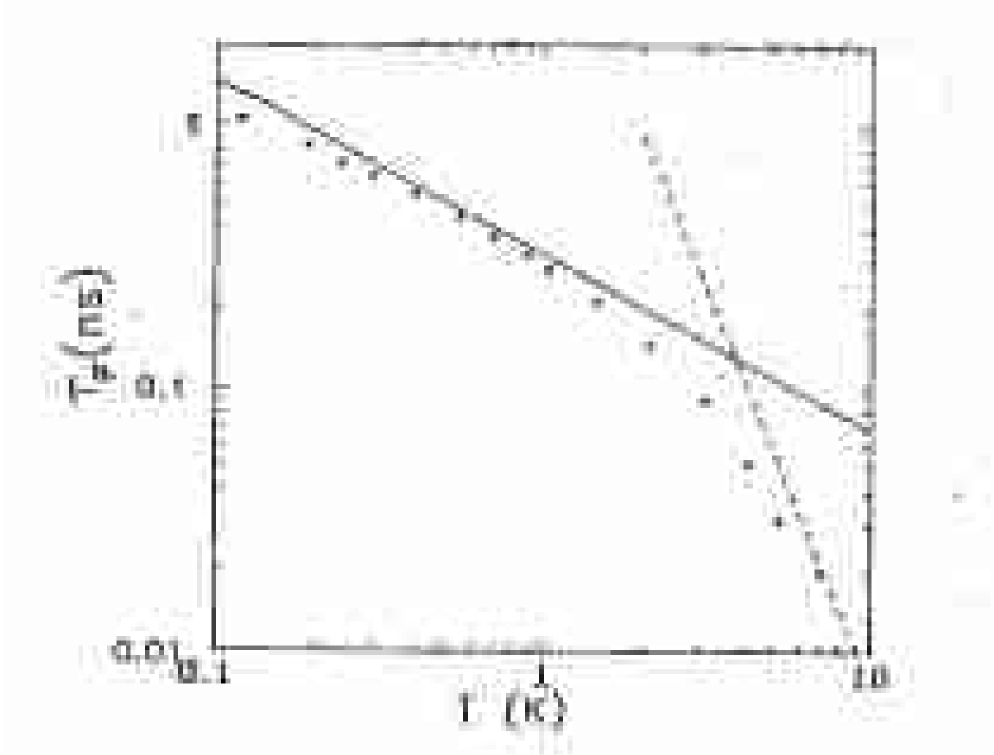




Deconvolution [

Energy relaxation

Temperature dependence of τ_f
(from magnetoresistance)



Echternach, Gershenson, Bozler, Bogdanov & Nilsson,

PRL 48, 11516 (1993)

Intrinsic Decoherence in Mesoscopic Systems

P. Mohanty, E. M. Q. Jariwala, and R. A. Webb
Center for Superconductivity Research, Department of Physics, University of Maryland, College Park, Maryland 20742
(Received 17 December 1996)

Saturation is not due to magnetic impurities

Saturation is not due to overheating

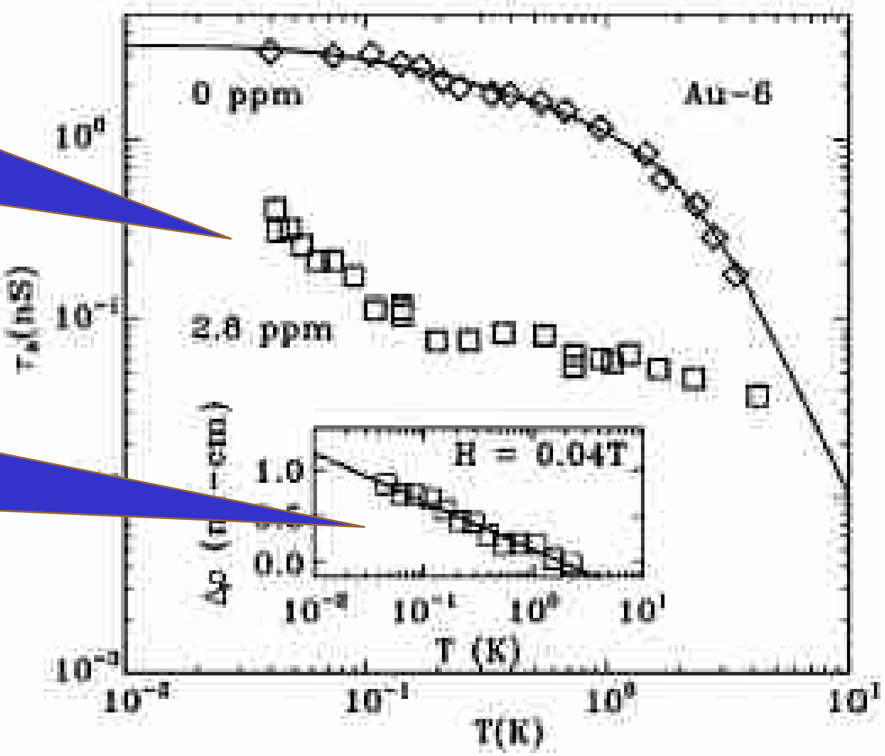
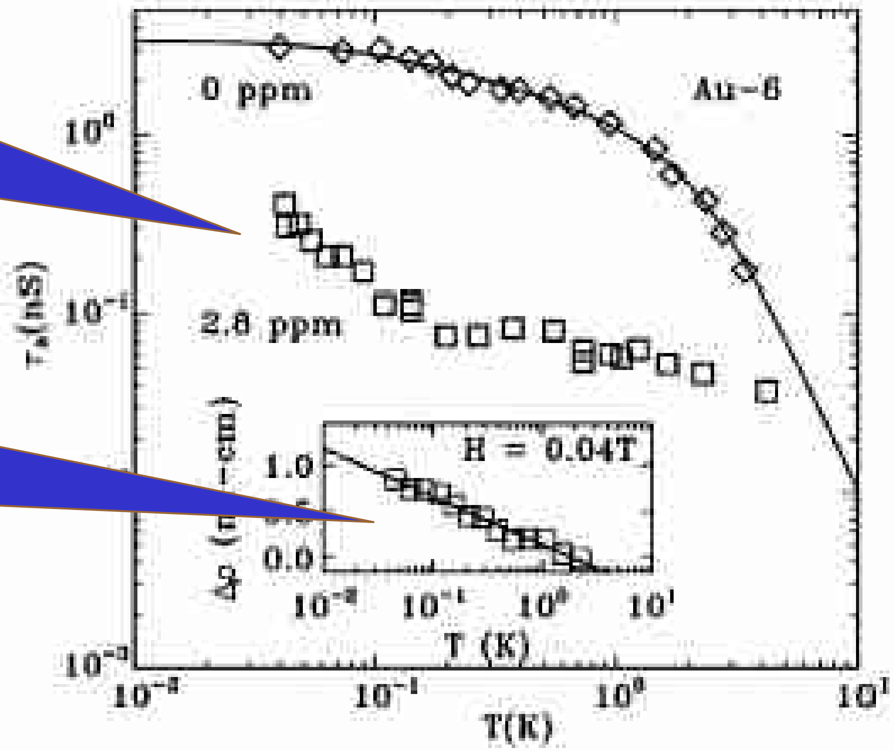


FIG. 3. Temperature dependence of τ_ϕ before (diamonds) and after (boxes) Fe implantation. The solid line is a fit to Eq. (1) with phonons. The inset shows the log(T) dependence of $\Delta\rho$ due to magnetic impurities with a theoretical fit.

?

Saturation is not due to magnetic impurities

Saturation is not due to overheating



?

It could be magnetic impurities with low Kondo temperature (Mn)

It could also be external radiation !

Dephasing without heating

Effect of microwave radiation on weak localization

Dephasing without heating

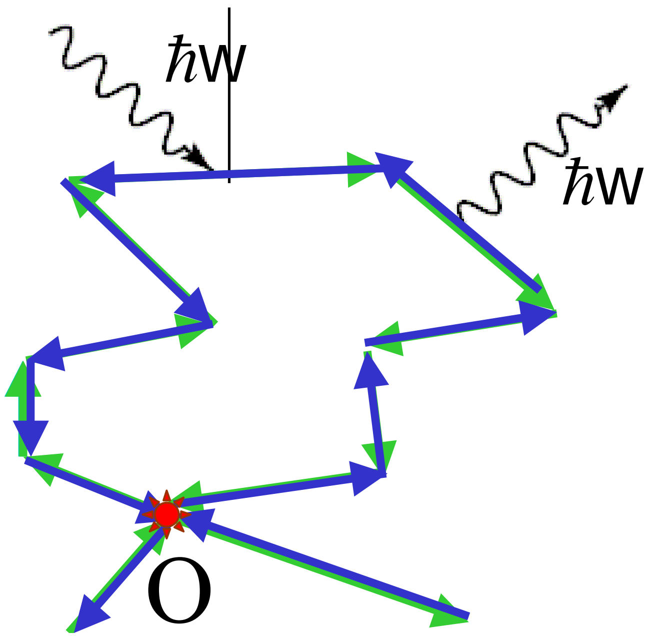
e-e interaction – Electric noise

Fluctuation- dissipation theorem:

Electric noise - randomly time and space - dependent electric field $E^{\alpha}(\vec{r},t) \Leftrightarrow E^{\alpha}(\vec{k},\omega)$. Correlation function of this field is completely determined by the conductivity $\sigma(\vec{k},\omega)$

$$\left\langle E^{\alpha}E^{\beta}\right\rangle_{\omega,\vec{k}} = \frac{\omega}{\sigma_{\alpha\beta}(\omega,\vec{k})}\coth\left(\frac{\omega}{2T}\right)\frac{k_{\alpha}k_{\beta}}{k^{2}} \propto \frac{T}{\sigma_{\alpha\beta}(\omega,\vec{k})}$$

Noise intensity increases with the temperature, $\boldsymbol{T}$, and with resistance



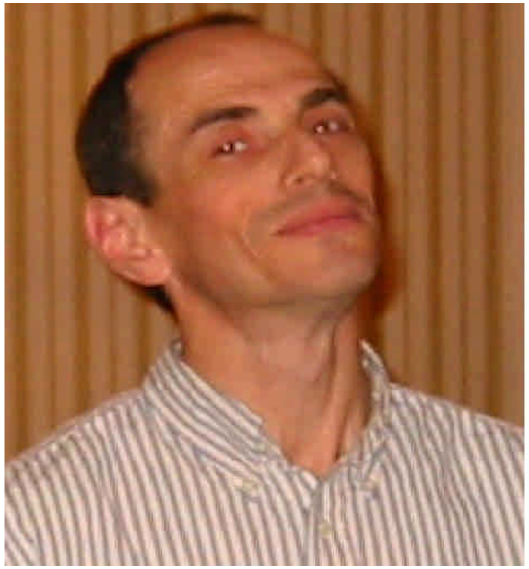
What is the effect of microwave radiation ?

External noise ?

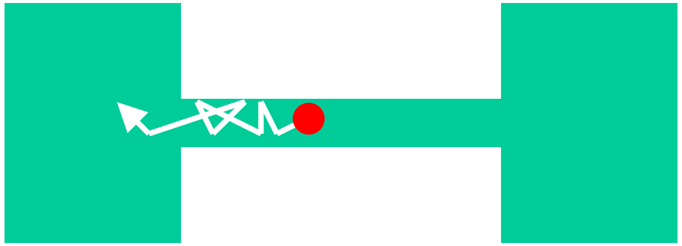
Microwave-Induced Dephasing in One-Dimensional Metal Wires

J. Wei, S. Pereverzev, and M. E. Gershenson*
Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08854
(Dated: August 4, 2005)

We report on the effect of monochromatic microwave (MW) radiation on the weak localization corrections to the conductivity of quasi-one-dimensional silver wires. Due to the improved electron cooling in the wires, the MW-induced dephasing was observed without a concomitant overheating of electrons over wide ranges of the MW power P_{MW} and frequency f. The observed dependences of the MW-induced dephasing rate on P_{MW} and f are in agreement with the theory by Altshuler, Aronov, and Khmelnitsky [1]. Our results suggest that the saturation of dephasing time, often observed at $T \leq 0.1$ K, may be caused by an insufficient screening of the sample from the external microwave noise.



Dephasing without Heating in 1D



$$P_{es} \approx \left(\frac{2\pi k_B}{e}\right)^2 \frac{T\Delta T}{R}$$

power that gets out of the sample due to diffusion of "hot" electrons to "cold" leads, given the overheating ΔT

$$P_{es} \approx \left(\frac{k_B T}{\hbar}\right)^2 \frac{e^2 R}{\hbar}$$

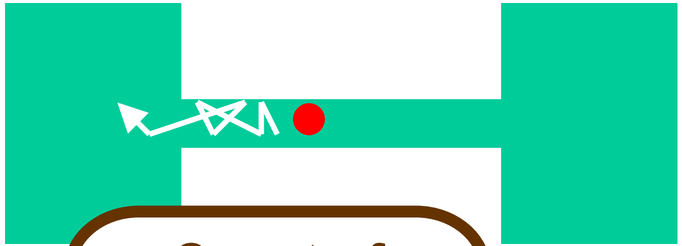
power at which dephasing effect of the radiation compares with the effect of the thermal noise

$$\frac{P_{es}}{P_\varphi} \approx \left(\frac{h}{e^2 R}\right)^2 \frac{\Delta T}{T}$$

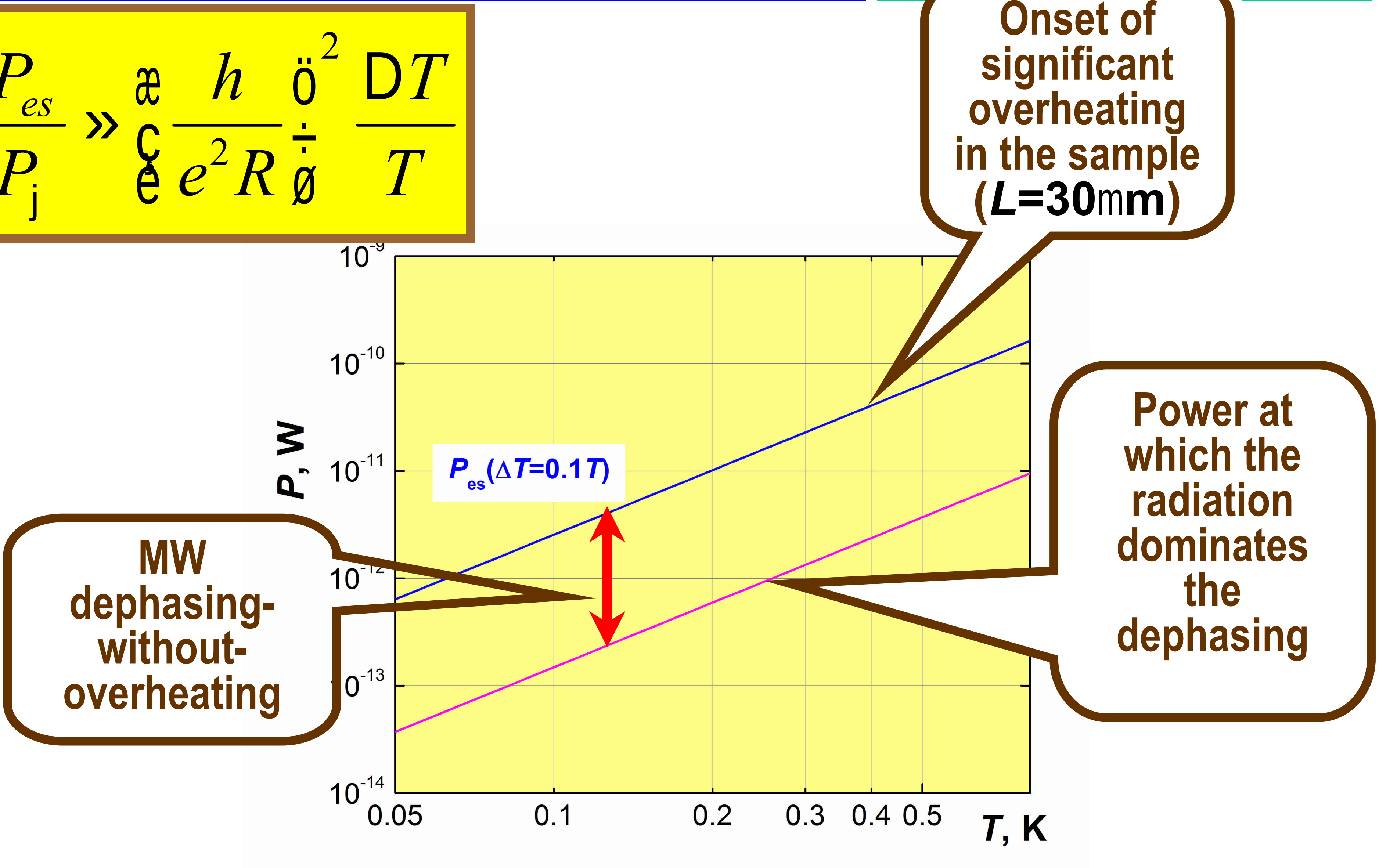
the shorter the wire, the larger resistance R*, the bigger the range of powers with* "Dephasing – without – overheating"

No "Dephasing – without – overheating" as long as $R >> R_q = h/e^2$

Dephasing without Heating in 1D



$$\frac{P_{es}}{P_j} \approx \left(\frac{h}{e^2 R}\right)^2 \frac{\Delta T}{T}$$



Solid State Communications, Vol.39, pp.619-623.
Pergamon Press Ltd. 1981. Printed in Great Britain.

0038-1098/81/290619-05$02.00/0

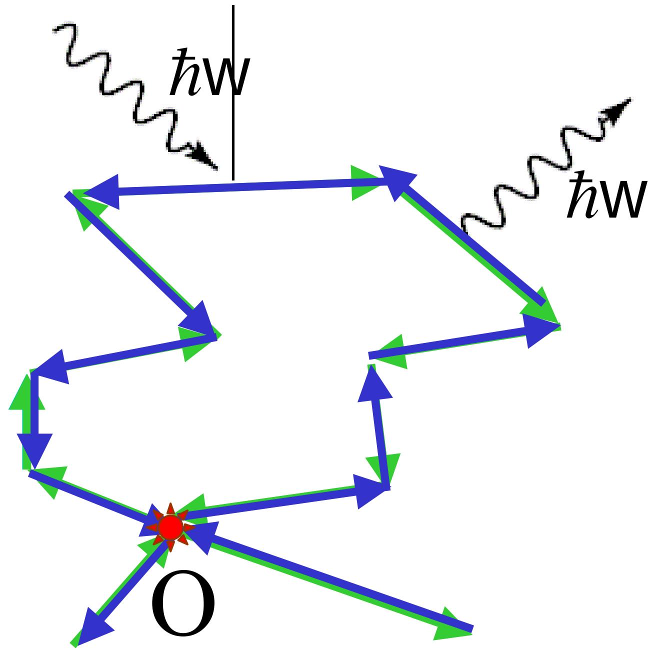
SUPPRESSION OF LOCALIZATION EFFECTS BY THE HIGH FREQUENCY FIELD AND THE NYQUIST NOISE

B.L.Altshuller, A.G.Aronov
Leningrad Nuclear Physics Institute, Gatchina,
Leningrad 188350, USSR

D.E.Khmelnitsky
L.D.Landau Institute for Theoretical Physics,
Chernogolovka, Moscow 142432, USSR

$$\left[\Delta\sigma_1\right]_{B=0} = \frac{2e^2}{\hbar}\sqrt{\frac{D}{\pi\Omega}}\int_{\Omega\tau_{im}}^{\infty}\frac{dx}{\sqrt{x}} I_0\left(\alpha f(x)\right) e^{-\alpha f(x) - \frac{2x}{\Omega}\frac{1}{\tau_\varphi}}$$

$$f(x) = x\left(1 + \frac{\sin 2x}{2x} - 2\frac{\sin^2 x}{x^2}\right) \qquad \alpha = \frac{2e^2}{\hbar^2}\frac{DE^2}{\Omega^3}$$



$$\left[\Delta\sigma_1\right]_{B=0} = \frac{2e^2}{\hbar}\sqrt{\frac{D}{\pi\Omega}}\int_{\Omega\tau_{im}}^{\infty}\frac{dx}{\sqrt{x}}\, I_0\left(\alpha f(x)\right) e^{-\alpha f(x) - \frac{2x}{\Omega}\frac{1}{\tau_\varphi}}$$

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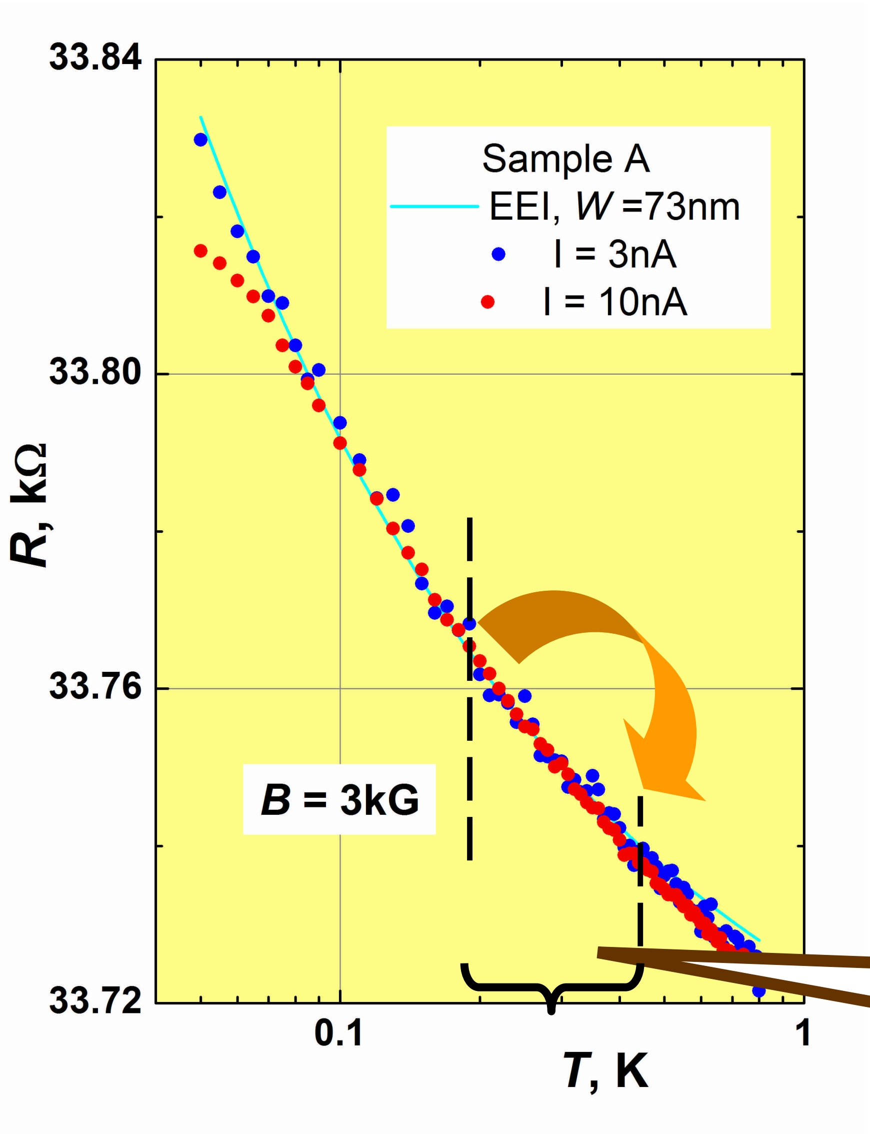
Interaction Corrections as a Built-in Thermometer



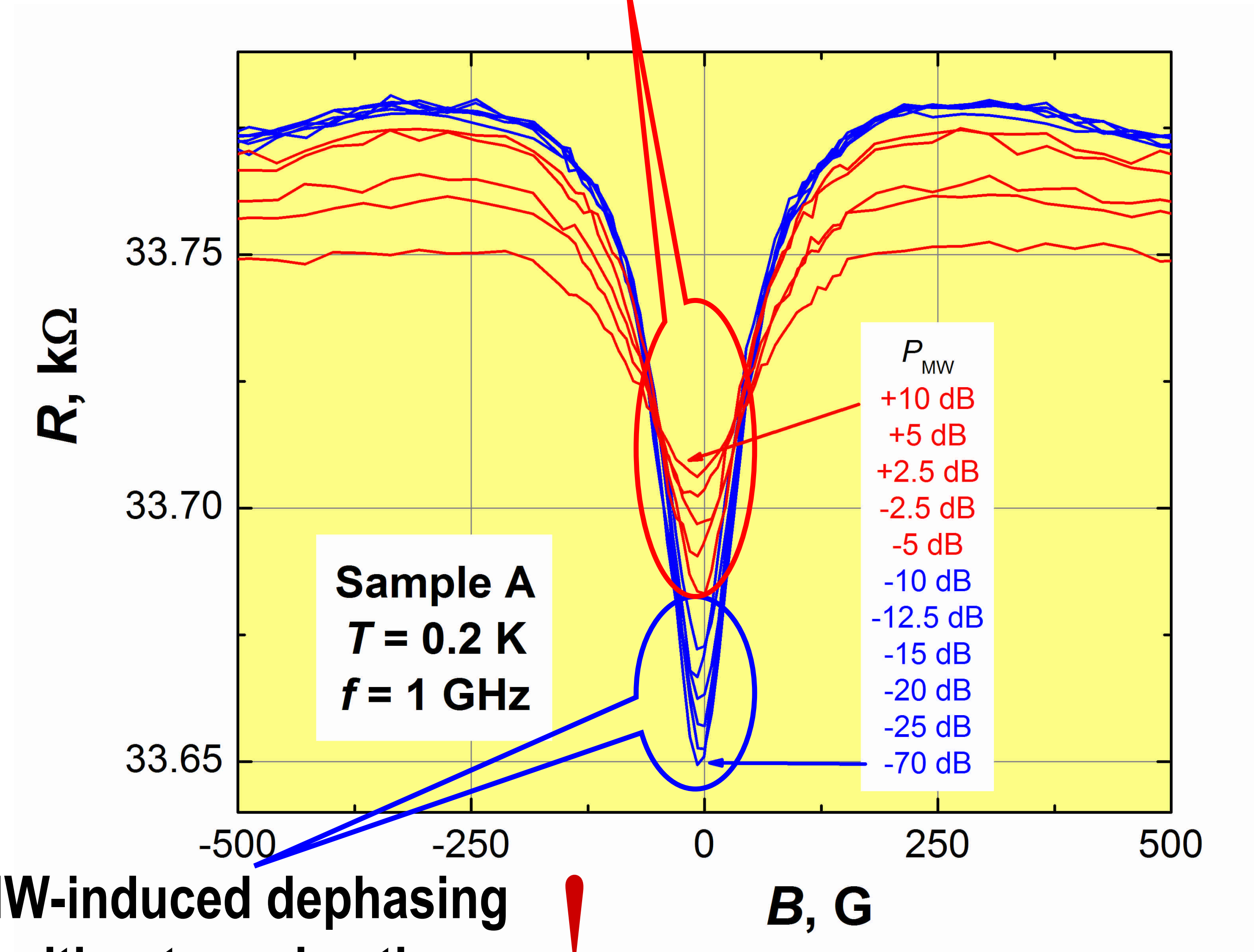
In a strong magnetic field ($L_H << L_j$), $R(T)$ is determined by the interaction corrections $D\sigma_{EEI}(T_e)$.

The measurements of R in strong B provide both the direct measurement of T_e and calibration of the MW power dissipated in the sample, P_{MW}.

Overheating that corresponds to P_{MW} =+10 dB at f = 1 GHz

Effect of Microwave Radiation on the WL MR

MW-induced dephasing
+ overheating



MW-induced dephasing
without overheating

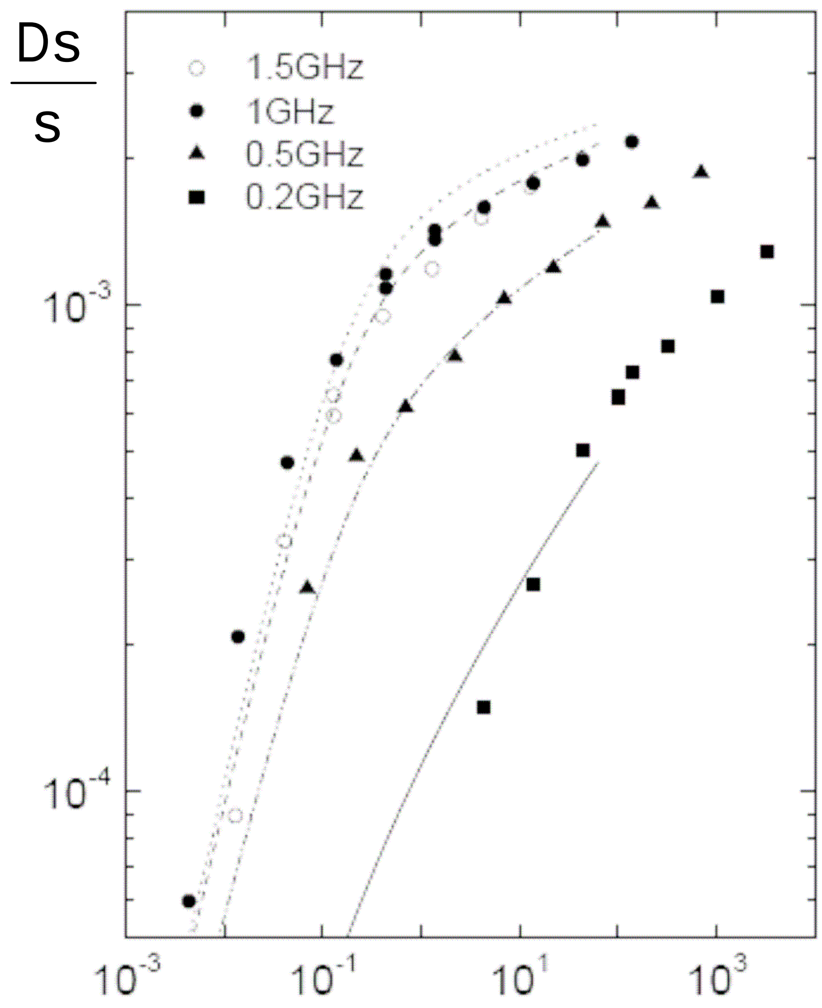
!

Effect of the radiation at zero magnetic field

$$[\mathrm{Ds}_1]_{B=0} = \frac{2e^2}{\hbar}\sqrt{\frac{D}{\mathrm{pW}}}\int_{\mathrm{Wt}_{im}}^{\infty}\frac{dx}{\sqrt{x}}I_0\left(\mathrm{a}\, f(x)\right)$$

$$exp\left(-\mathrm{a}\, f(x) - \frac{2x}{\mathrm{W}}\frac{1}{\mathrm{t}_j}\right)$$

$$f(x) = x\left(1 + \frac{\sin 2x}{2x} - 2\frac{\sin^2 x}{x^2}\right)$$



$$\mathrm{a} = \frac{2e^2 D E^2}{\hbar^2 \mathrm{W}^3}$$

s_1 - conductivity per unit length

W - microwave frequency

D – diffusion constant of electrons

$\boldsymbol{W}$ – width of the wire

$\boldsymbol{I_0(z)}$ – Bessel function of a complex argument

$\boldsymbol{E}$ – amplitude of the el. field in the microwave

Radiation effect on magnetoresistance

$$\left[\frac{\partial^2 \sigma_1}{\partial B^2}\right]_{B=0} = \frac{8e^4}{3\hbar^3}\sqrt{\frac{D^3}{\pi\Omega^3}}W^2 \int_{\Omega\tau_{im}}^{\infty} dx\, I_0\left(\alpha f(x)\right)$$

$$\sqrt{x}\, exp\left(-\alpha f(x) - \frac{2x}{\Omega}\frac{1}{\tau_\varphi}\right)$$

$$f(x) = x\left(1 + \frac{sin\, 2x}{2x} - 2\frac{sin^2 x}{x^2}\right)$$

σ_1 - conductivity per unit length

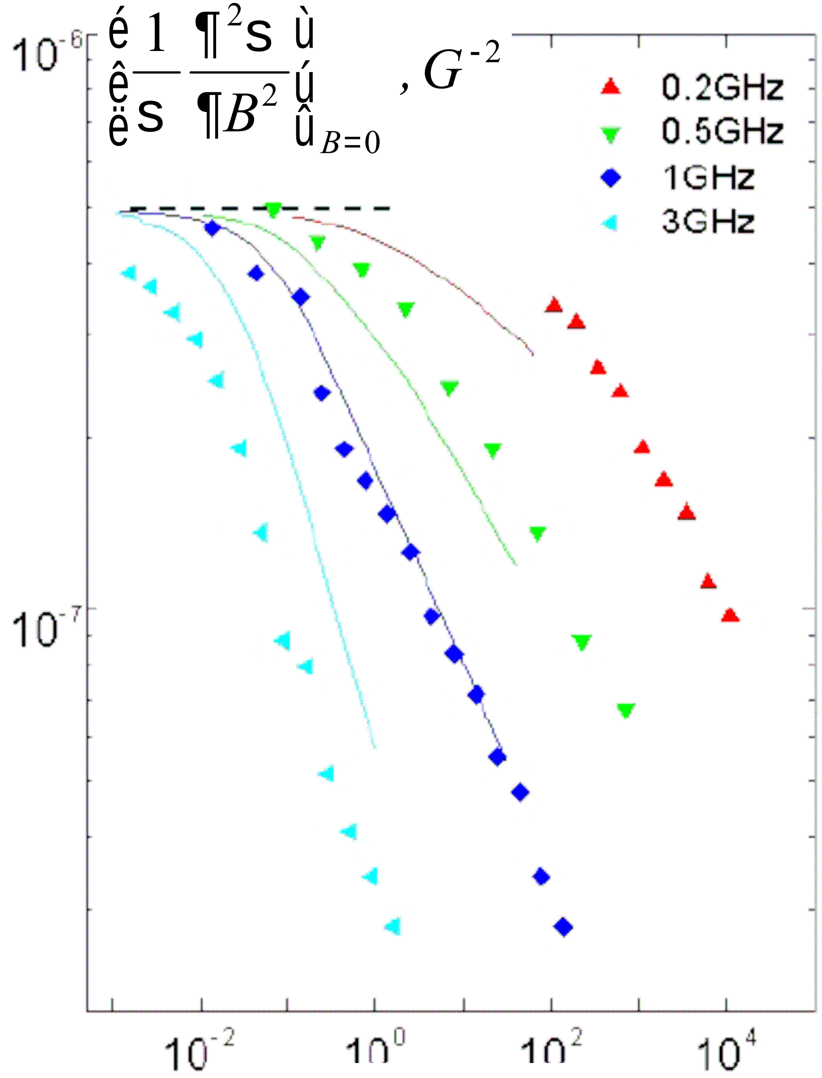
Ω - microwave frequency

D – diffusion constant of electrons

W – width of the wire

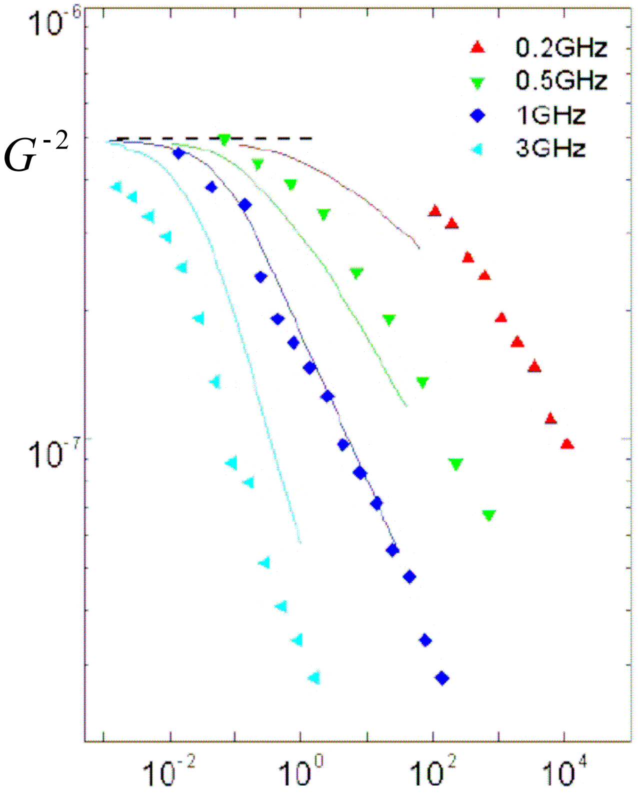
$I_0(z)$ – Bessel function of a complex argument

E – amplitude of the el. field in the microwave



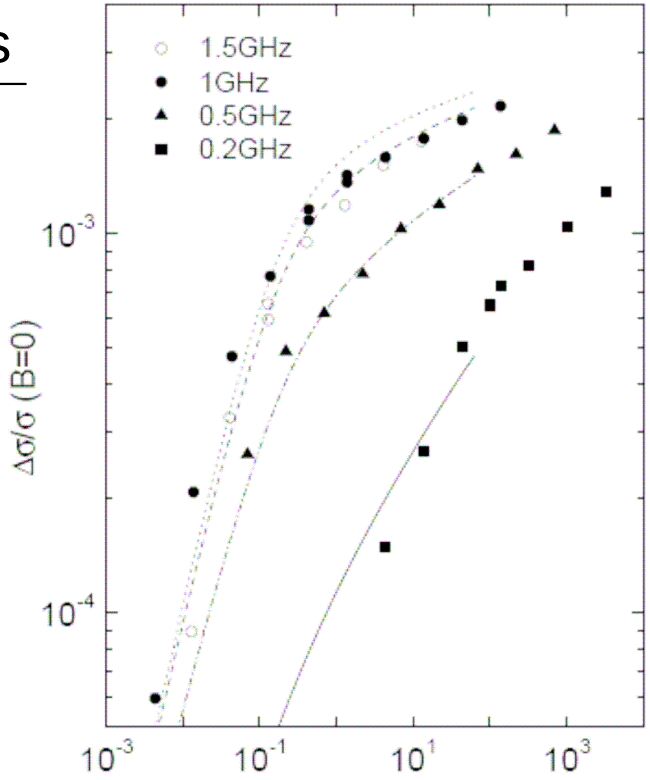
$$\alpha = \frac{2e^2 D E^2}{\hbar^2 \Omega^3}$$

$$\left[\frac{1}{\sigma}\frac{\partial^2 \sigma}{\partial B^2}\right]_{B=0},\ G^{-2}$$



$$\alpha = \frac{2e^2 D E^2}{\hbar^2 \Omega^3}$$

$$\frac{\Delta\sigma}{\sigma}$$

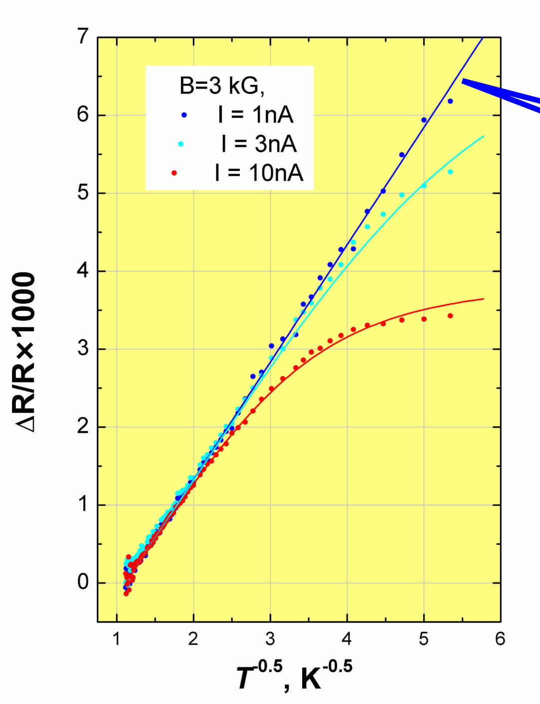


$$\alpha = \frac{2e^2 D E^2}{\hbar^2 \Omega^3}$$

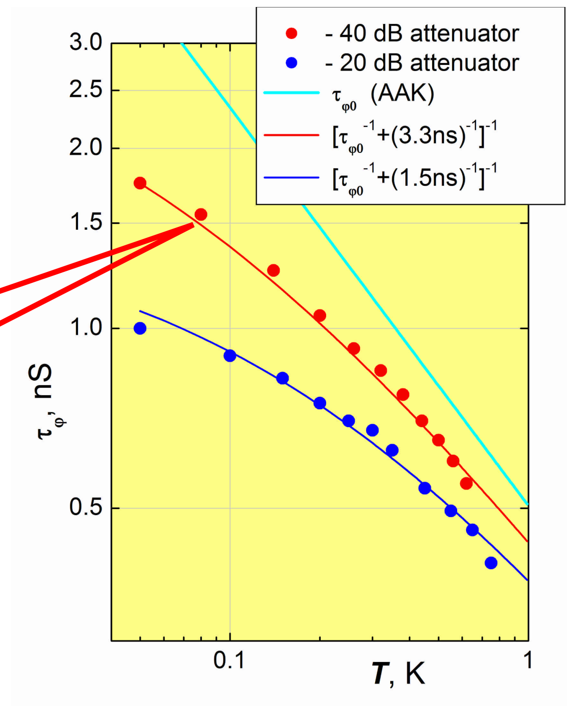
$$\Delta\sigma_1 = \frac{2e^2}{\hbar}\sqrt{\frac{D}{\pi\Omega}}\int_{\Omega\tau_{im}}^{\infty}\frac{dx}{\sqrt{x}}I_0\left(\alpha f(x)\right)exp\left(-\alpha f(x)-\frac{2x}{\Omega}\left(\frac{1}{\tau_\varphi}+\frac{D}{3\hbar^2c^3}(eBW)^2\right)\right)$$

No adjustable parameters !

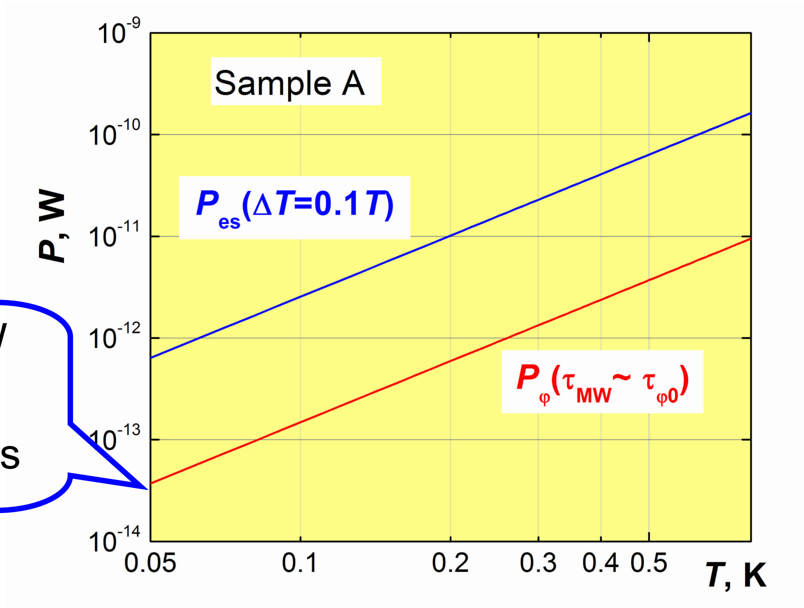
Low-T saturation of τ_φ



the upper bound on the external noise power ~ $3\cdot10^{-14}$ W



$$\tau_\varphi^{-1}(T)=\tau_{\varphi0}^{-1}(T)+(3.3\text{ ns})^{-1}$$



$P_{MW} = 3\cdot10^{-14}$ W leads to $\tau_{MW}\sim \tau_\varphi$(50 mK)=3.7 ns

Conclusion: at least in this experiment, the reason for $\tau_\varphi(T)$ saturation may be the external electromagnetic noise

Saturation of the dephasing rate

- Birge, Pothier et al – Magnetic impurities
- Gershenson et al - External noise
- Mohanty & Webb - ????

No reason to expect zero-temperature dephasing by zero-point oscillations

One-particle excitations in finite closed systems

Can one localize a quasiparticle

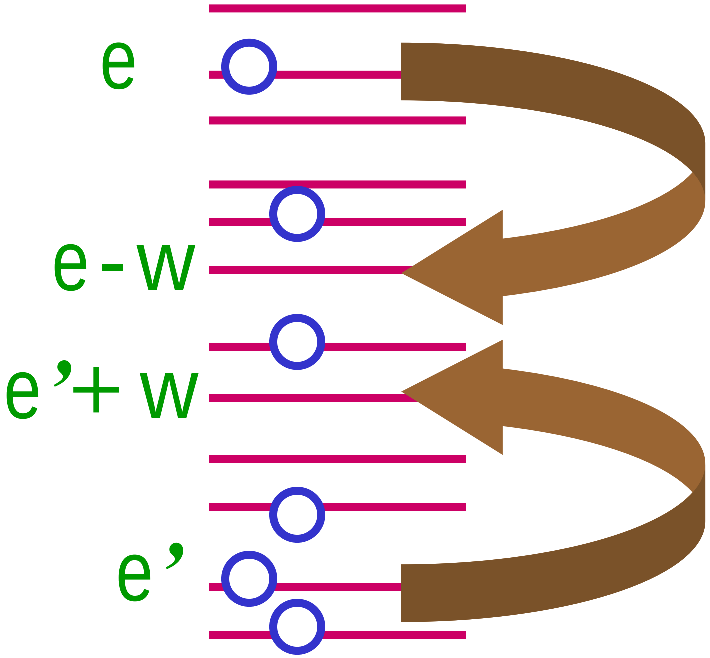
Wigner-Dyson random matrix statistics follows from the delocalization.

Q: ***Why the random matrix theory (RMT) works so well for nuclear spectra*** **?**

Spectra of Many-Body excitations !

Quasiparticle relaxation rate in 0D case

T=0



Offdiagonal matrix element

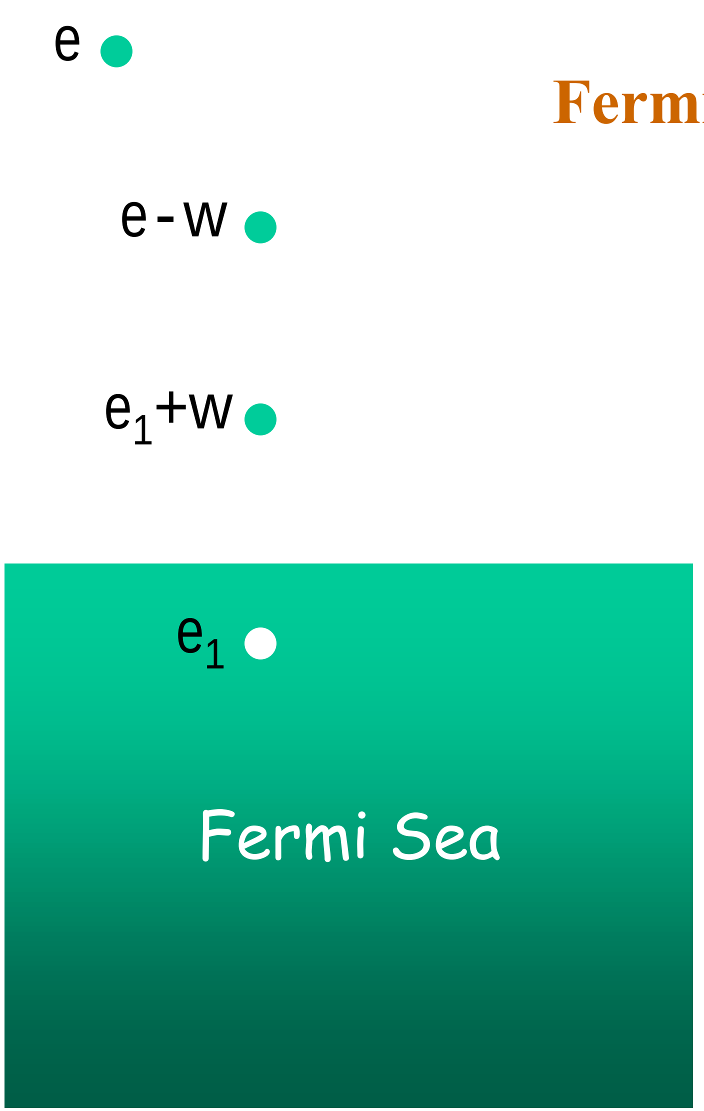
$$M(\omega,\varepsilon,\varepsilon') \propto \frac{\delta_1}{g} << \delta_1$$

Problem: in a discrete random spectrum it is impossible to satisfy the conservation law exactly ! Probability that

$$\varepsilon_\alpha + \varepsilon_\beta = \varepsilon_\gamma + \varepsilon_\delta$$

equals to zero

Decay of a quasiparticle with an energy ϵ in Landau Fermi liquid



Fermi Golden rule: $\gamma(\epsilon) \propto \delta_1 \left(\frac{\epsilon}{E_T}\right)^2$

Mean level spacing (δ_1)

Thouless energy (E_T)

zero-dimensional case

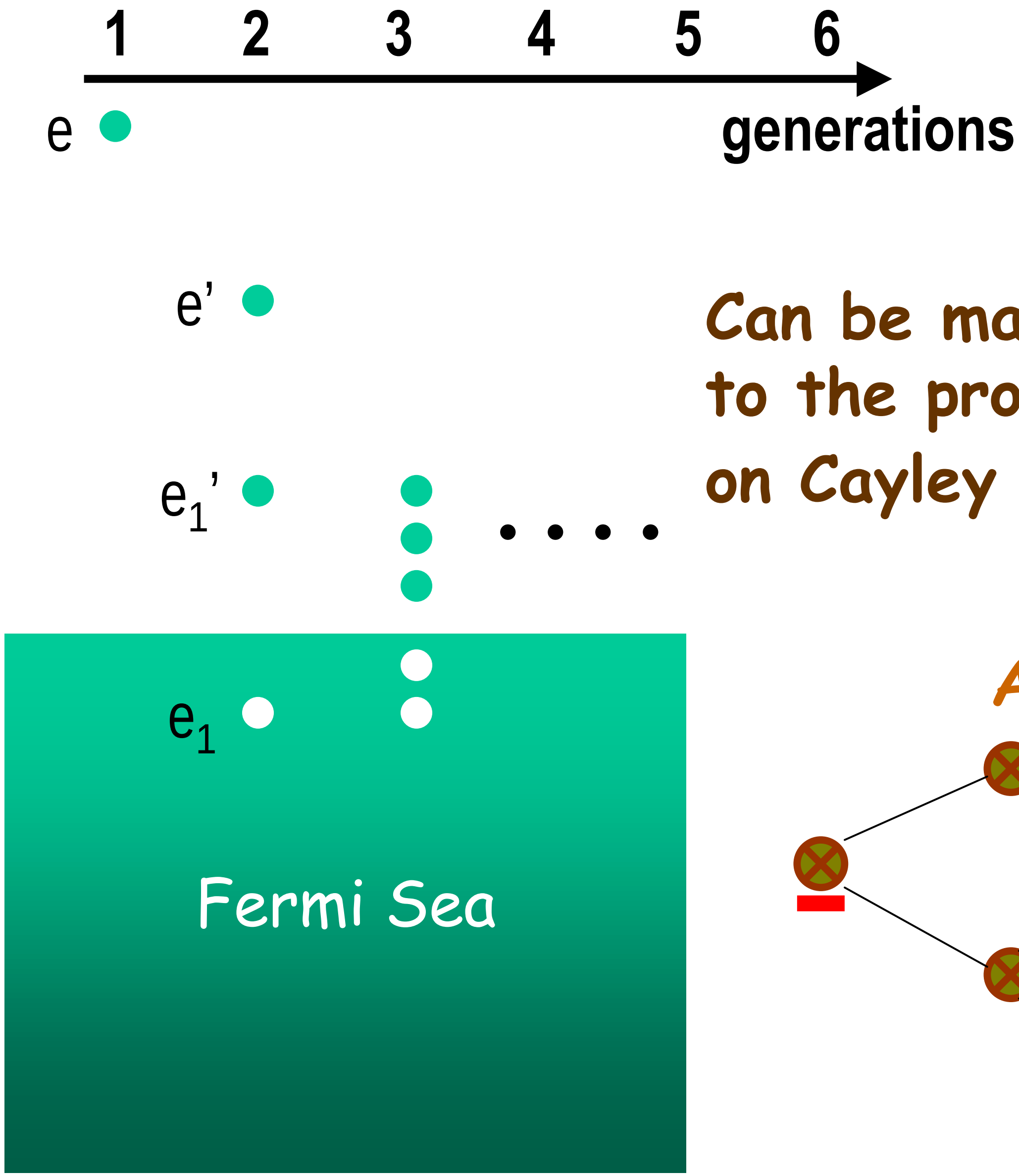
↓

one-particle spectrum is **discrete**

↓

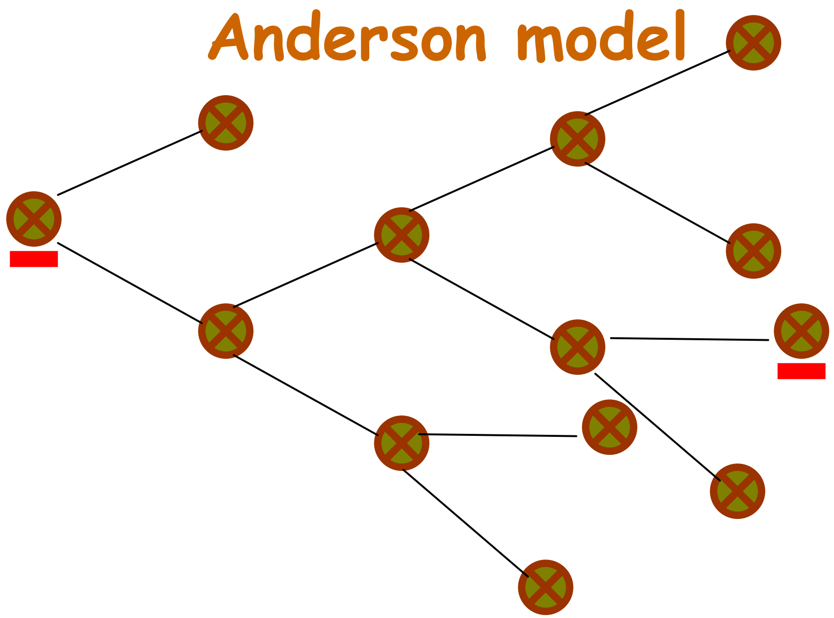
equation $\epsilon_1 + \epsilon_2 = \epsilon'_1 + \epsilon'_2$ can not be satisfied exactly

Chaos in Nuclei – Delocalization?

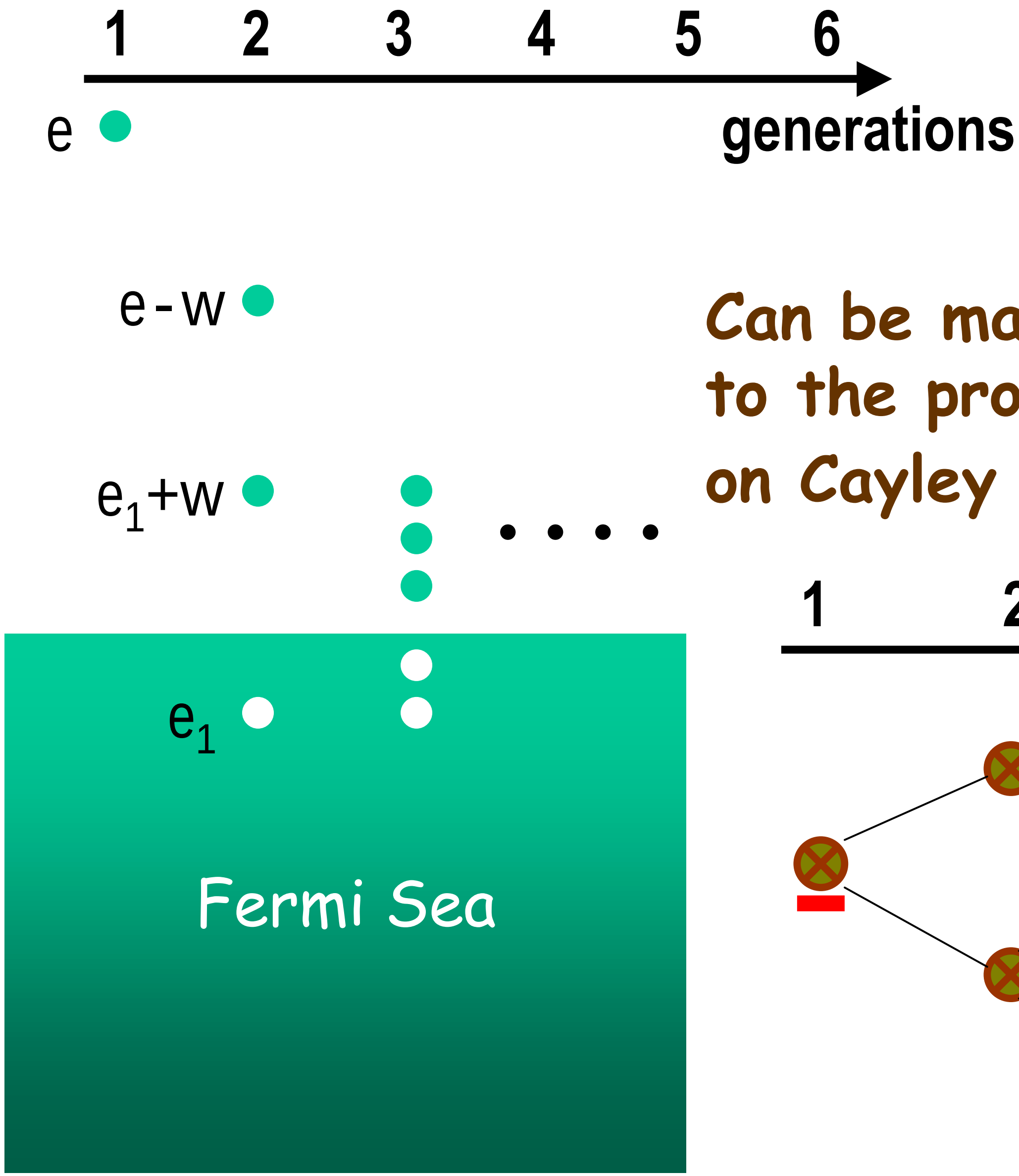


Delocalization in Fock space

Can be mapped (approximately) to the problem of localization on Cayley tree



Chaos in Nuclei – Delocalization?



Delocalization in Fock space

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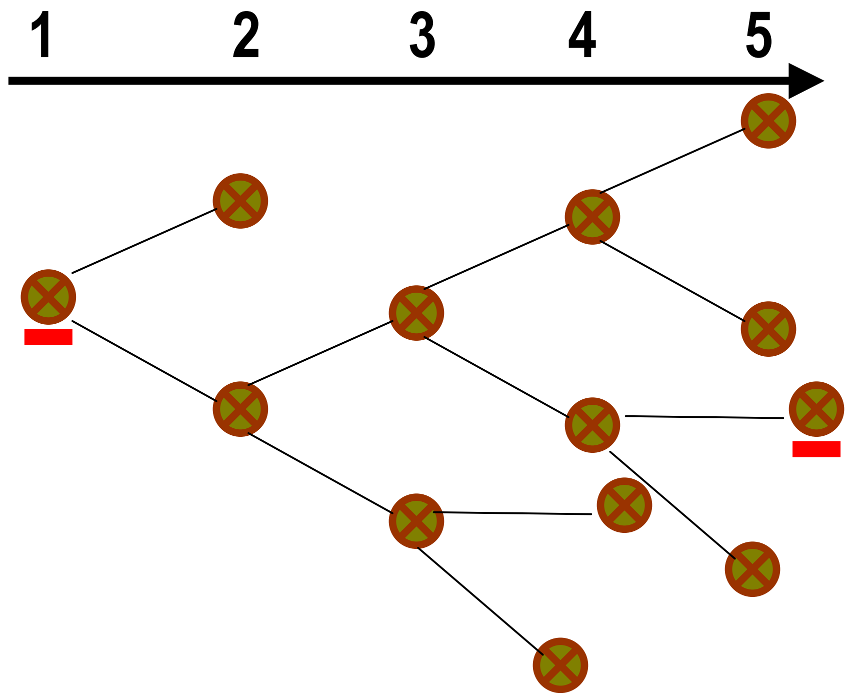

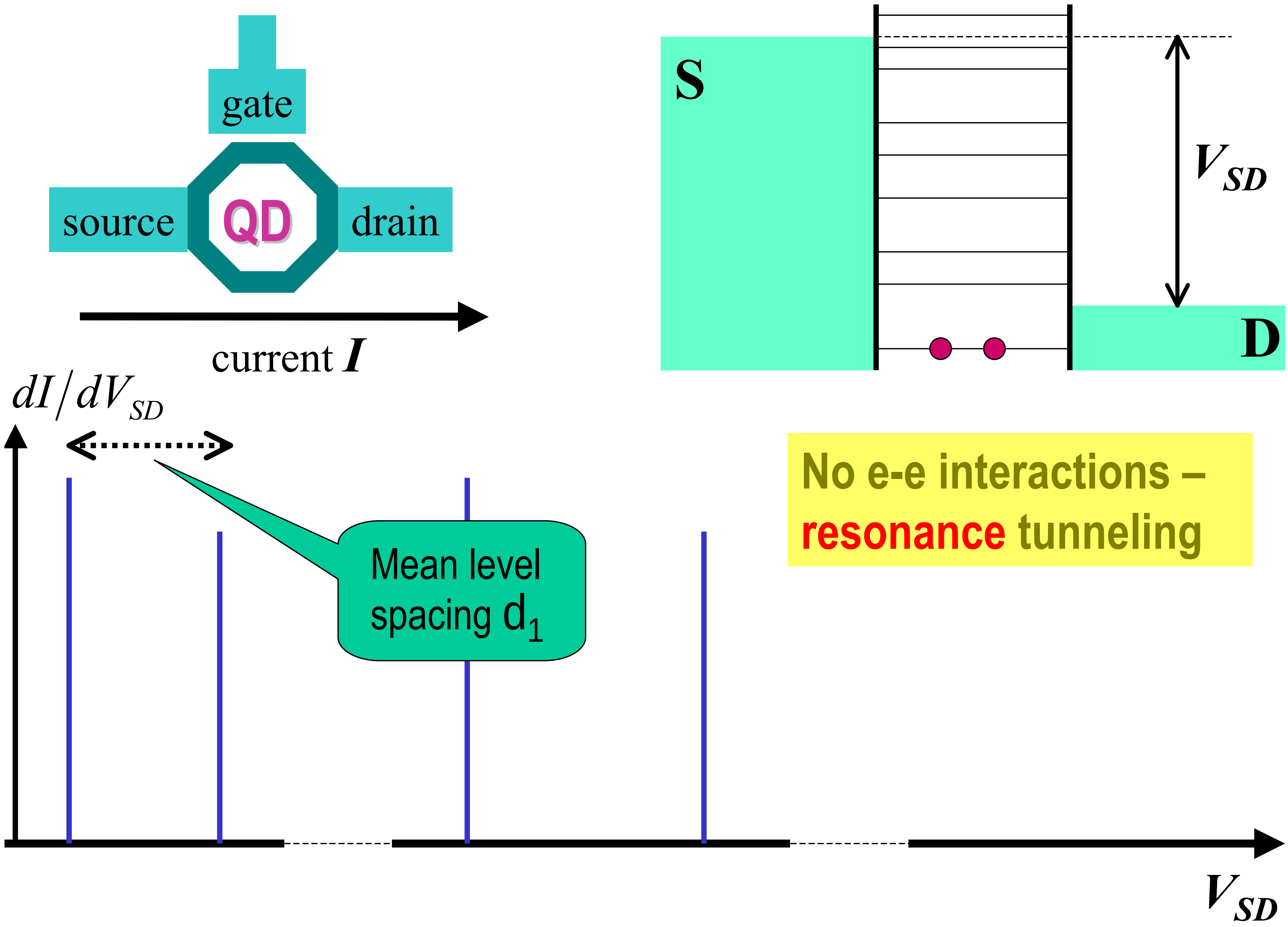
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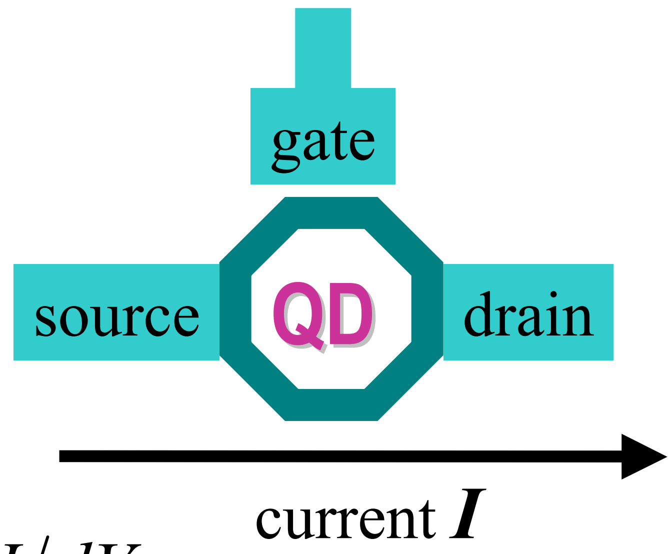
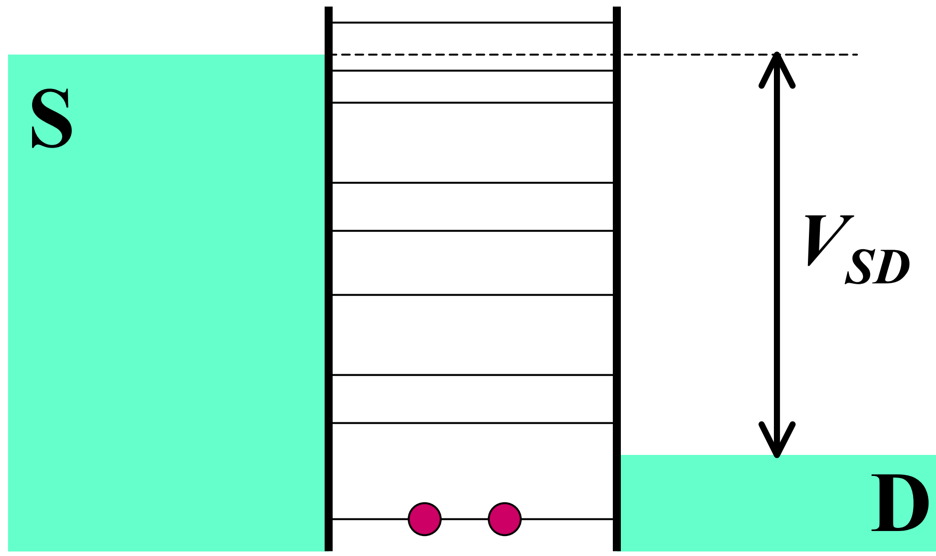
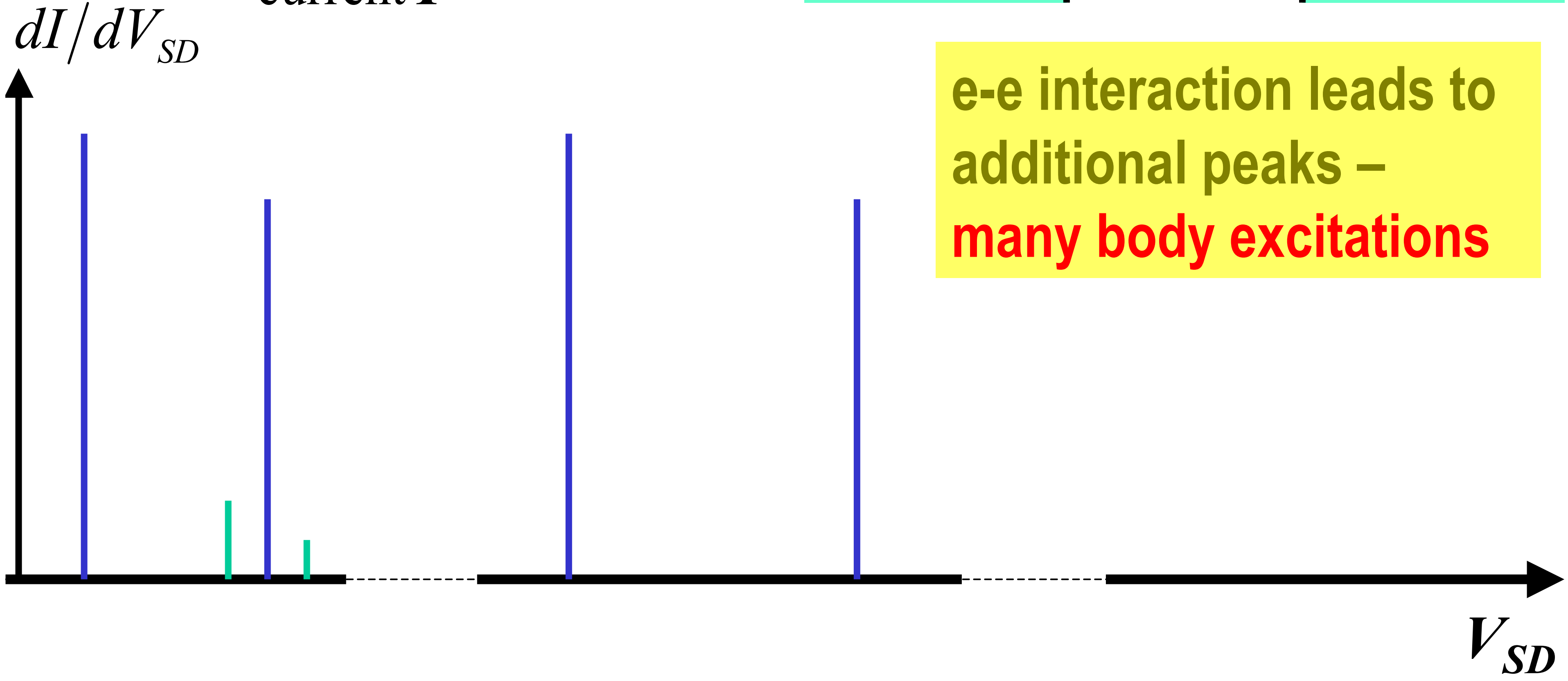
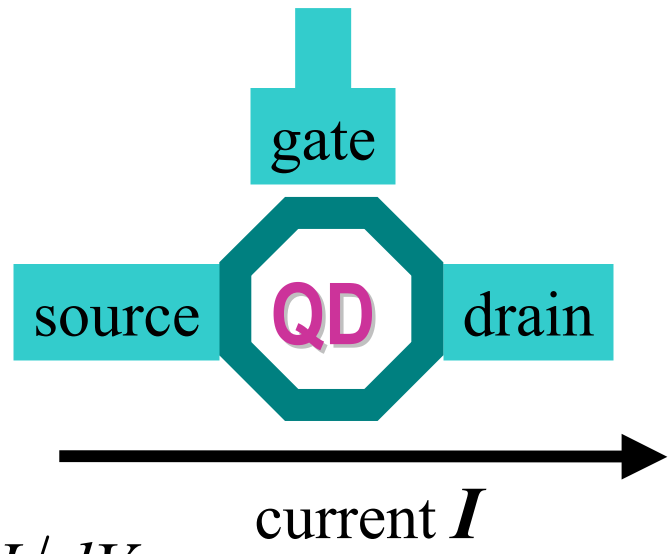
source
QD
drain
current I
S
D
V_SD
dI/dV_SD
Mean level spacing d1
No e-e interactions –
resonance tunneling
V_SD

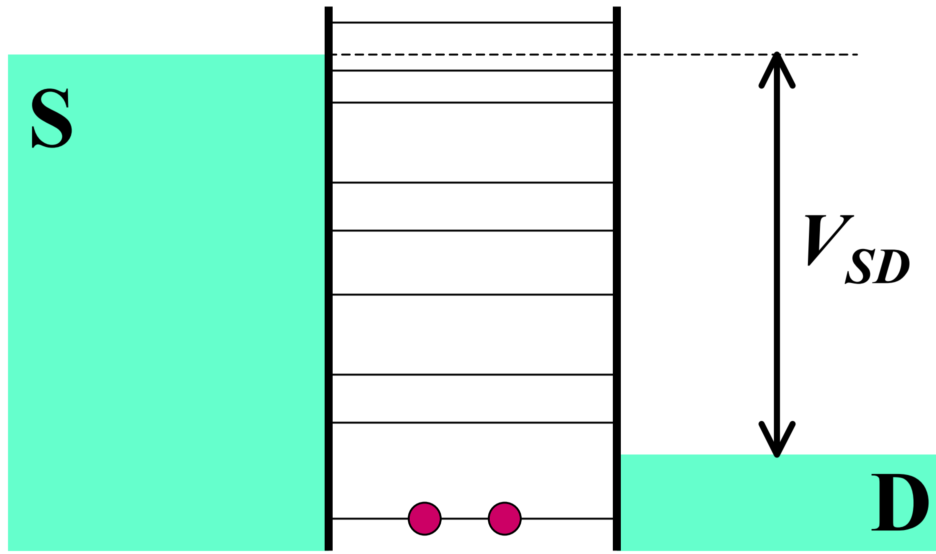


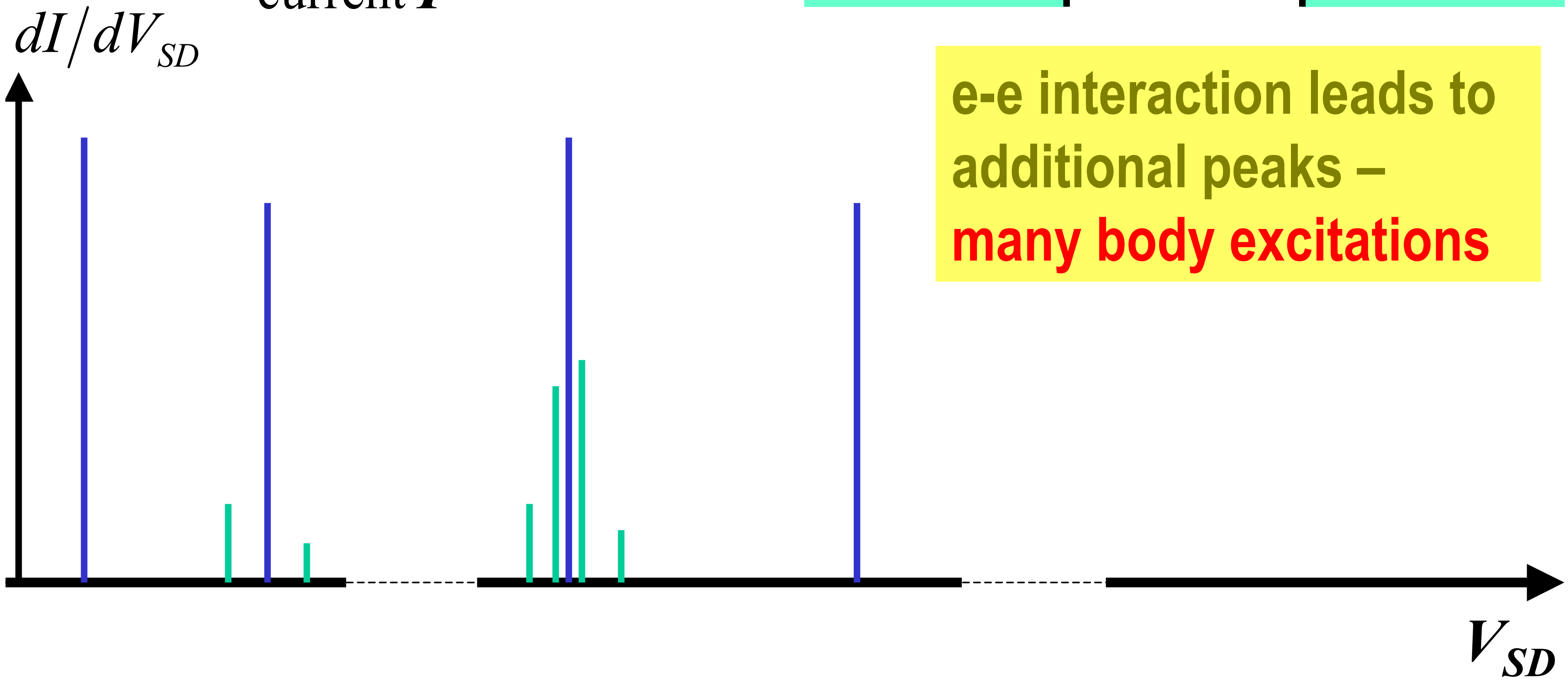


e-e interaction leads to additional peaks –
many body excitations

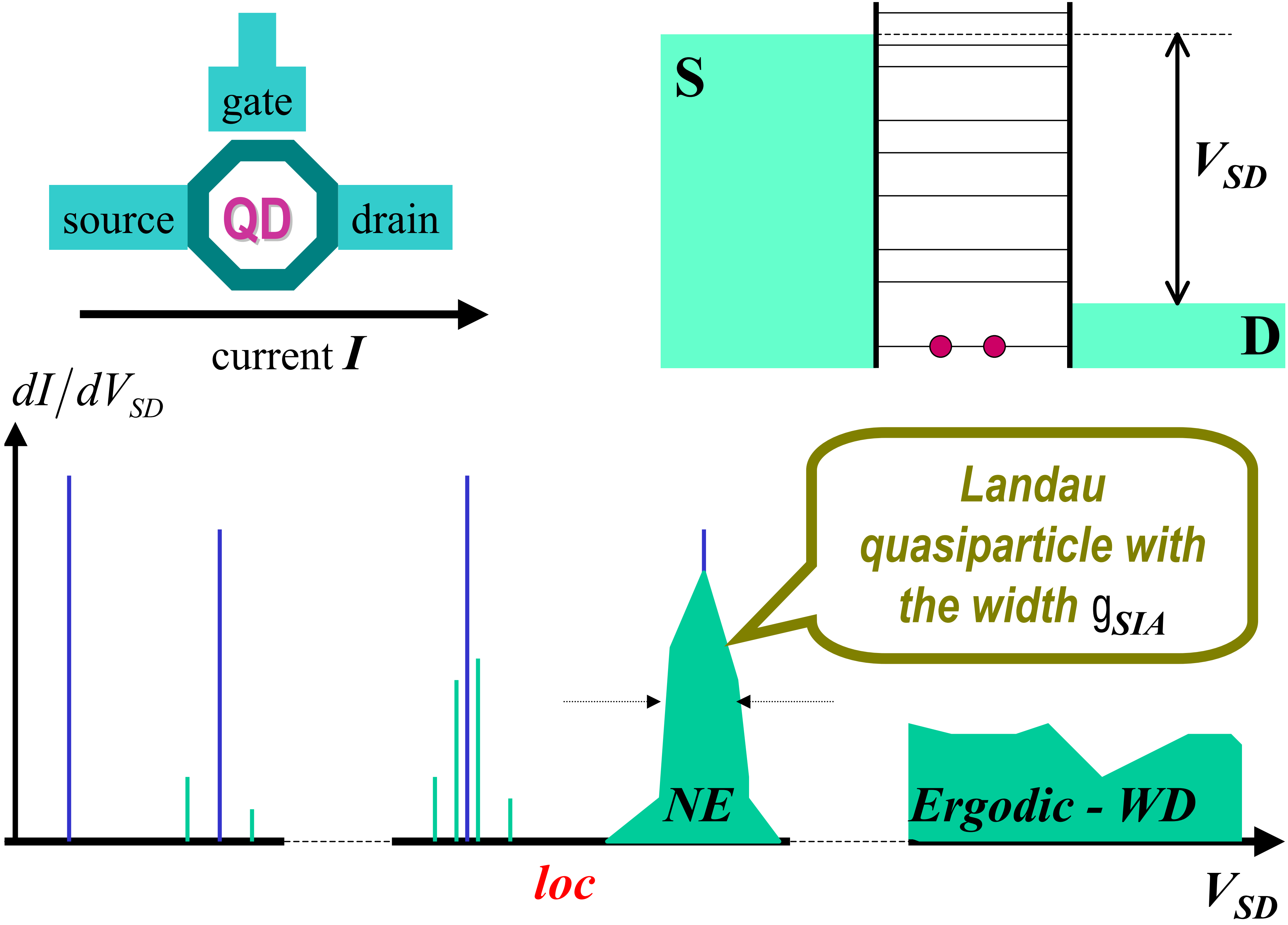








e-e interaction leads to additional peaks – many body excitations


gate
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dI/dV_SD
Landau quasiparticle with the width g_SIA
NE
Ergodic - WD
loc
V_SD

Many Body Localization

Can hopping conductivity exist without phonons ?

Problem: can *e-e* interaction **alone** sustain hopping conduction in a localized system?

Given:

1. All one-electron states are localized
2. Electrons interact with each other
3. The system is closed (no phonons)
4. Temperature is low but finite

Find: DC conductivity $s(T, w=0)$
(zero or finite?)

"All states are *localized* "

means

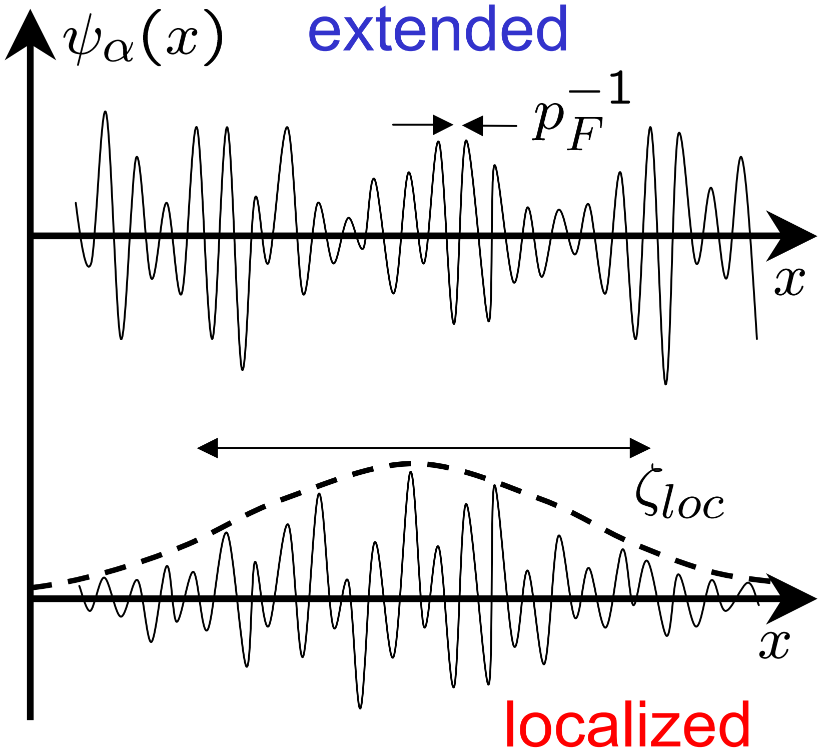
Probability to find an extended state:

$$\mathcal{P}_{ext} \propto \exp\left(-\#\frac{L}{\zeta_{loc}}\right)$$

System size

1. Localization of single-electron wave-functions:

$$\left[-\frac{\boldsymbol{\nabla}^2}{2m}+U(\boldsymbol{r})-\epsilon_F\right]\psi_\alpha(\boldsymbol{r})=\xi_\alpha\psi_\alpha(\boldsymbol{r})$$



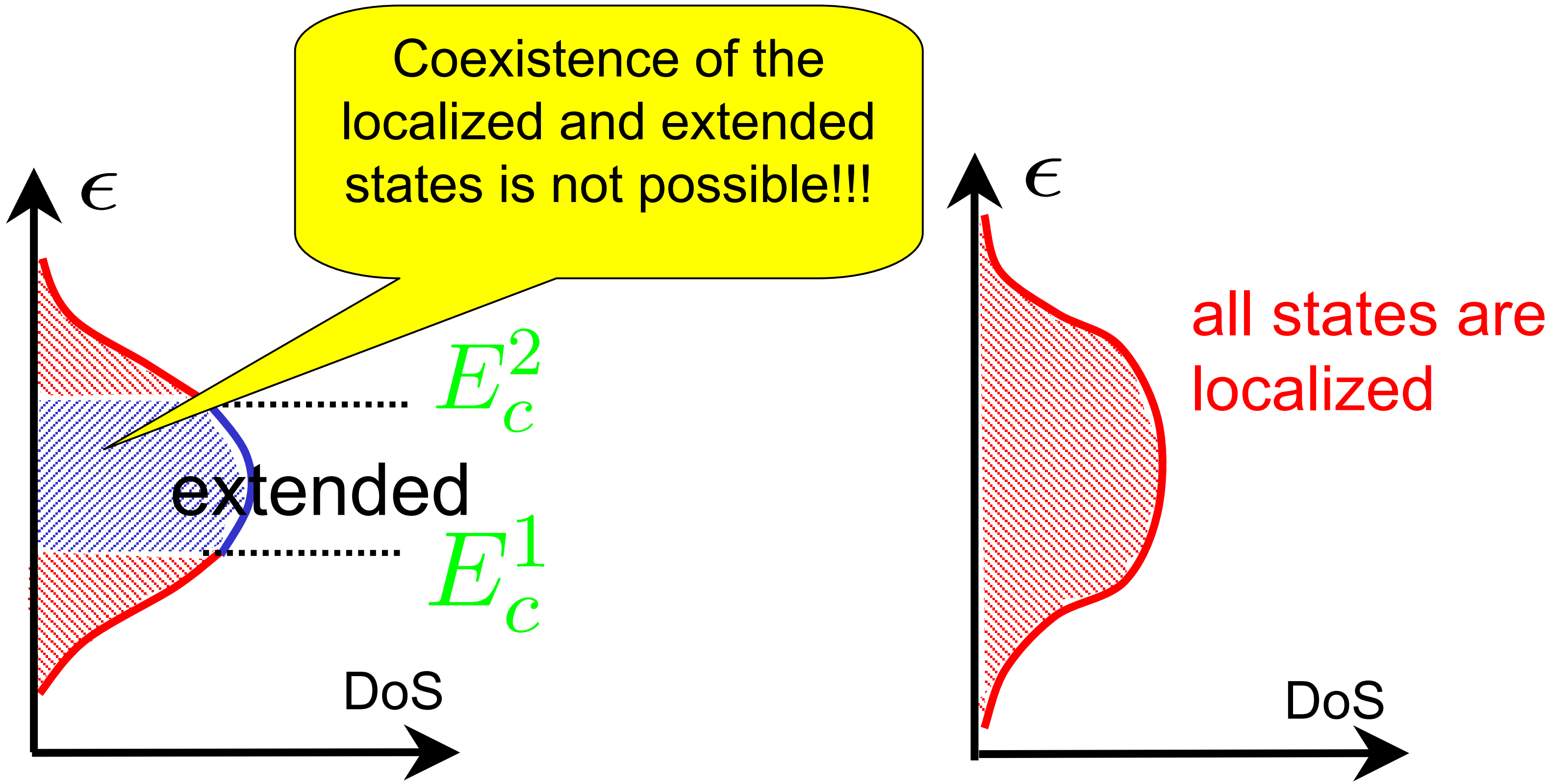
d=1; All states are *localized*

d=2; All states are *localized*

d>2; ***Anderson transition***

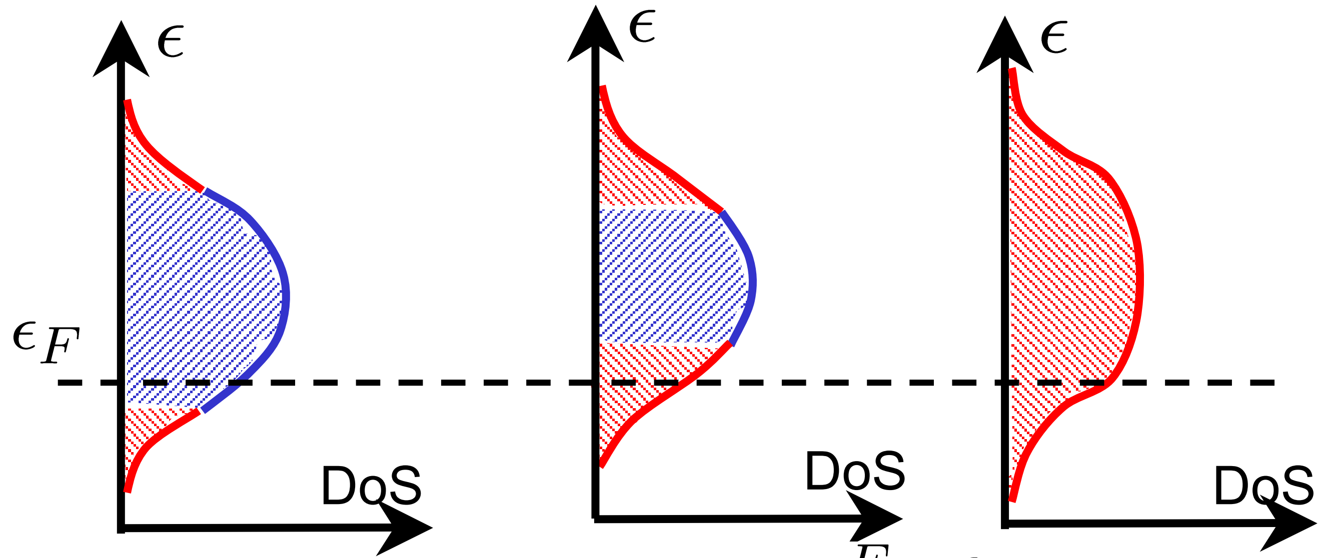
Anderson model; Anderson Transition

$I > I_c$ $\qquad\qquad\qquad\qquad$ $I < I_c$



E_c - mobility edges (one particle)

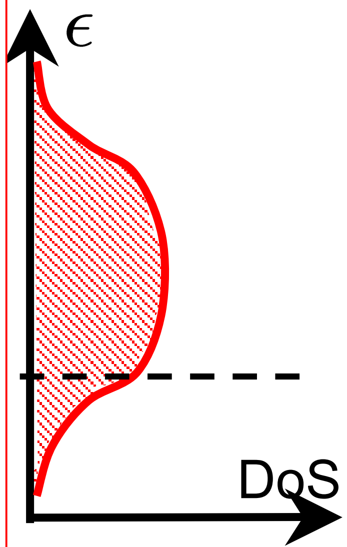
Temperature dependence of the conductivity of noninteracting electrons



$\sigma(T \to 0) > 0$ $\sigma(T) \propto e^{-\frac{E_c - \epsilon_F}{T}}$ $\sigma(T) = 0$

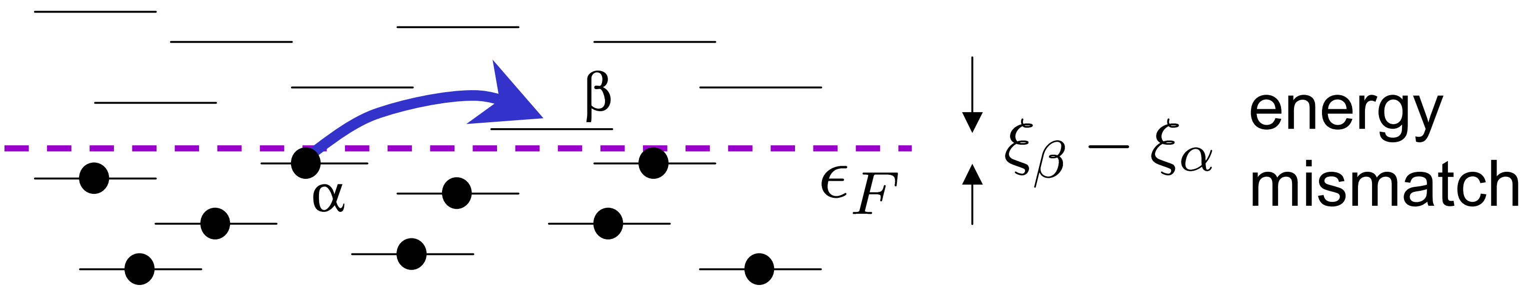
Temperature dependence of the conductivity of noninteracting electrons

Assume that all the states are *localized*



$$\sigma(T) = 0$$

Inelastic processes) transitions between localized states

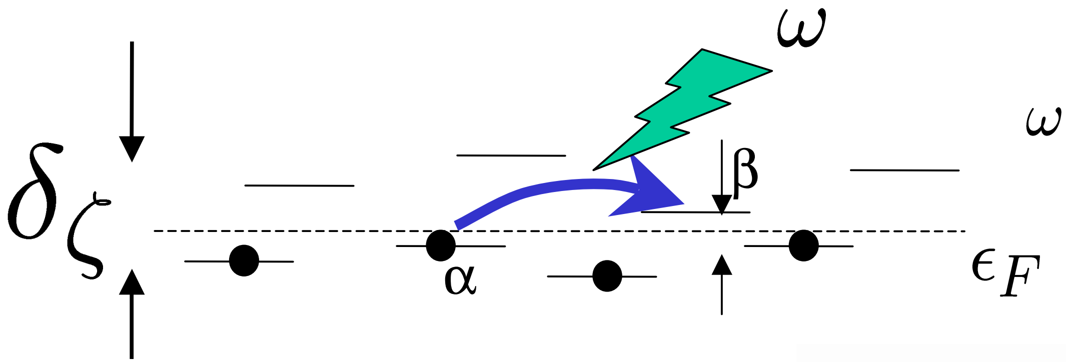


$\sigma(T) \propto \Gamma_\alpha$ **(inelastic lifetime)$^{-1}$**

$T = 0 \quad \Rightarrow \quad \sigma = 0$ **(any mechanism)**

$T > 0 \quad \Rightarrow \quad \sigma = ?$

Phonon-induced hopping



$$\omega = \xi_\beta - \xi_\alpha \ll \delta_\zeta$$

energy difference can be matched by a phonon

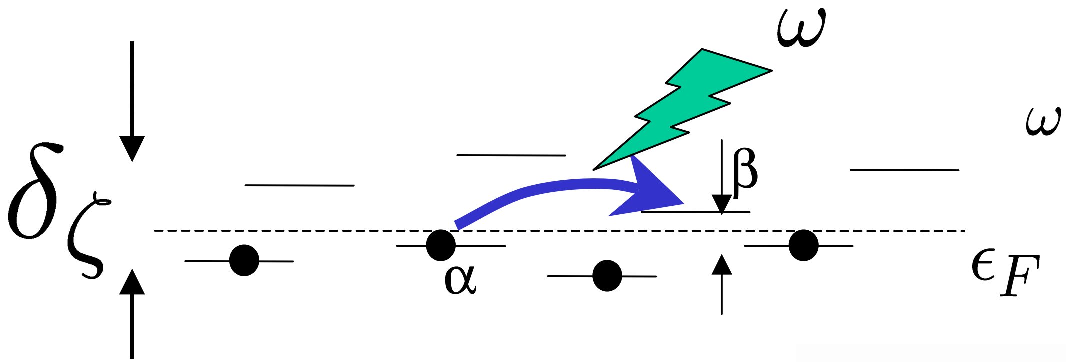
Variable Range Hopping
Sir N.F. Mott (1968)

$$\sigma(T) \propto T^\gamma \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

Mechanism-dependent prefactor

Without Coulomb gap
A.L.Efros, B.I.Shklovskii (1975)

Phonon-induced hopping



$$\omega = \xi_\beta - \xi_\alpha \ll \delta_\zeta$$

Variable Range Hopping
Sir N.F. Mott (1968)

$$\sigma(T) \propto T^\gamma \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

Mechanism-dependent prefactor

Optimized phase volume

Any bath with a continuous spectrum of delocalized excitations down to w = *0* will give the same exponential

Q: Can we replace phonons with e-h pairs and obtain **phonon-less** *VRH?*

A#1: Sure

Easy steps:

1) Recall phonon-less AC conductivity:

Sir N.F. Mott (1970)

$$\sigma\left(\omega\right)\simeq\frac{e^2\zeta_{loc}^{d-2}}{\hbar}\left(\frac{\hbar\omega}{\delta_\zeta}\right)^2\ln^{d+1}\left|\frac{\delta_\zeta}{\hbar\omega}\right|$$

2) Calculate the Nyquist noise.

3) Use the electric noise instead of phonons.

4) Do self-consistency (whatever it means).

Q: Can we replace phonons with e-h pairs and obtain **phonon-less** *VRH?*

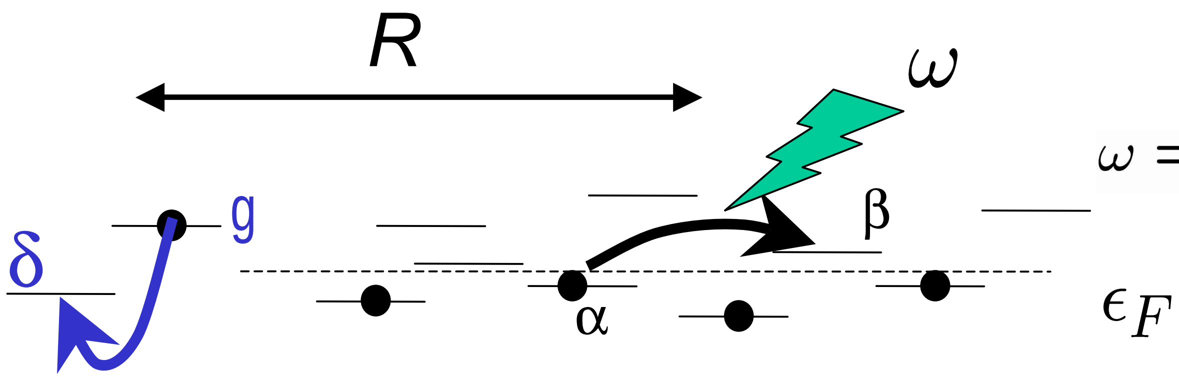
A#1: Sure

A#2: No way [L. Fleishman. P.W. Anderson (1980)]

(for Coulomb interaction in 3D – may be)

$$\sigma(\omega) \simeq \frac{e^2 \zeta_{loc}^{d-2}}{\hbar} \left(\frac{\hbar\omega}{\delta_\zeta}\right)^2 \ln^{d+1}\left|\frac{\delta_\zeta}{\hbar\omega}\right|$$

is contributed by rare resonances



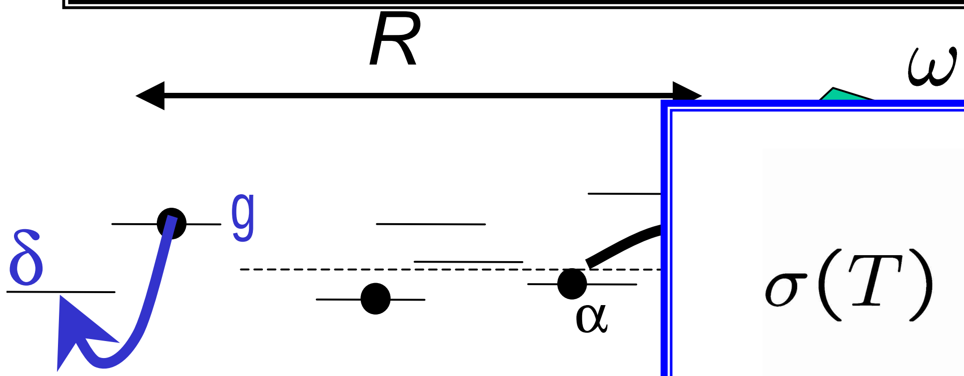
$$\omega = \xi_\beta - \xi_\alpha = \xi_\gamma - \xi_\delta$$

Q: Can we replace phonons with e-h pairs and obtain **phonon-less** *VRH?*

A#1: Sure

A#2: No way [L. Fleishman. P.W. Anderson (1980)]
(for Coulomb interaction in 3D – may be)

$R \longrightarrow \infty$ ***Thus, the matrix element vanishes !!!***



$$\sigma(T) \propto 0 * \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

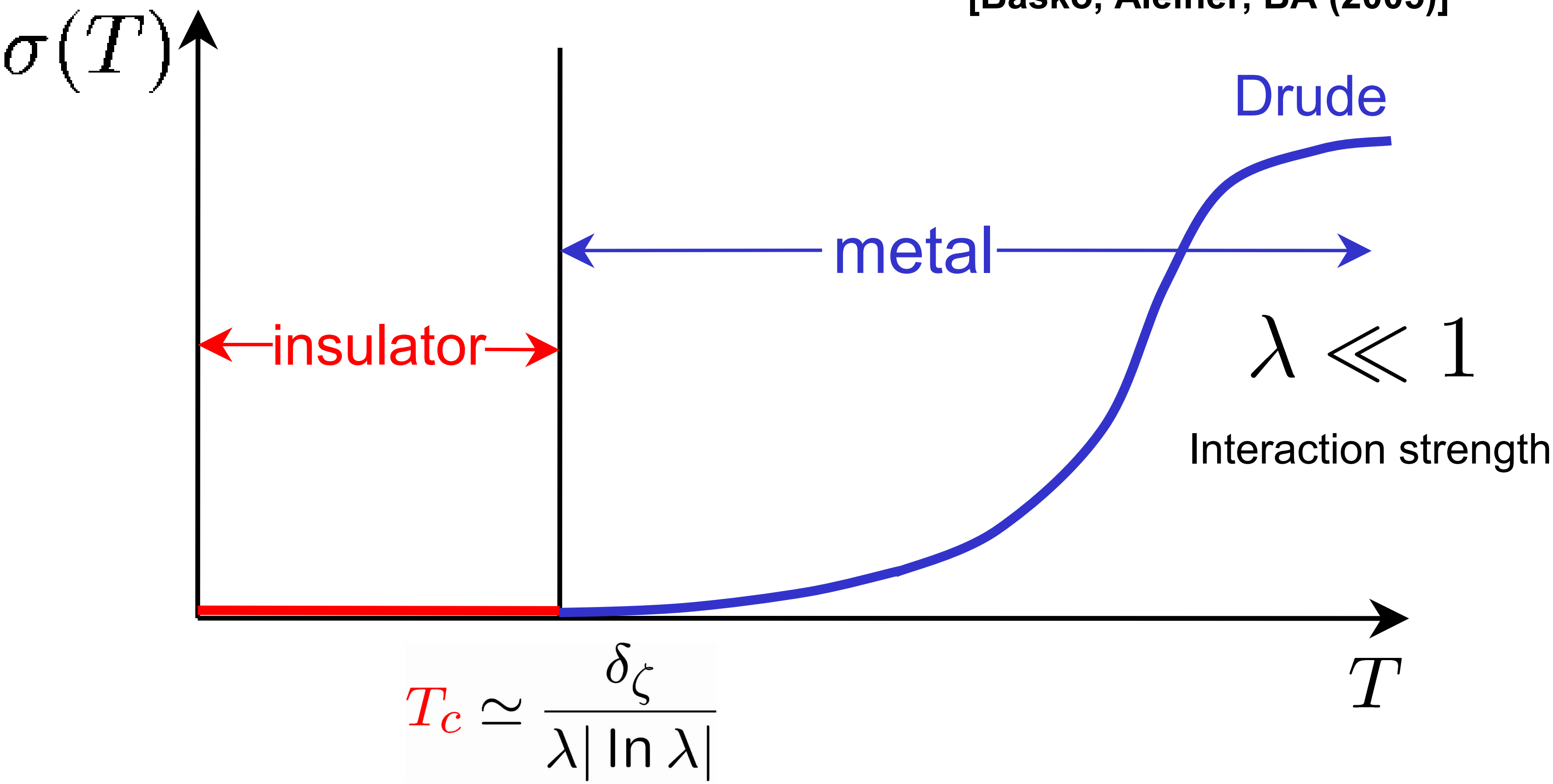
Q: Can we replace phonons with e-h pairs and obtain **phonon-less** *VRH?*

A#1: Sure [a person from the street (2005)]:

A#2: No way [L. Fleishman. P.W. Anderson (1980)]

A#3: Finite T *Metal-Insulator Transition*

[Basko, Aleiner, BA (2005)]

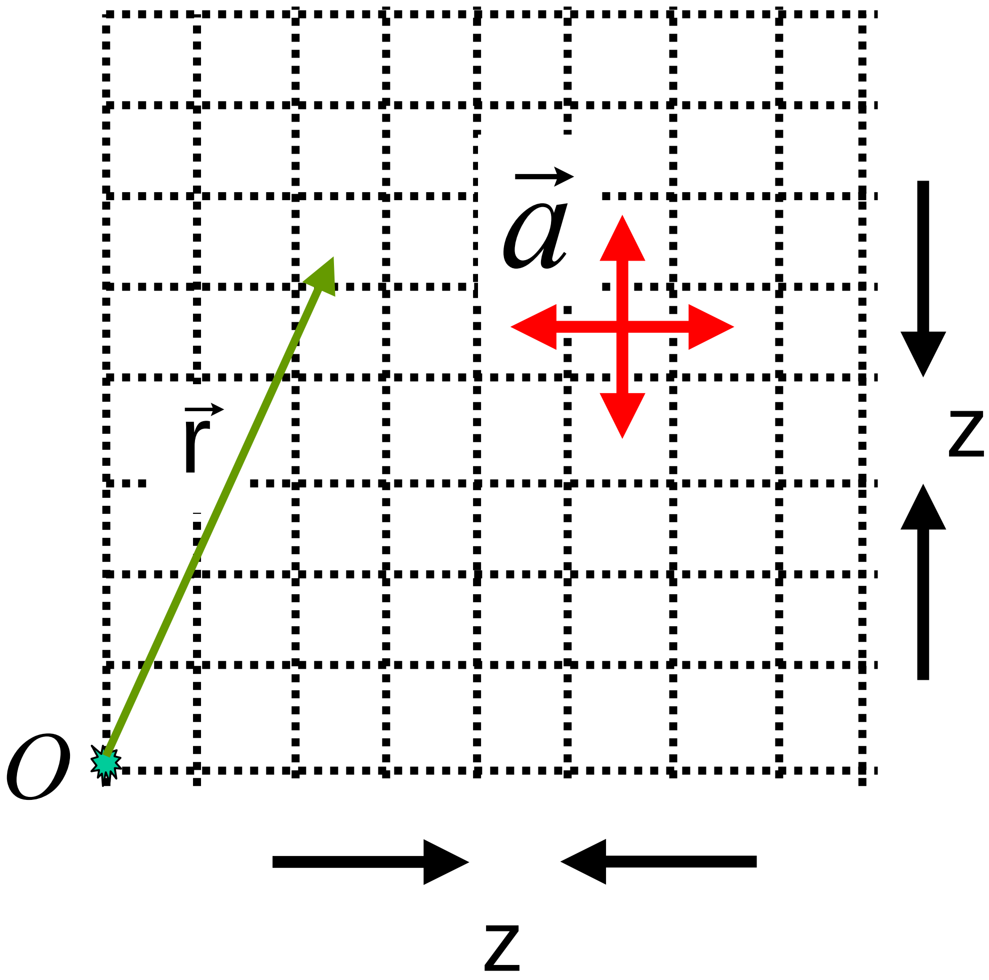


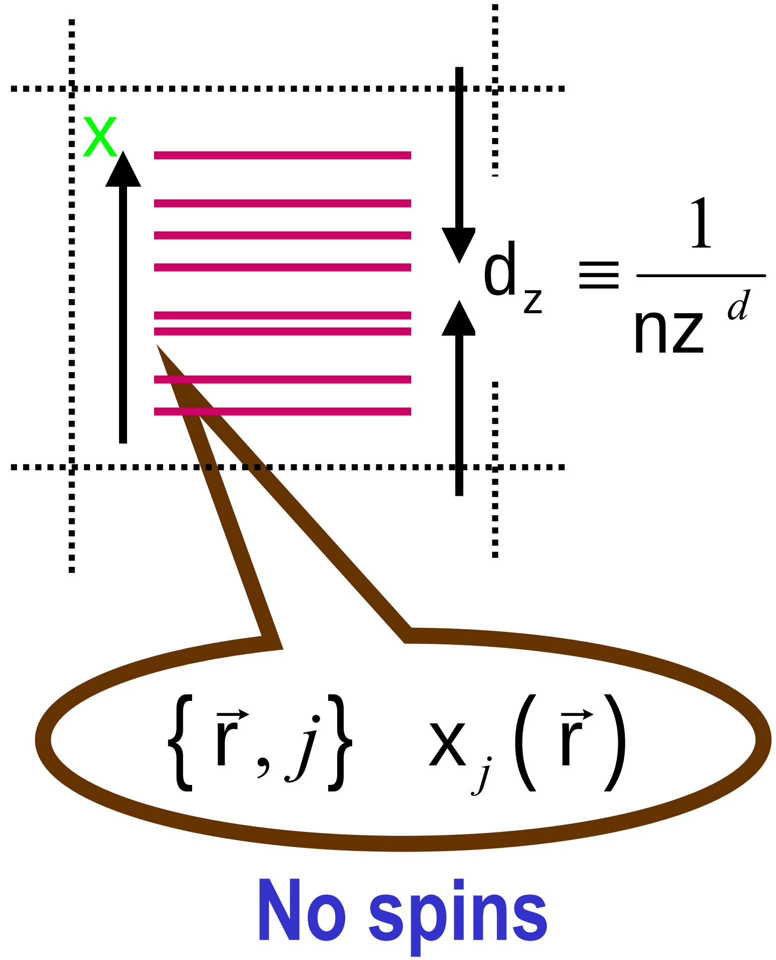
We have to take into account that

1. A one-electron wave function decays exponentially as a function of the distance from its center.

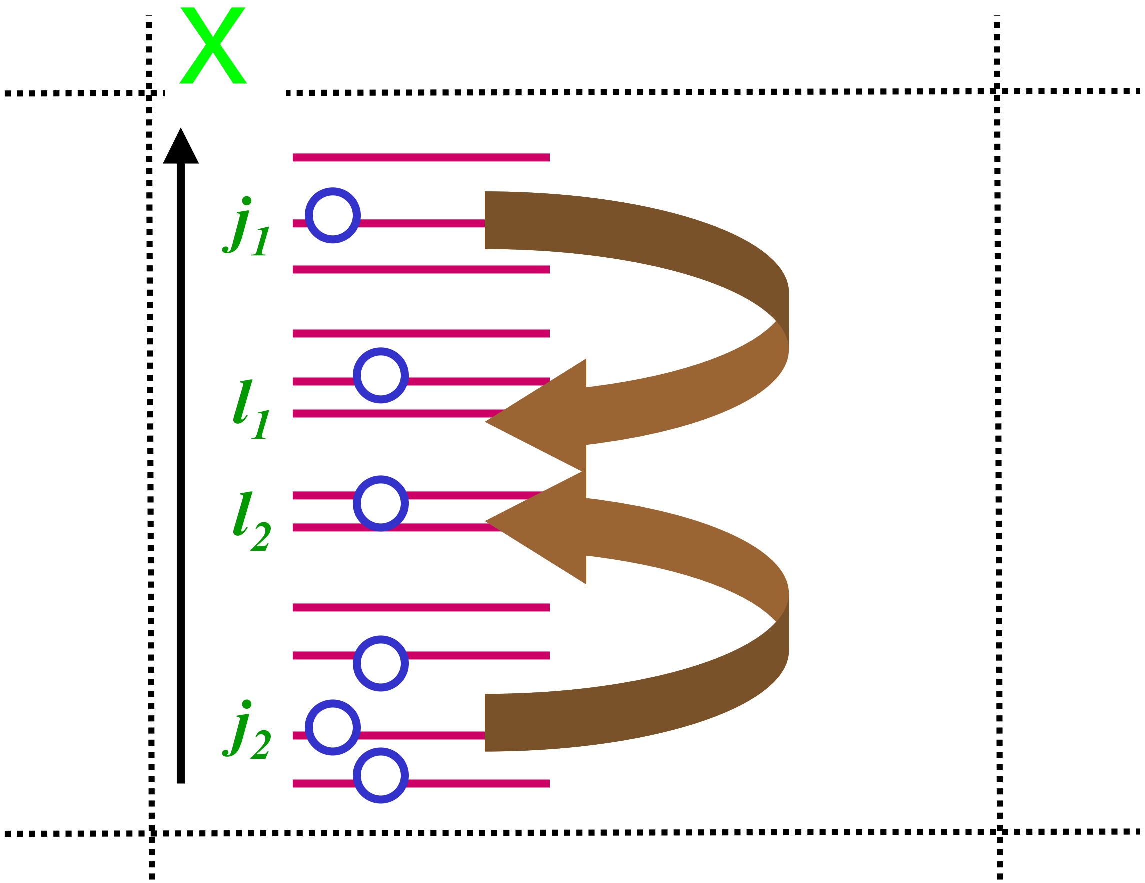
2. Matrix elements of the interaction decay (probably as a power law) when differences between the energies of involved quasiparticles is increased.

3. These matrix elements have random sign.





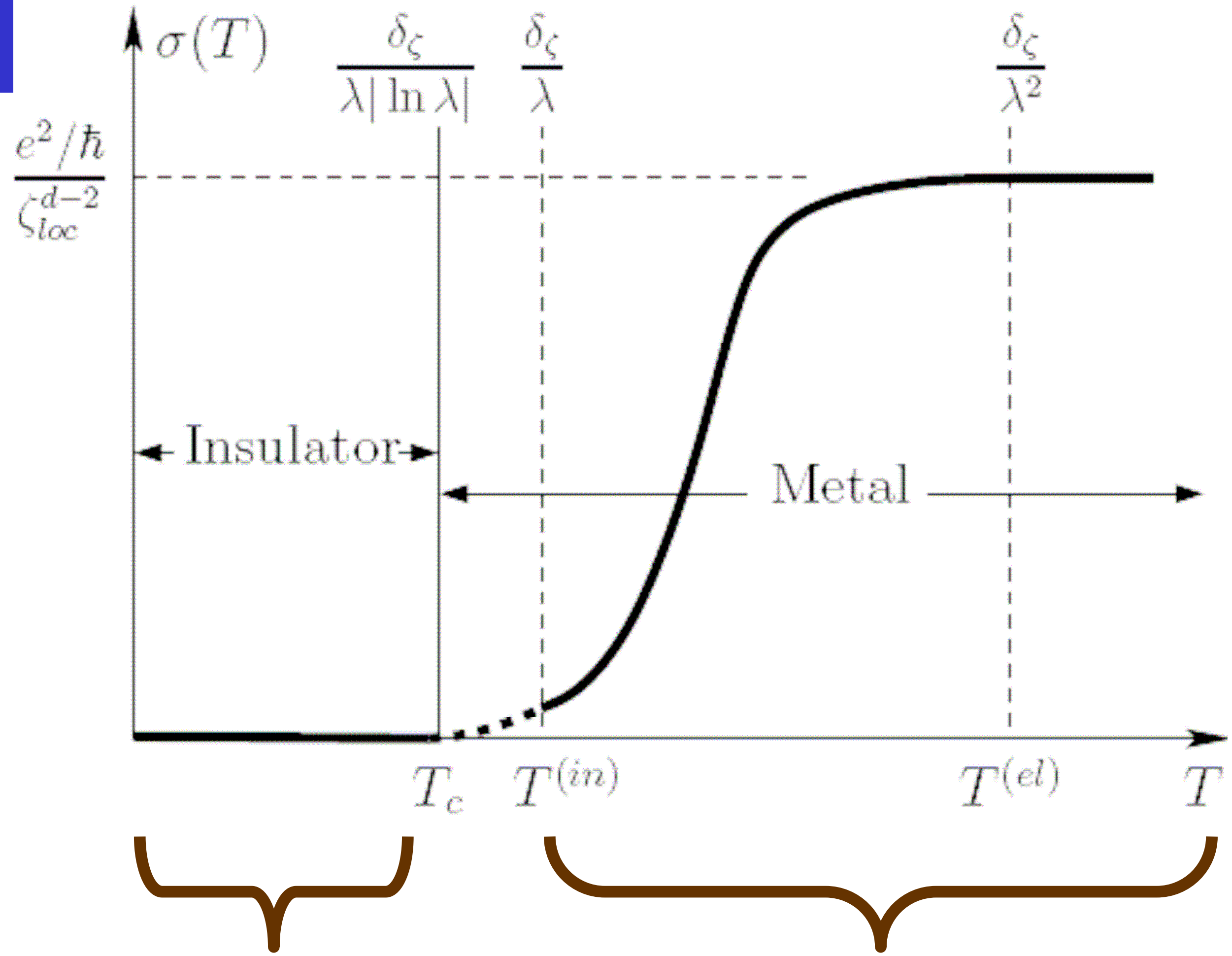
$$\hat H_0 = \sum_{\vec r, l} \hat c_l^\dagger(\vec r)\left[\xi_l(\vec r)\hat c_l(\vec r) + I\delta_\zeta \sum_{\vec a, m} \hat c_m(\vec r + \vec a)\right]$$



$$\hat{V}_{\text{int}} = \frac{1}{2} \sum_{\vec{r};l_1,l_2;j_1,j_2} V^{j_1,j_2}_{l_1,l_2}(\vec{r})\,\hat{c}^{\dagger}_{l_1}(\vec{r})\,\hat{c}^{\dagger}_{l_2}(\vec{r})\,\hat{c}_{j_1}(\vec{r})\,\hat{c}_{j_2}(\vec{r})$$

Interaction only within the same cell;
no diagonal matrix elements

Technique



Self-Consistent Born Approximation

Boltzmann Equation

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

J. Phys. C: Solid State Phys., Vol. 6, 1973. Printed in Great Britain.

A selfconsistent theory of localization

R Abou-Chacra†, P W Anderson‡§ and D J Thouless†
† Department of Mathematical Physics, University of Birmingham, Birmingham, B15 2TT
‡ Cavendish Laboratory, Cambridge, England and Bell Laboratories, Murray Hill, New Jersey, 07974, USA

Received 12 January 1973

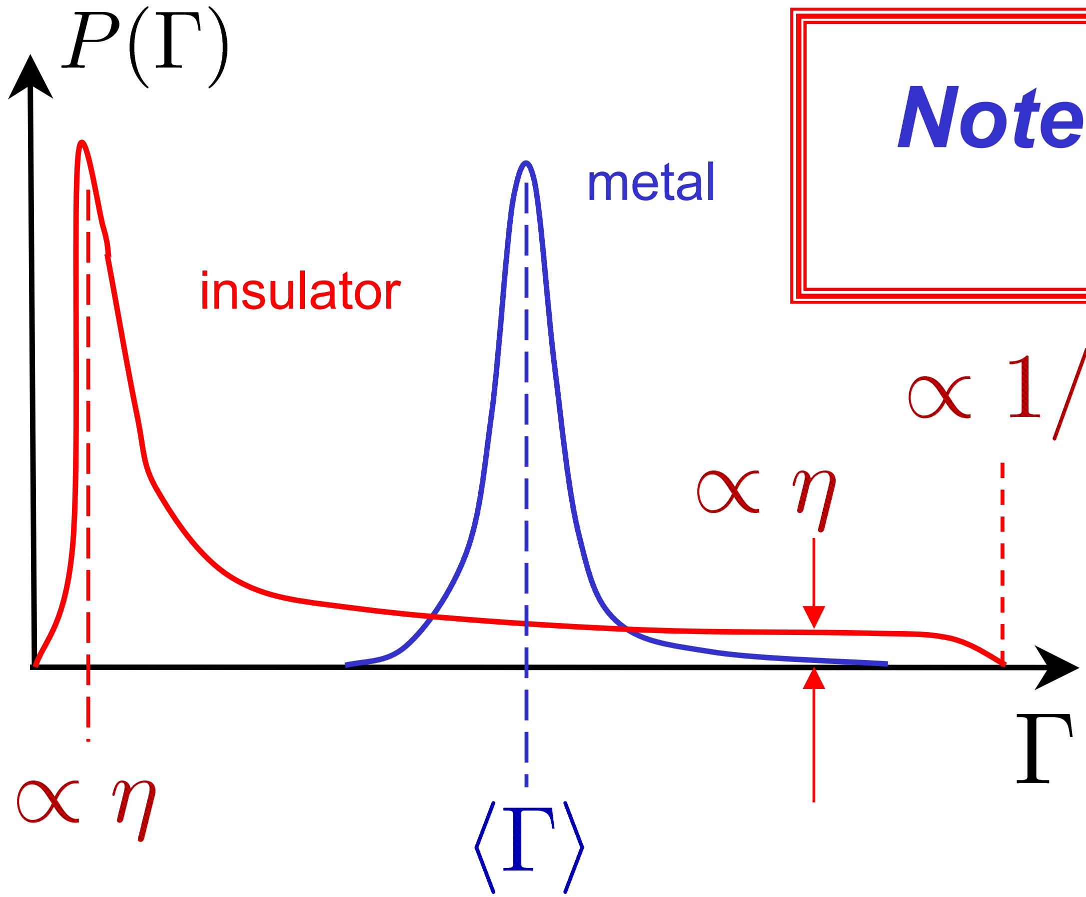
Idea of the calculation:

1. Start with some infinitesimal width h (***Im*** part of the self-energy due to a bath) of each one-electron eigenstate
2. Consider ***Im*** part of the self-energy G in the presence of tunneling and *e-e* interaction.
3. Calculate the probability distribution function $P(G)$
4. Consider the limit: $\lim_{h \to 0; W \to \infty} P(G) \equiv P_0(G)$

W is the volume of the system

$$P_0(G) \; \begin{cases} = d(G) & \text{- insulator} \\ \neq 0 \; for \; G \neq 0 & \text{- metal} \end{cases}$$

Probability Distribution



Note: $\langle\Gamma\rangle = \langle\Gamma\rangle$

Look for:

$$\lim_{\eta\to+0}\lim_{\mathcal{V}\to\infty} P(\Gamma > 0) = \begin{cases} > 0; & metal \\ 0; & insulator \end{cases}$$

Stability of the insulating phase: NO spontaneous generation of broadening

- $\Gamma_\alpha(\epsilon) \equiv 0$ **is always a solution**
- $\epsilon \to \epsilon + i\eta$ - ***linear*** **stability analysis**:

$$\frac{\Gamma}{(\epsilon-\xi_\alpha)^2+\Gamma^2} \to \pi\delta(\epsilon-\xi_\alpha) + \frac{\Gamma}{(\epsilon-\xi_\alpha)^2}$$

- **after** n **iterations of SCBA equations:**

$$P_n(\Gamma) \propto \frac{\eta}{\Gamma^{3/2}}\left(\text{const}\cdot\frac{\lambda T}{\delta_\zeta}\ln\frac{1}{\lambda}\right)^n$$

first $n \to \infty$
then $\eta \to 0$

$(\ldots) < 1$ – **insulator is stable !**

Stability of the metallic phase: Finite broadening is self-consistent

- $$P(\Gamma) = \frac{1}{\sqrt{2\pi\langle\delta\Gamma^2\rangle}} \exp\left[-\frac{(\Gamma - \langle\Gamma\rangle)^2}{2\langle\delta\Gamma^2\rangle}\right]$$

  $\sqrt{\langle\delta\Gamma^2\rangle} \ll \langle\Gamma\rangle$ **as long as** $T \gg \frac{\delta_\zeta}{\lambda}$

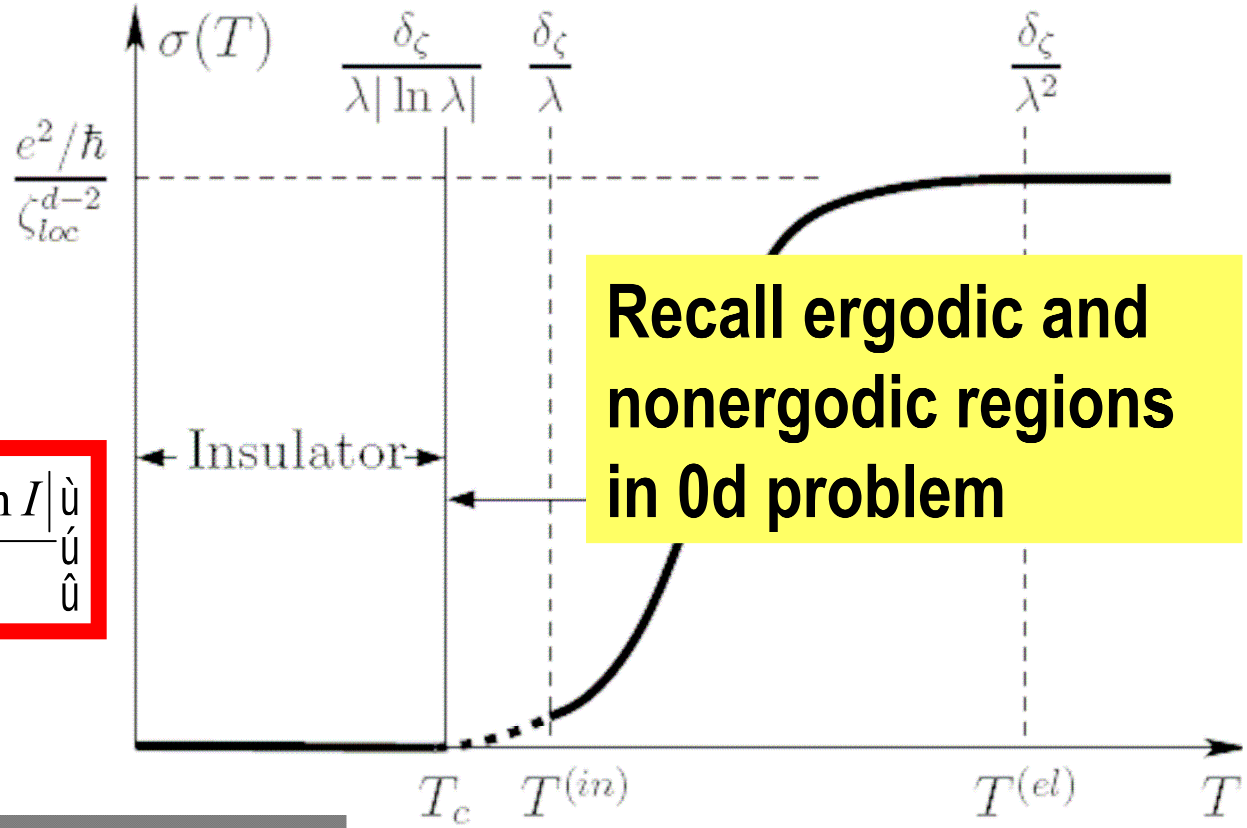
- $\langle\Gamma\rangle \ll \delta_\zeta$ **(levels well resolved)**

- **quantum kinetic equation for transitions between localized states**

  $\sigma(T) \propto \lambda^2 T^\alpha$ **(model-dependent)**

T_c:

$$\frac{12\lambda M T_c}{\delta_\zeta}\ln\left(\frac{1}{\lambda}\right) = \exp\left[\frac{\delta_\zeta \left|\ln\lambda\right|}{T_c}\right]$$



Recall ergodic and nonergodic regions in 0d problem

$$T^{(in)} = \frac{\delta_\zeta}{6\pi M\lambda}$$

$$\Downarrow$$

$$\left\langle\left(\delta\Gamma^{(in)}\right)^2\right\rangle = \left\langle\Gamma^{(in)}\right\rangle^2$$

$$T^{(el)} = \frac{\delta_\zeta}{16\pi^2 M d\lambda^2}$$

$$\Downarrow$$

$$\left\langle\left(\delta\Gamma^{(el)}\right)^2\right\rangle = \left\langle\Gamma^{(el)}\right\rangle^2$$

$$T >> T^{(el)} = \frac{d_z}{16p^2 Md\, l^2}$$

$$\sigma(T \gg \sqrt{\delta_\zeta T_{el}}) \approx \sigma_\infty \left(1 - \frac{2}{3}\frac{\delta_\zeta T_{el}}{T^2}\right);$$

$$\kappa(T \gg \sqrt{\delta_\zeta T_{el}}) \approx \kappa_\infty(T)\left[1 - \left(\frac{14}{5} - \frac{24}{\pi^2}\right)\frac{\delta_\zeta T_{el}}{T^2}\right]$$

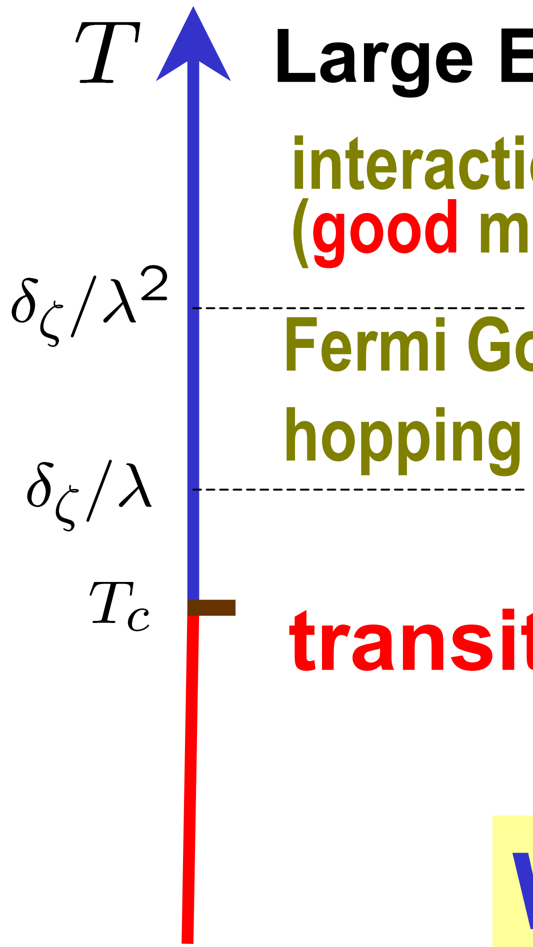
$$\sigma_\infty \equiv \frac{2\pi e^2 I^2 \zeta_{loc}^{2-d}}{\hbar}, \quad \kappa_\infty(T) \equiv \frac{2\pi^3 e^2 T I^2 \zeta_{loc}^{2-d}}{3\hbar}.$$

$$T^{el} >> T >> T^{(in)} = \frac{d_z}{6p\, M\, l}$$

$$\sigma(T \ll \sqrt{\delta_\zeta T_{el}}) = \sigma_\infty \frac{\pi}{4}\left(\frac{T^2}{\delta_\zeta T_{el}}\right),$$

$$\kappa(T \ll \sqrt{\delta_\zeta T_{el}}) = \kappa_\infty(T)\,\frac{48G^2}{\pi^3}\left(\frac{T^2}{\delta_\zeta T_{el}}\right)$$

Many-body mobility edge



Large E_k) high T: extended states

interaction ! dephasing ! cutoff of WL
(good metal)

Fermi Golden Rule

hopping (bad metal)

transition ! mobility edge

Why no activation?

Many-body mobility edge

T

Large E_k → high T: extended states

interaction → dephasing → cutoff of WL
(**good** metal)

δ_ζ/λ^2

Fermi Golden Rule
hopping (**bad** metal)

δ_ζ/λ

T_c

transition → mobility edge

$E, E_c \propto$ *volume*

No activation:

$$E_c \propto \frac{T_c^2}{\delta_\zeta \zeta^d}, \qquad E \propto \frac{T^2}{\delta_\zeta \zeta^d} \qquad \times \textit{volume}$$

$$\exp\left(\frac{E(T)-E_c}{T}\right) \xrightarrow{\textit{volume}\to\infty} 0$$

Conclusions & Some speculations

Conductivity exactly vanishes at finite temperature. **Finite temperature phase transition without any apparent symmetry change!**
Is it an ordinary thermodynamic phase transition or low temperature phase is a glass?

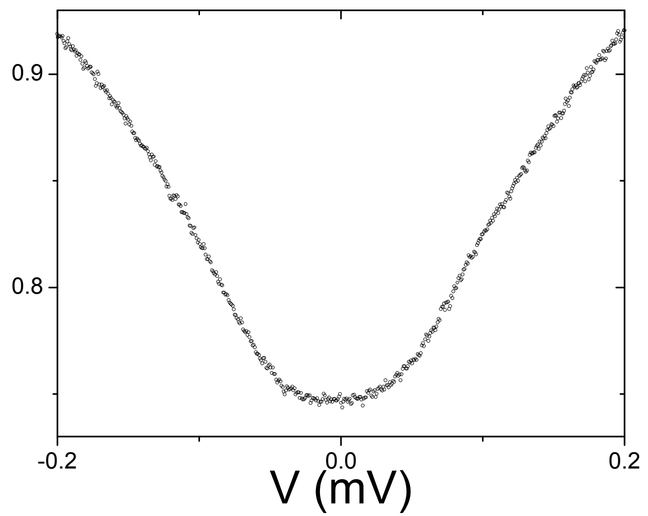
We considered weak interaction.
What about strong electron-electron interactions?
Melting of a pined Wigner crystal?

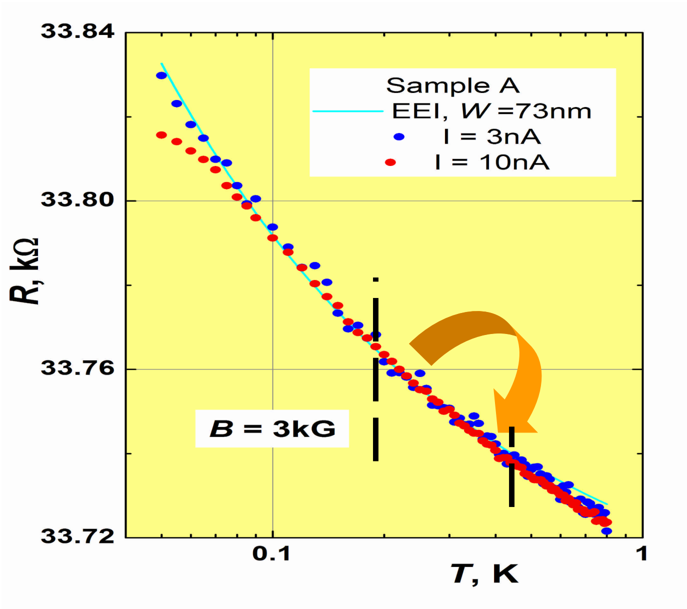
What if we now turn on phonons?
Cascades.
Is conventional hopping conductivity picture ever correct?

Orthogonality catastrophe

Electron-electron interaction effects other than inelastic collisions

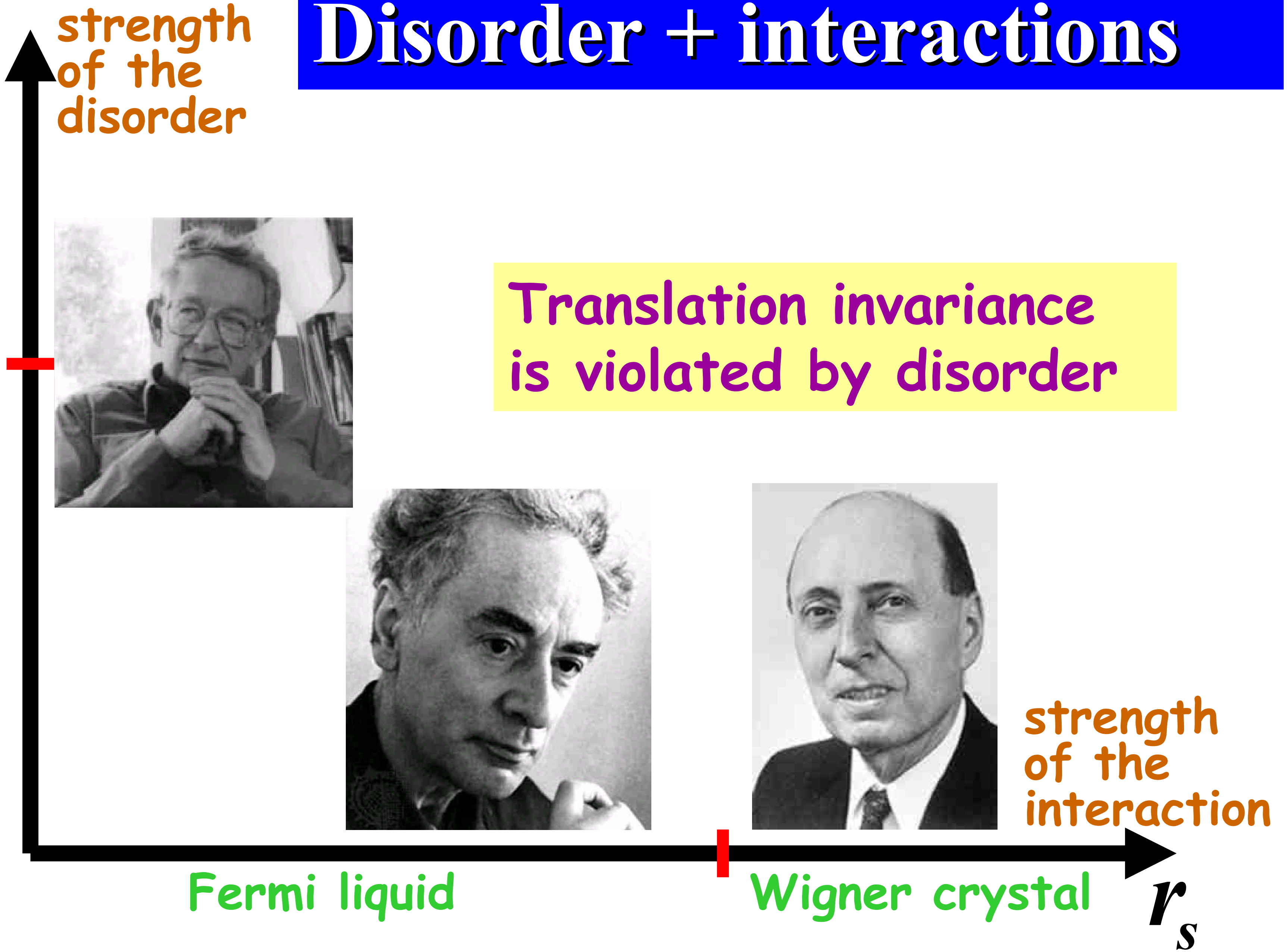
- Anomalies in the tunneling density of states
- Temperature dependence of the conductivity





Disorder + interactions

Translation invariance is violated by disorder

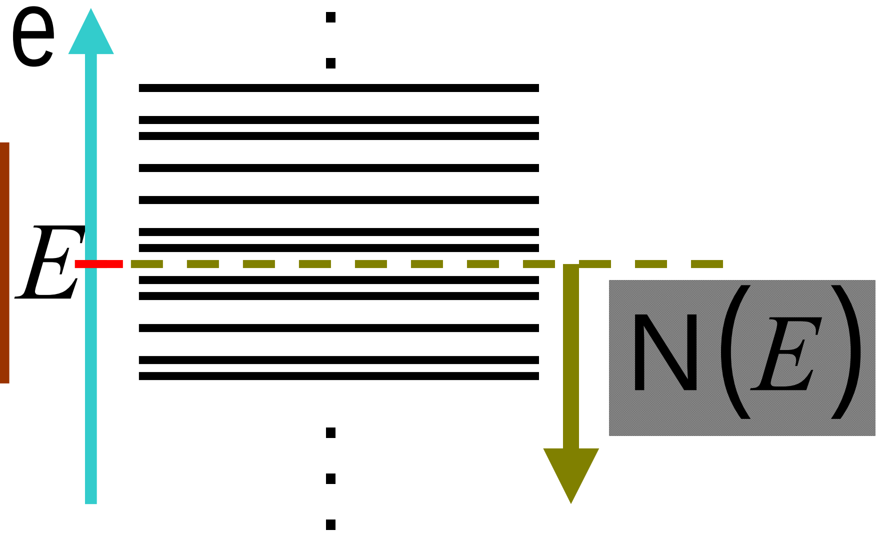


One-particle Density of States

0) Free fermions

$$\nu(E) \equiv \frac{N(E+\Delta) - N(E-\Delta)}{2\Delta * volume}$$

$volume \rightarrow \infty \qquad \Delta \rightarrow 0$



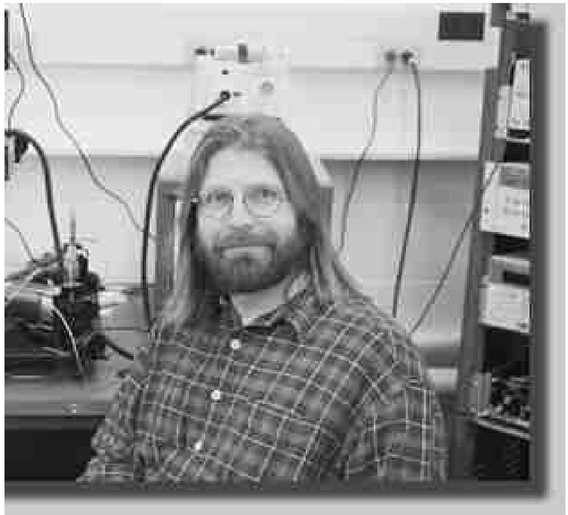
Observables are determined by DoS at the chemical potential, μ :

Compressibility	$dn/d\mu = \nu(\mu)$
Specific heat	$c/T = \nu(\mu)$
Magnetic succeptibility	$\chi/(g\mu_B) = \nu(\mu)$
Conductivity	$\sigma = e^2 \nu(\mu)\, D$

$\nu(\mu) = const > 0$

"Carbon Nanoelectronics"
talk at ITP UCSB, Aug. 2001

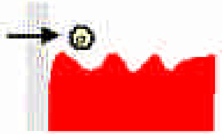
Tunneling into a Luttinger Liquid



Fermi Liquid

$dI/dV \sim const$

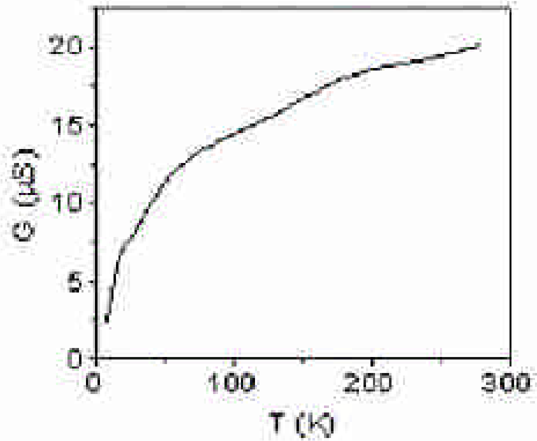
$G(T) \sim const$



Luttinger Liquid

$dI/dV \sim V^{\alpha}$

$G(T) \sim T^{\alpha}$



Expt:
Bockrath et al. (99)
Yao et al. (99)
Postma et al. (01)
Theory
Kane Balents and Fisher (97)
Egger and Gogolin (97)

I. *Excitations are* ***similar*** *to the excitations in a disordered* ***Fermi-gas.***

II. *Small decay rate*

III. *Substantial renormalizations*

AND

These always are infrared singularities

Tunneling Density of States

Tunneling Phenomena in Solids

Lectures presented at the 1967 NATO Advanced Study Institute at Risö, Denmark, 1967

Edited by

ELIAS BURSTEIN
Department of Physics University of Pennsylvania Philadelphia, Pennsylvania

and

STIG LUNDQVIST
Institute of Theoretical Physics Chalmers Tekniska Högskola Göteborg, Sweden

Chapter 3

Metal–Insulator–Metal Tunneling

I. Giaever
General Electric Research and Development Center Schenectady, New York

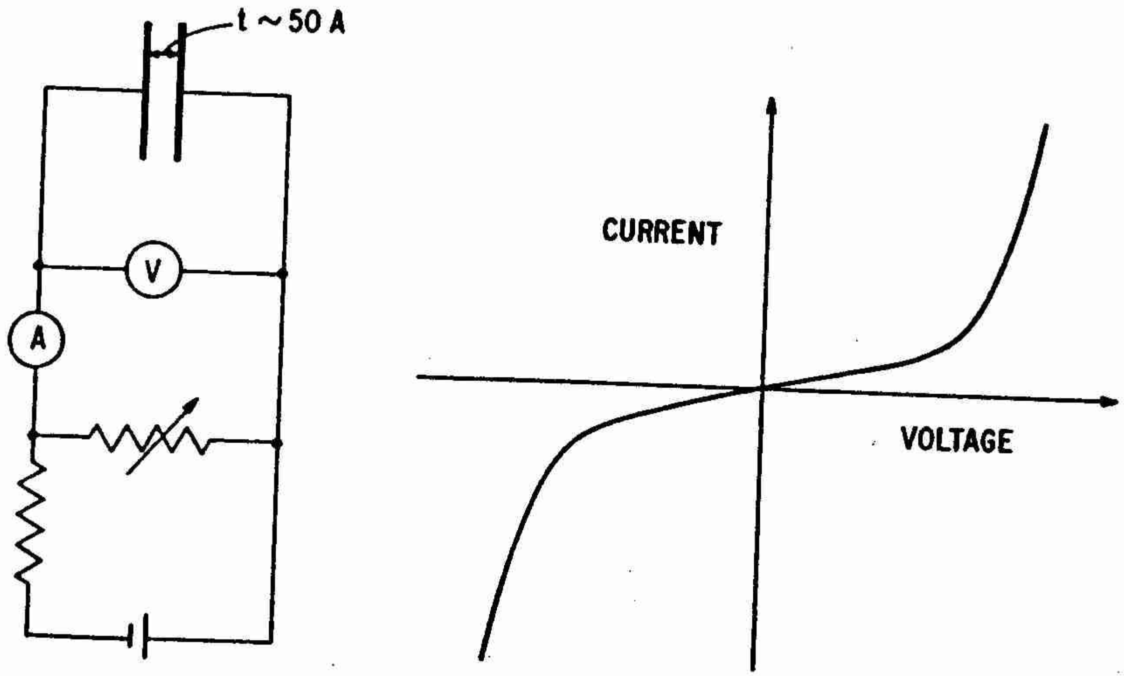
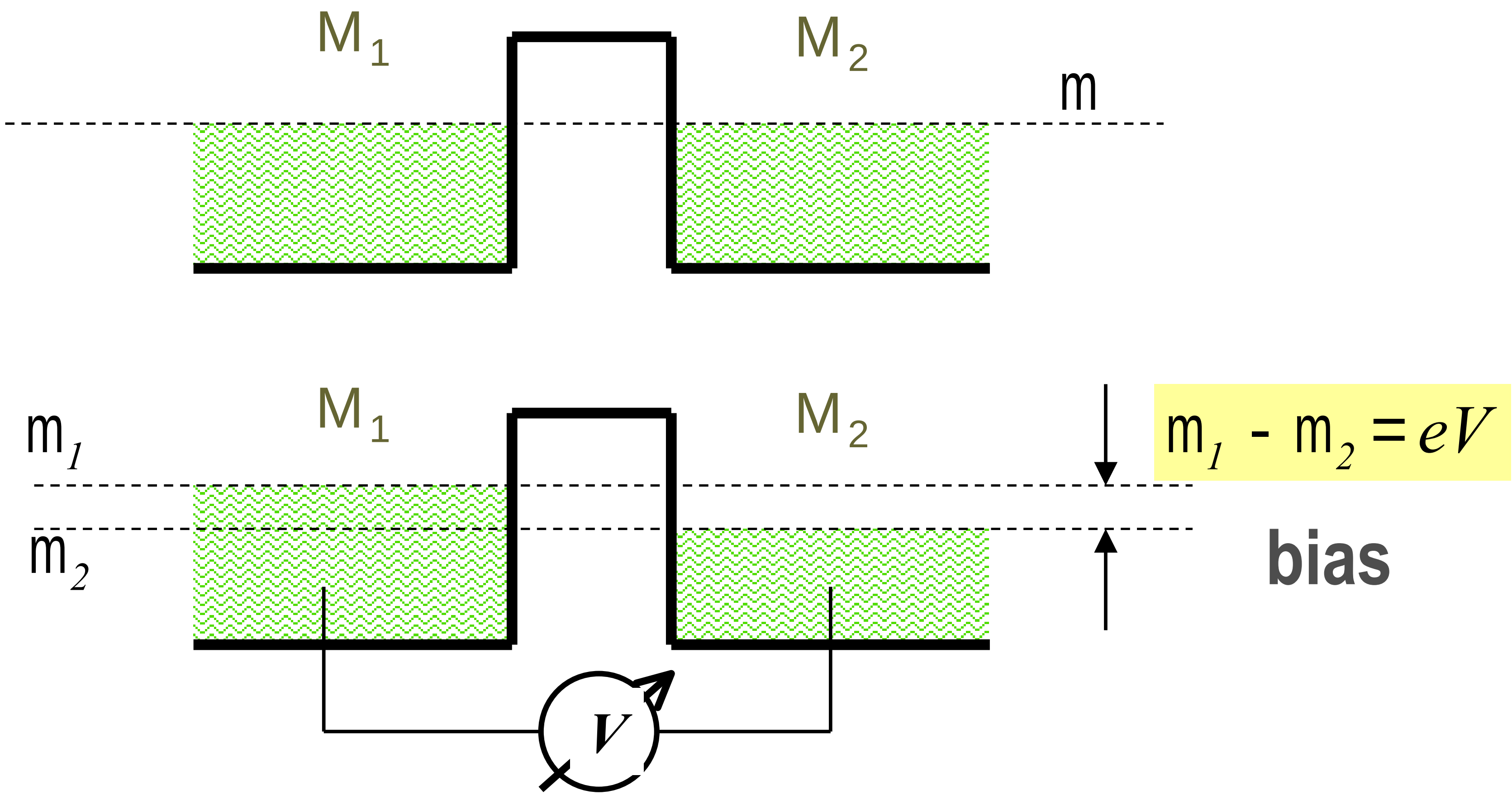
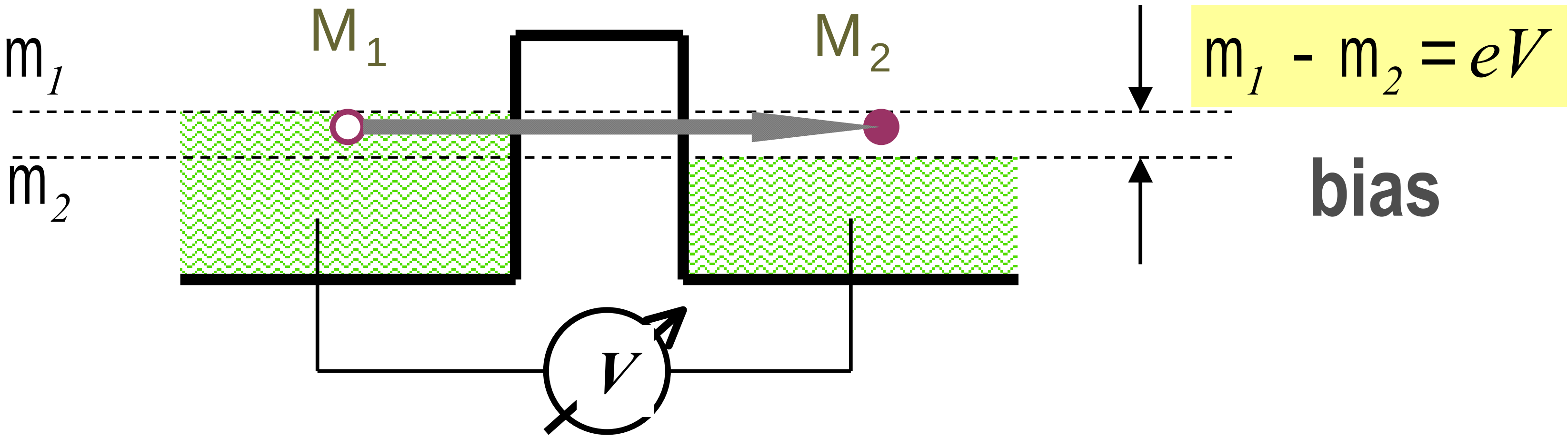


Fig. 1. Schematic drawing of a tunneling experiment. If the capacitor plates are spaced about 50 Å apart or less, a tunnel current will be easily observable. The current–voltage characteristic will be nearly symmetric about zero, linear at low voltages (below ~ 0.1 V), and nonlinear at higher voltages.

Tunneling Density of States



Tunneling Density of States

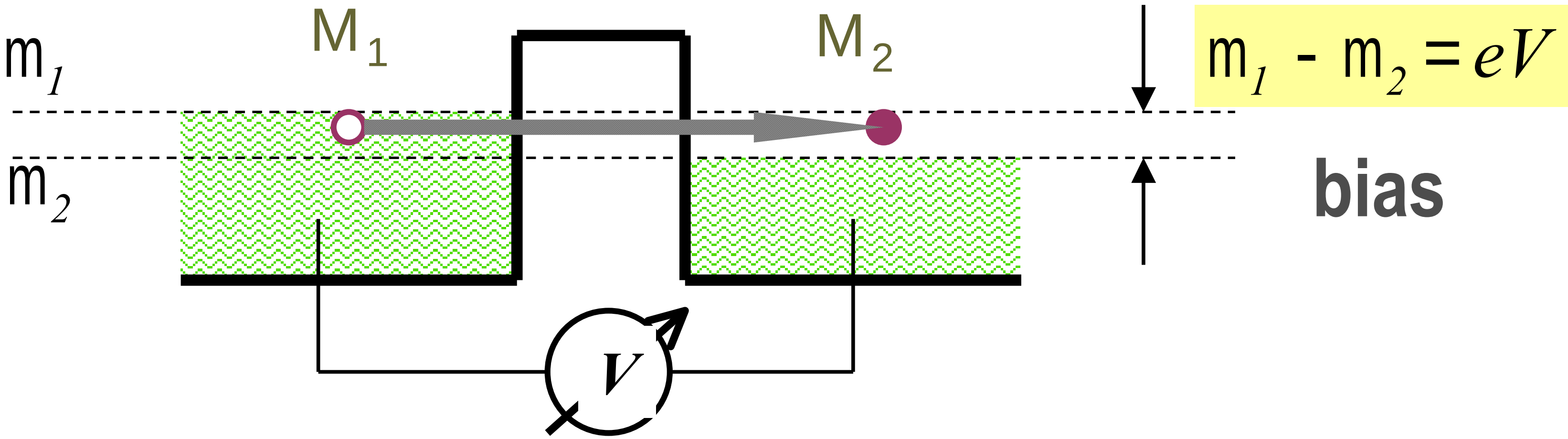


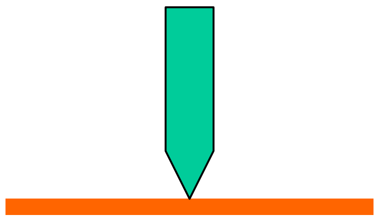
$$G(V) \equiv \frac{d\,I(V)}{dV} \propto \nu_1(\mu)\,\nu_2(\mu) \approx const$$

tunneling probability

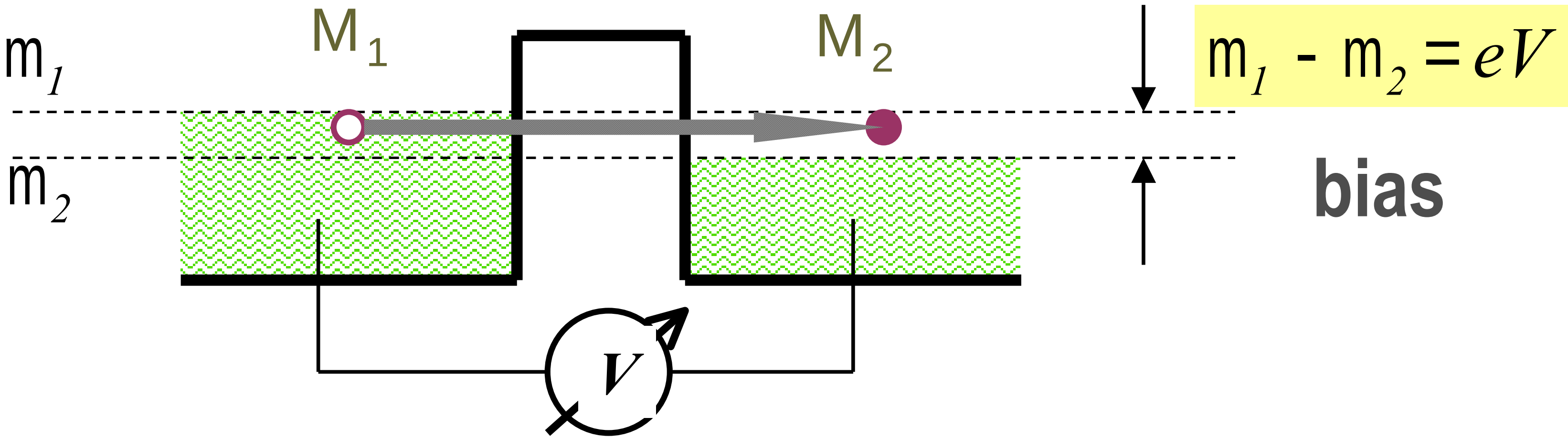
Depends on the bias only on the scale of the Fermi energy ?

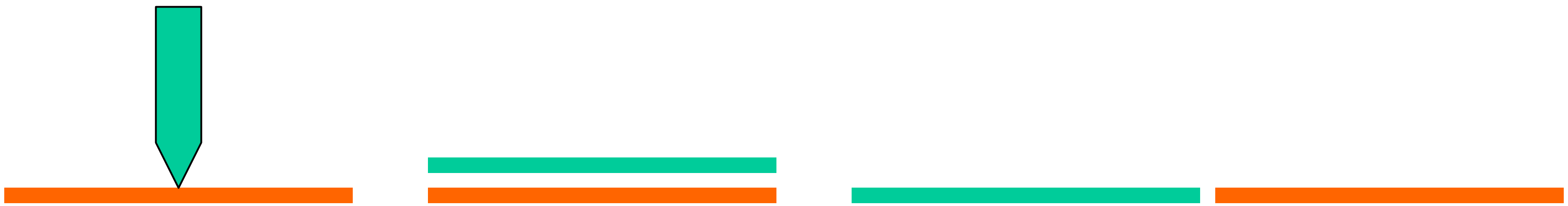
Tunneling Density of States





Tunneling Density of States





A charge is created at $t=0$

DIRECT OBSERVATION OF POLARONS AND PHONONS DURING TUNNELING IN GROUP 3-5 SEMICONDUCTOR JUNCTIONS

R. N. Hall, J. H. Racette, and H. Ehrenreich
General Electric Research Laboratory, Schenectady, New York

(Received April 6, 1960)

First observation of the Zero Bias Anomaly

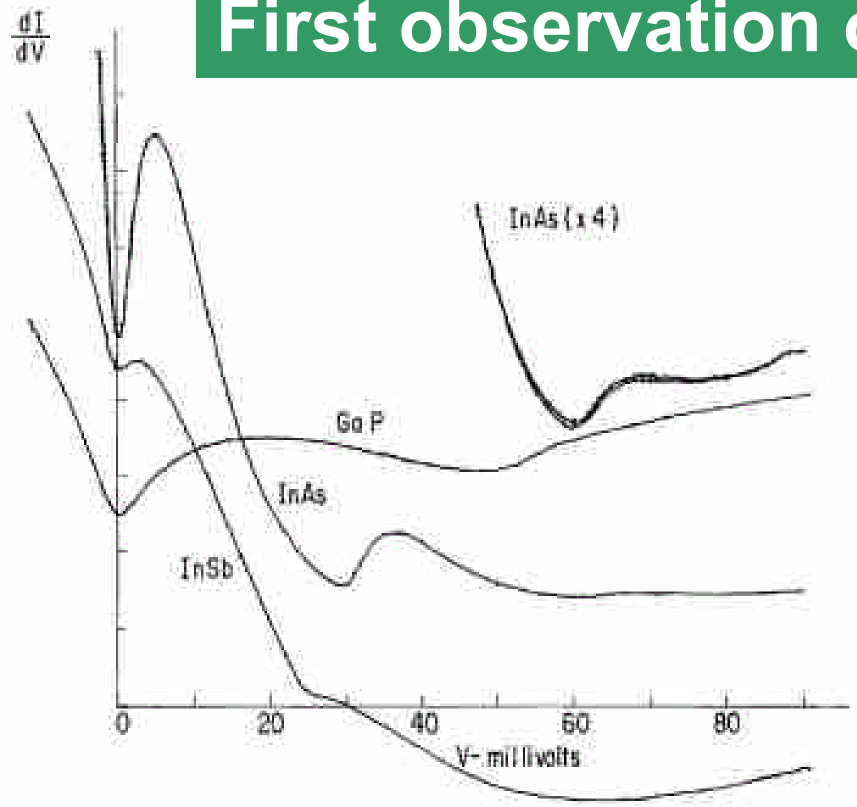


FIG. 1. Conductance (dI/dV) in arbitrary units vs voltage for several group 3-5 junctions.

Zero Bias Anomaly (ZBA)

Tunneling conductance, G_t,
is determined by the product of the tunneling probability, W,
and the densities of states in the electrodes, $n_t(\varepsilon = eV)$.

Originally ZBA was attributed to W:
- Paramagnetic impurities inside the barrier (Appelbaum-Andersdon theory) for the maximum of G_t.
- Phonon assisted tunneling for the minimum.

Now it is accepted that in most of cases

ZBA is a hallmark of the interactions between the electrons.

In other words, it is better to speak in terms of anomalies in the tunneling DoS.

VOLUME 4, NUMBER 9 PHYSICAL REVIEW LETTERS MAY 1, 1960

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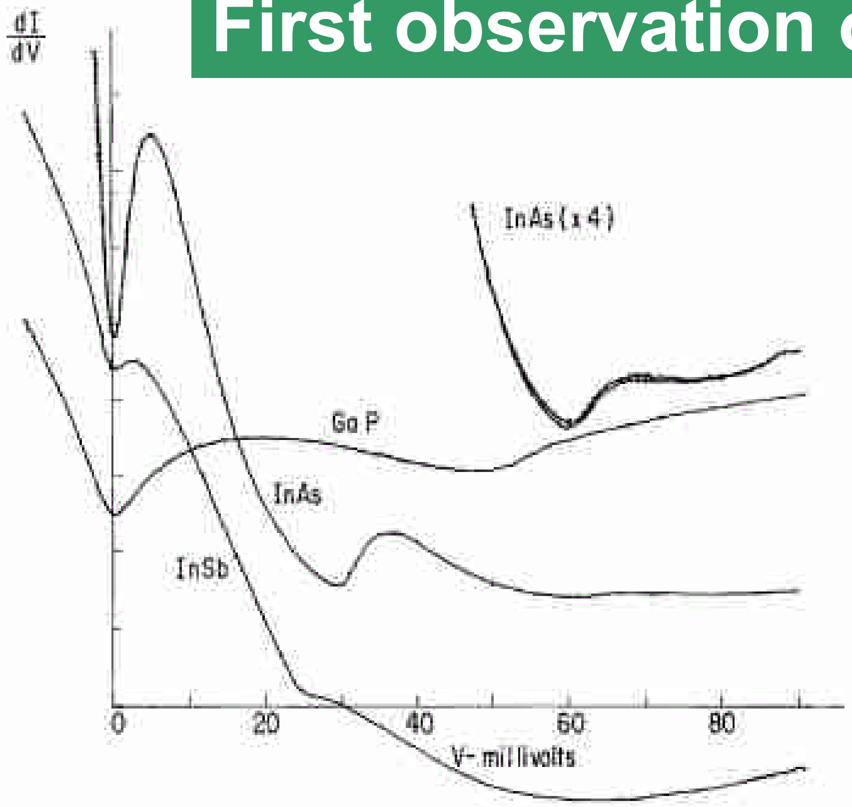


FIG. 1. Conductance (dI/dV) in arbitrary units vs voltage for several group 3-5 junctions.

Minimum in the conductance at zero bias

⇓

Minimum in the density of states at the Fermi level

⇓

Tunneling is suppressed at small energies

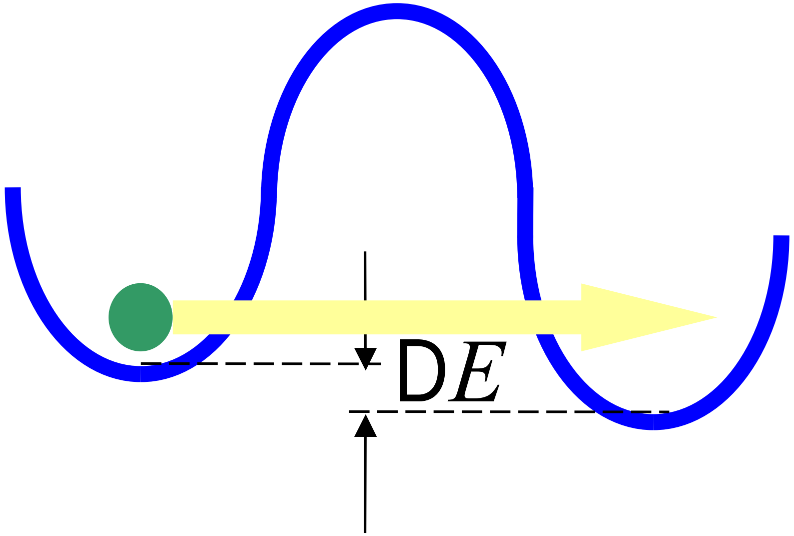
VOLUME 18, NUMBER 24 PHYSICAL REVIEW LETTERS 12 JUNE 1967

INFRARED CATASTROPHE IN FERMI GASES WITH LOCAL SCATTERING POTENTIALS

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 27 March 1967)

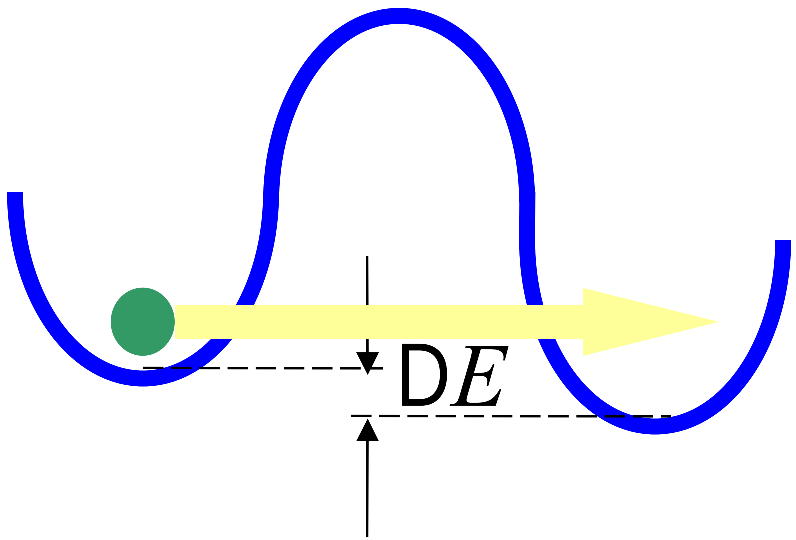
We prove that the ground state of a system of *N* fermions is to the ground state in the presence of a finite range scattering potential, as *N* ® ¥. This implies that the responce to application of such a potential involves only emission of excitations into the continuum, and that certain processes in Fermi gases may be blocked by orthogonality in a low - ***T***, low - energy limit.

Orthogonality catastrophe

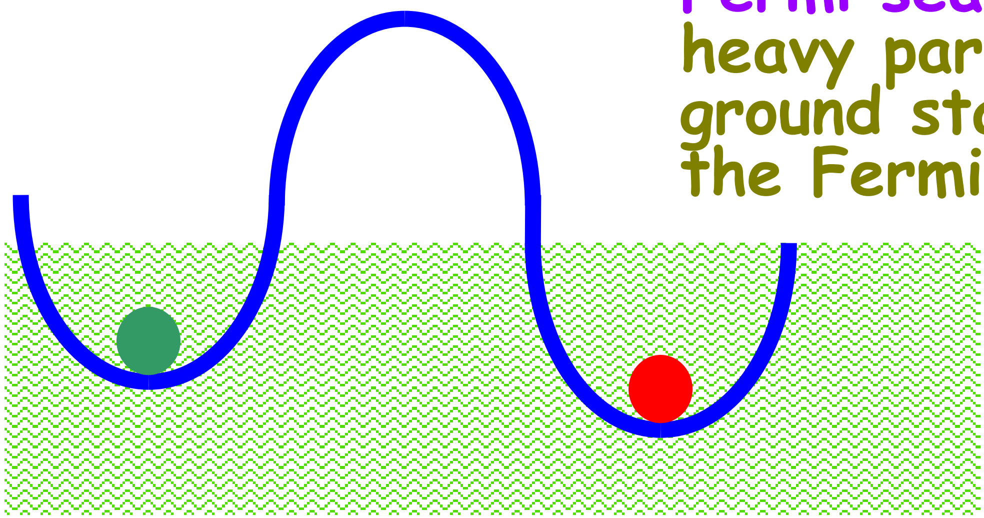


Tunneling probability is more or less independent on D*E*

Orthogonality catastrophe



Tunneling probability is more or less independent on DE



Tunneling in the presence of the Fermi sea: the fact that the heavy particle found itself in the ground state does not mean that the Fermi sea also remains in its ground state – the Fermi sea also has to tunnel

Soft pairs are created

VOLUME 18, NUMBER 24 PHYSICAL REVIEW LETTERS 12 JUNE 1967

INFRARED CATASTROPHE IN FERMI GASES WITH LOCAL SCATTERING POTENTIALS

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We believe this theorem is related to Fermi-surface anomalies both in tunneling and in impurity resistance,[2,3] and a paper on this application is being prepared.

[2] J. M. Rowell and L. Y. L. Shen, Phys. Rev. Letters 17, 15 (1966).

[3] B. R. Coles, Phys. Letters 8, 243 (1964). M. P. Sarachik, to be published; I am grateful to Mrs. Sarachick for seeing her preliminary data.

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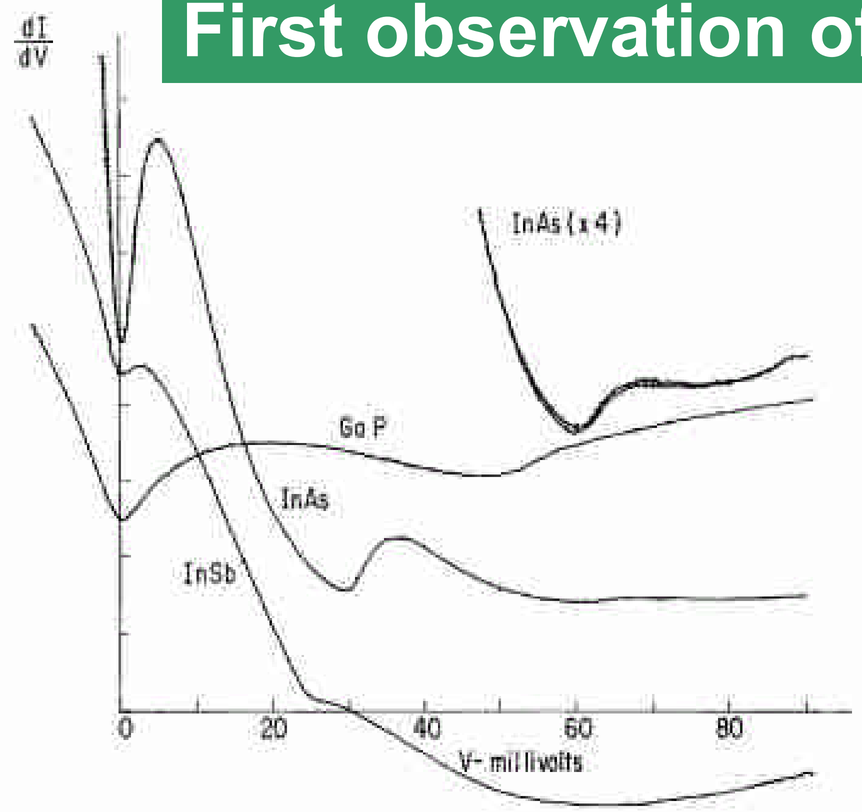

FIG. 1. Conductance (dI/dV) in arbitrary units vs voltage for several group 3-5 junctions.

Minimum in the conductance at zero bias

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Minimum in the density of states at the Fermi level

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VOLUME 17, NUMBER 1 PHYSICAL REVIEW LETTERS 4 JULY 1966

ZERO-BIAS ANOMALIES IN NORMAL METAL TUNNEL JUNCTIONS

J. M. Rowell and L. Y. L. Shen
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 20 May 1966)

We have investigated the current flow through thin chromium-oxide layers from 1°K to 290°K. We believe that current flows by means of a tunneling mechanism, but the dependence of the dynamic resistance of the junction on voltage and temperature is completely anomalous in terms of expected tunneling behavior. Some new results on other metal-oxide junctions strongly suggest that properties of the oxide layer are responsible for the anomaly observed by Wyatt in tantalum oxide junctions.

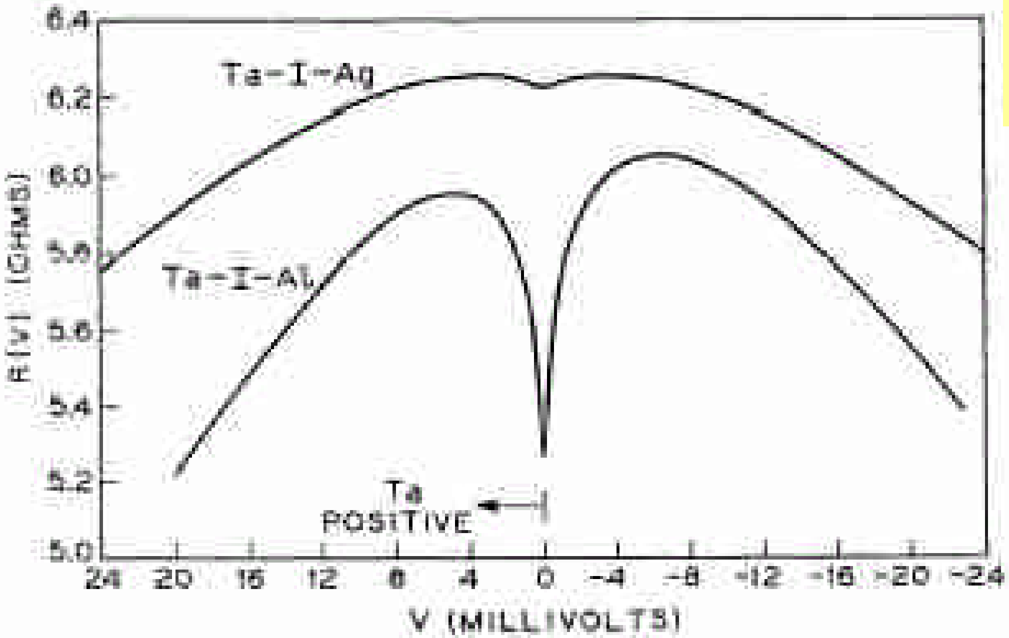

FIG. 3. Dynamic resistance versus voltage for Ta-I-Al and Ta-I-Ag junctions at 0.9°K. A field of 3 kG was used to drive the tantalum normal.

Minimum in the resistance at zero bias

↓

Maximum in the density of states at the Fermi level

↓

Tunneling is enhanced at small energies

A NEW TYPE OF LOW-TEMPERATURE RESISTANCE ANOMALY IN ALLOYS

B. R. COLES
Dept. of Physics, Imperial College, London S.W.7

Received 27 January 1964

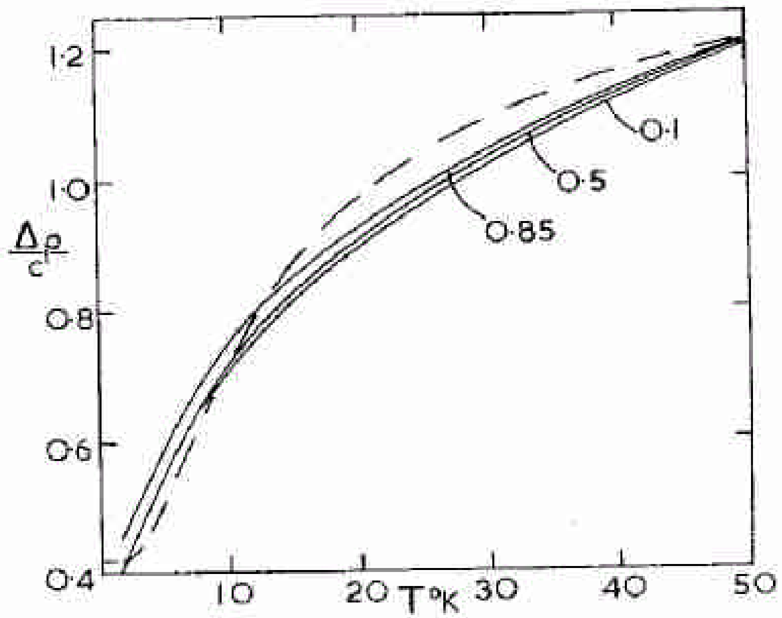

Fig. 2. Resistivity increment ($\Delta\rho/c = (\rho - \rho_{Rh})$/% Fe in microhm·cm per % iron) for dilute rhodium-iron alloys.
Dotted curve is $0.42 + \exp(-12/T)$
(Small arbitrary adjustments in the iron content will make the experimental curves lie even more closely together.)

Again:

Maximum in the density of states at the Fermi level

Correction to the DoS in the disordered case:

BA & A.G. Aronov, Solid St. Comm. 30, 115 (1980).
BA, A.G. Aronov, & P.A. Lee, PRL, 44, 1288 (1980).

Effect appears already in the first order in the perturbation theory
⇒ its sign is not determined

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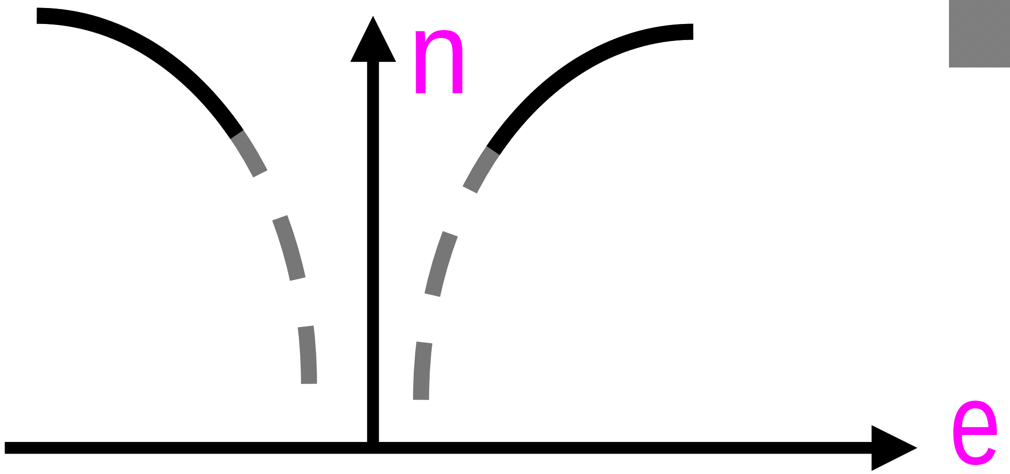
$$dn(e) = \frac{l_d}{e(\hbar D/e)^{d/2}} \propto \begin{cases} -\sqrt{e} & d=3 \\ \log e & d=2 \\ \frac{1}{\sqrt{e}} & d=1 \end{cases}$$

e electron energy counted from the Fermi level

D diffusion constant of the electrons

d # of the dimensions

l effective coupling constant;
l >0 -repulsion



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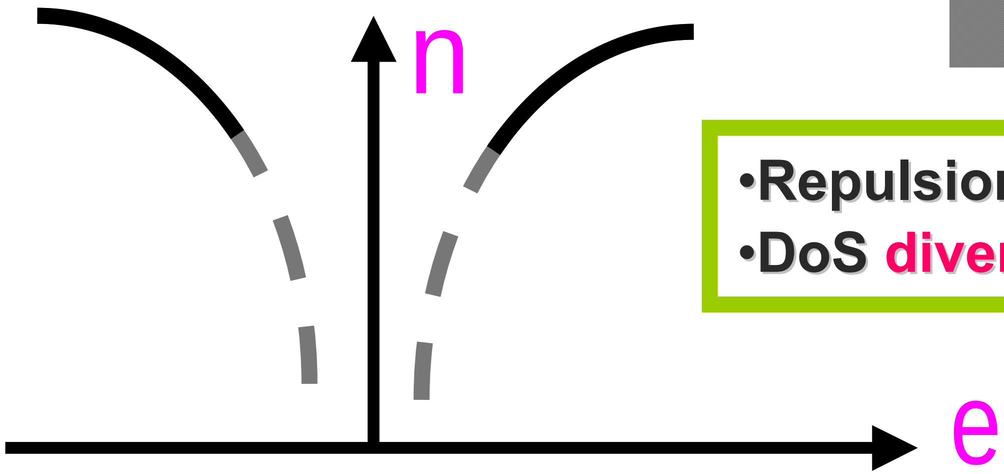
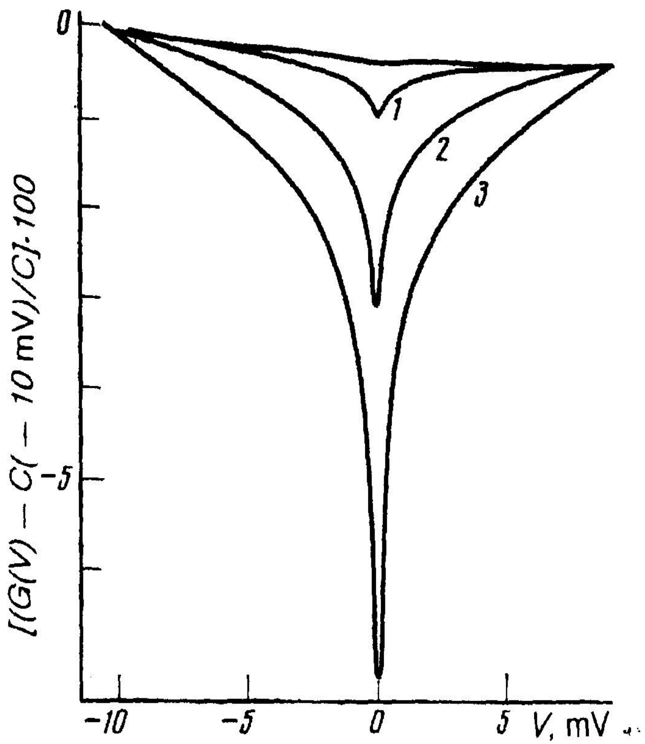
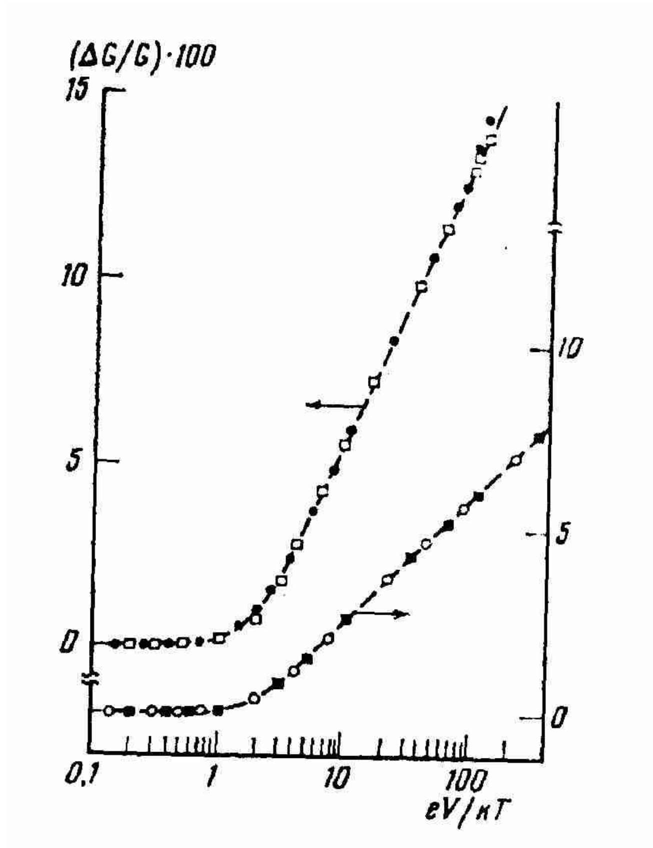


- **Repulsion - minimum in the DoS;**
- **DoS diverges at low dimensions**

Zero Bias Tunneling Anomaly





The conductivity of the tunnel junctions *Al-I-Al* (T=0.4K, B=3.5T) for 2D films with different R_o: 1 – 40 W, 2 – 100 W, 3 - 300 W.
Right panel: comparison with the theoretical prediction for the interaction-induced ZBA.

Gershenson et al, *Sov. Phys. JETP* 63, 1287 (1986)

Tunneling Density of States (DoS) $\nu(\varepsilon)$

Role of the Friedel Oscillations

K.A. Matveev, D.Yue, and L.I. Glazman Phys. Rev. Lett., v.71, p.3351 (1993)
A.M. Rudin, I.L. Aleiner, and L.I. Glazman; Phys. Rev. v.B71, #15 (1997)

$$\nu(\varepsilon) = \frac{1}{\text{volume}} \sum_{\alpha} \delta(\varepsilon - \varepsilon_{\alpha}) = -\frac{2}{\pi} \int \operatorname{Im} G^{R}(\vec{r}, \vec{r}) \frac{d\vec{r}}{\text{volume}}$$

Tunneling Density of States (DoS) $\nu(\varepsilon)$

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DoS at a given point $\vec{R}$ in space is determined by the quantum mechanical amplitude to come back to this point

Tunneling Density of States (DoS) $n(e)$

DoS at a given point $\vec{R}$ in space is determined by the quantum mechanical amplitude to come back to this point

0) No disorder
No interactions between the electrons

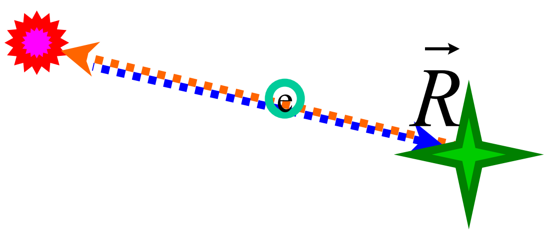
→ Non of the classical trajectories returns to the original point

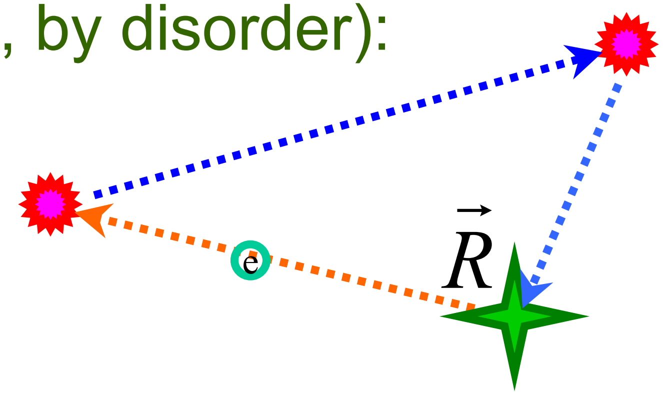
→ DoS is a smooth function of the energy

$$n(e) \propto (e + e_F)^{-1+d/2} \approx const$$

(Energy e is counted from the Fermi level)

1) Such classical trajectories appear as soon as translation invariance is violated (e.g., by disorder):





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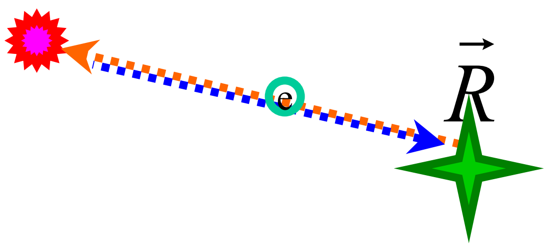
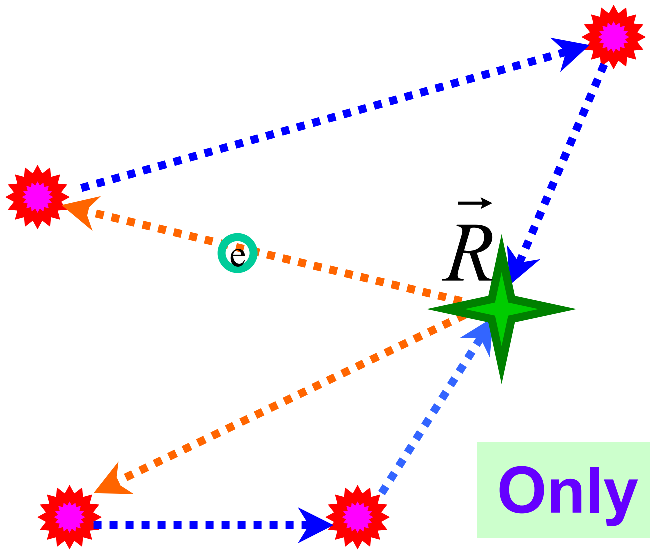


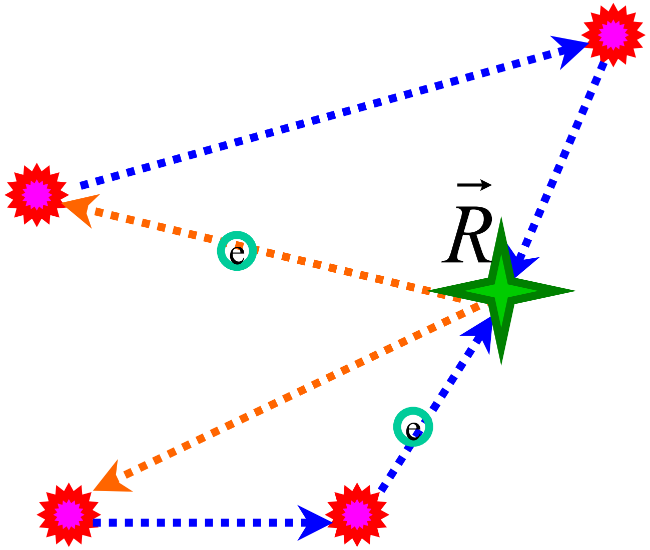
The return amplitude contains the phase factor. The phase $\varphi = 2k_F R$ is large (if the distance between the original point and the impurity exceeds the Fermi wavelength). The correction to the DoS vanishes when averaged over the sample volume



Different trajectories are characterized by different phase factors

$$\left\langle e^{i\varphi} \right\rangle_{disorder} = 0$$

Only mesoscopic fluctuations



Different trajectories have different phase factors

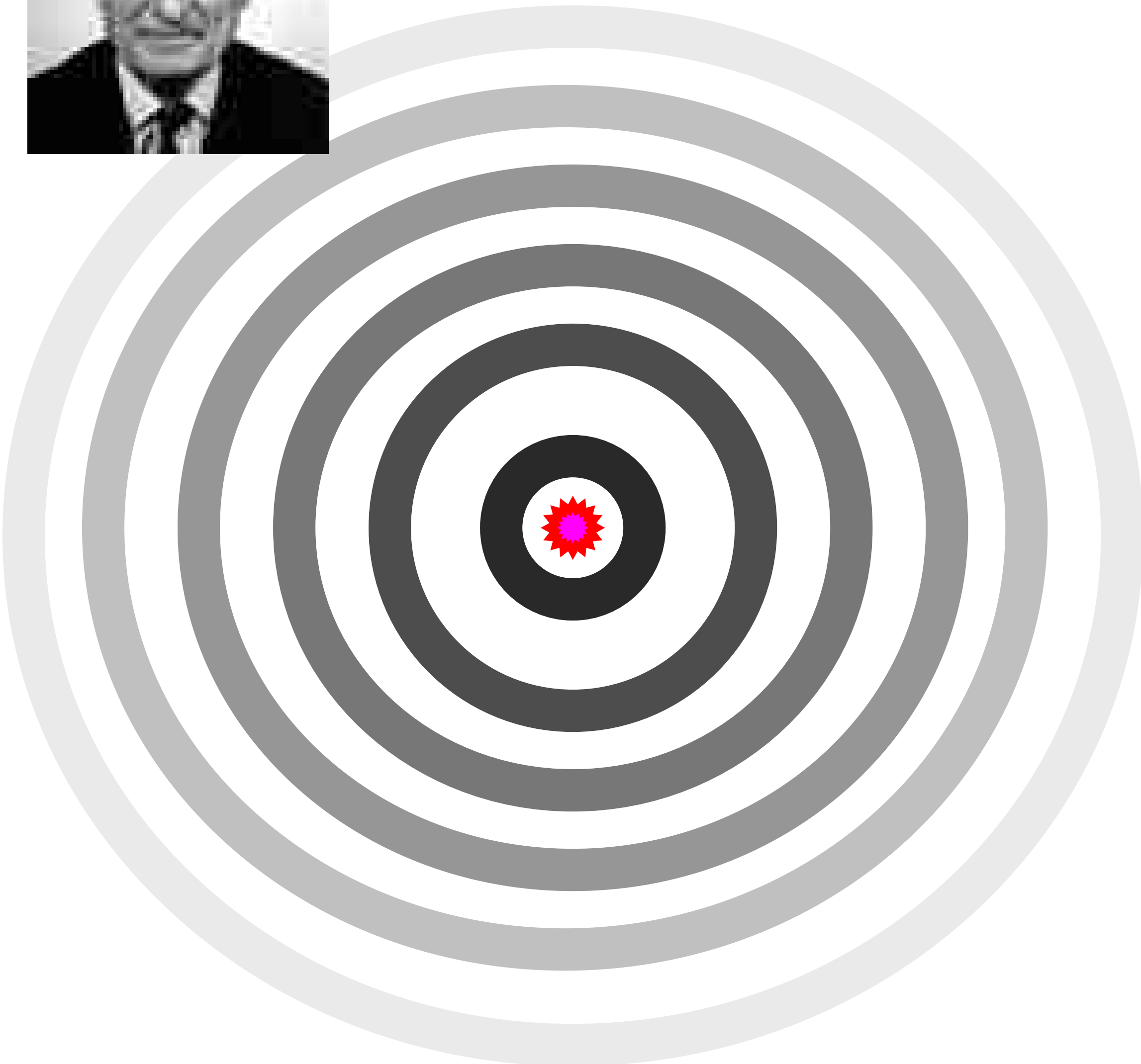
$$\left\langle e^{i\varphi} \right\rangle_{disorder} = 0$$

Without electron-electron interactions (averaged) DoS is not effected by the disorder.

Only mesoscopic fluctuations

Friedel Oscillations

$$\mathrm{dr}(\vec{r}) \propto \frac{\sin(2k_F r)}{r^d}$$



Electron density oscillates as a function of the distance from an impurity.

The period of these oscillations is determined by the Fermi wave length.

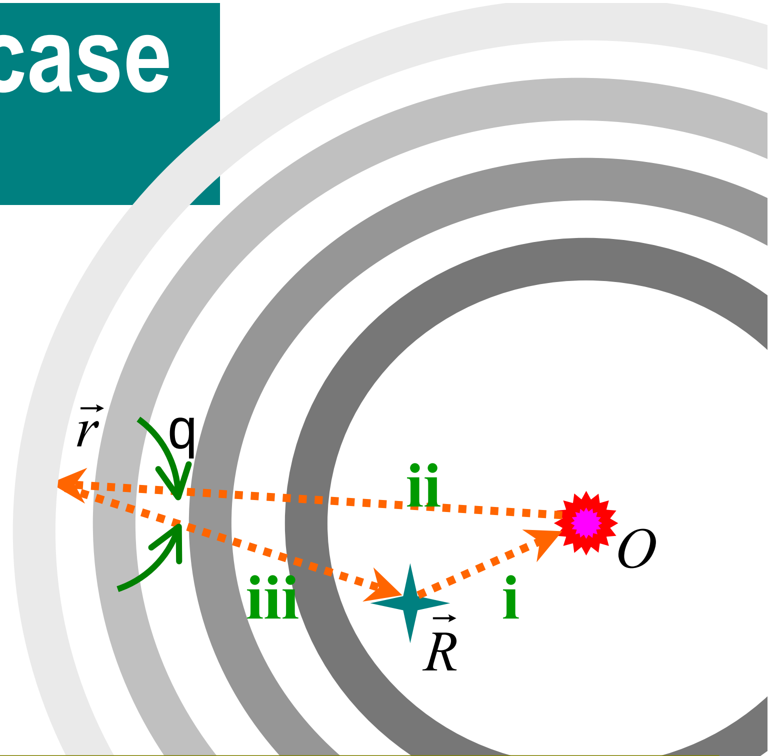
The amplitude of the oscillations decays only algebraically.

These oscillations are not screened

Single impurity (ballistic) case
Compensation of Phases

An electron right after the tunneling finds itself at a point R. It moves, then

- **(i)** gets scattered off an impurity at a point O,
- **(ii)** gets scattered off the Friedel oscillation **created by the same impurity** **(interaction !!!)**, and
- **(iii)** returns to the point R .



Phase factor at small angle q :

$$\sin(2k_F r)e^{ik\left|\vec{R}-\vec{r}\right|}e^{ikr}e^{ikR} \approx$$

$$e^{2i(k-k_F)r} = \exp\left(2ie r / v_F\right)$$

No oscillations in the limit

$$e \to 0;\ r_e \to \infty$$

ZBA !

An electron right after the tunneling finds itself at a point R. It moves, then

(i) gets scattered off an impurity at a point O,

(ii) gets scattered off the Friedel oscillation created by the same impurity, and

(iii) returns to the point R .

No oscillations in the limit $\varepsilon \to 0$

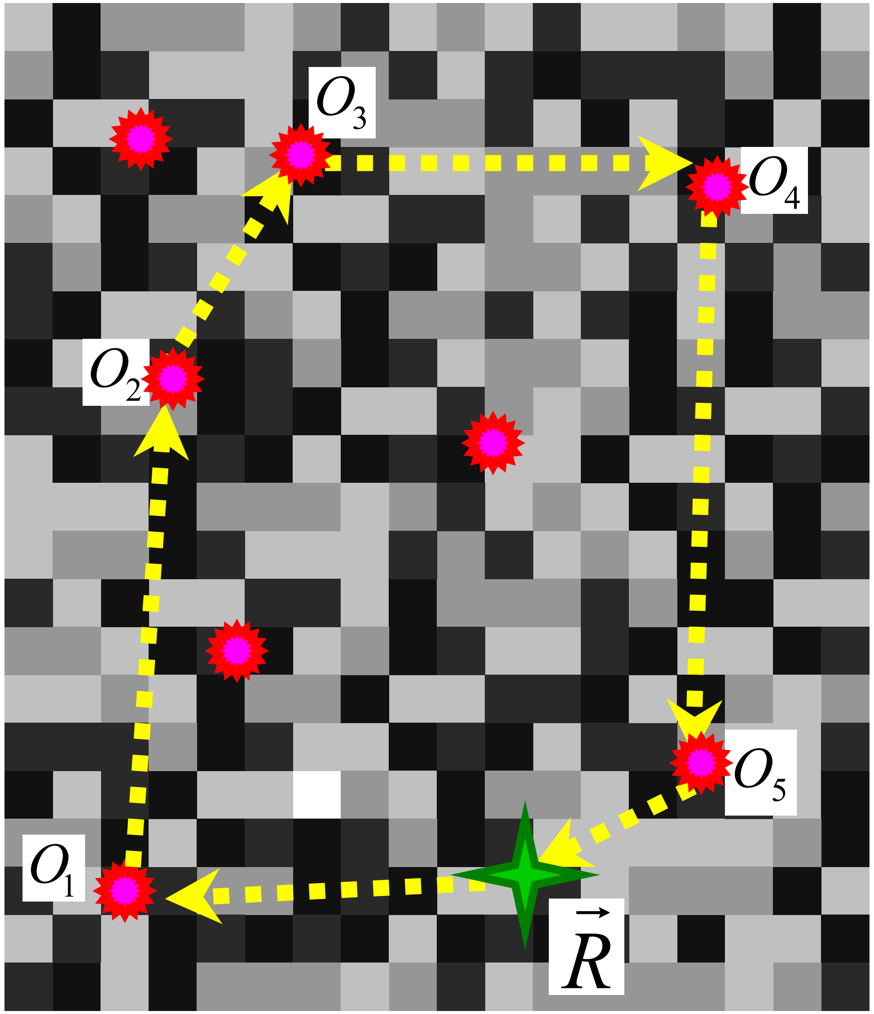
Phase fluctuates only when $r > r_\varepsilon$ where

$$r_\varepsilon \approx v_F / \varepsilon \to \infty$$

→ ZBA !

Important: this effect exists already in the first order of the perturbation theory in the interaction between the electrons (between the probe electron and the Friedel oscillation), i.e., in the Hartree-Fock approximation. As a result the DoS correction as well as ZBA can have arbitrary sign.

Multiple impurity scattering - diffusive case.
Compensation of Phases



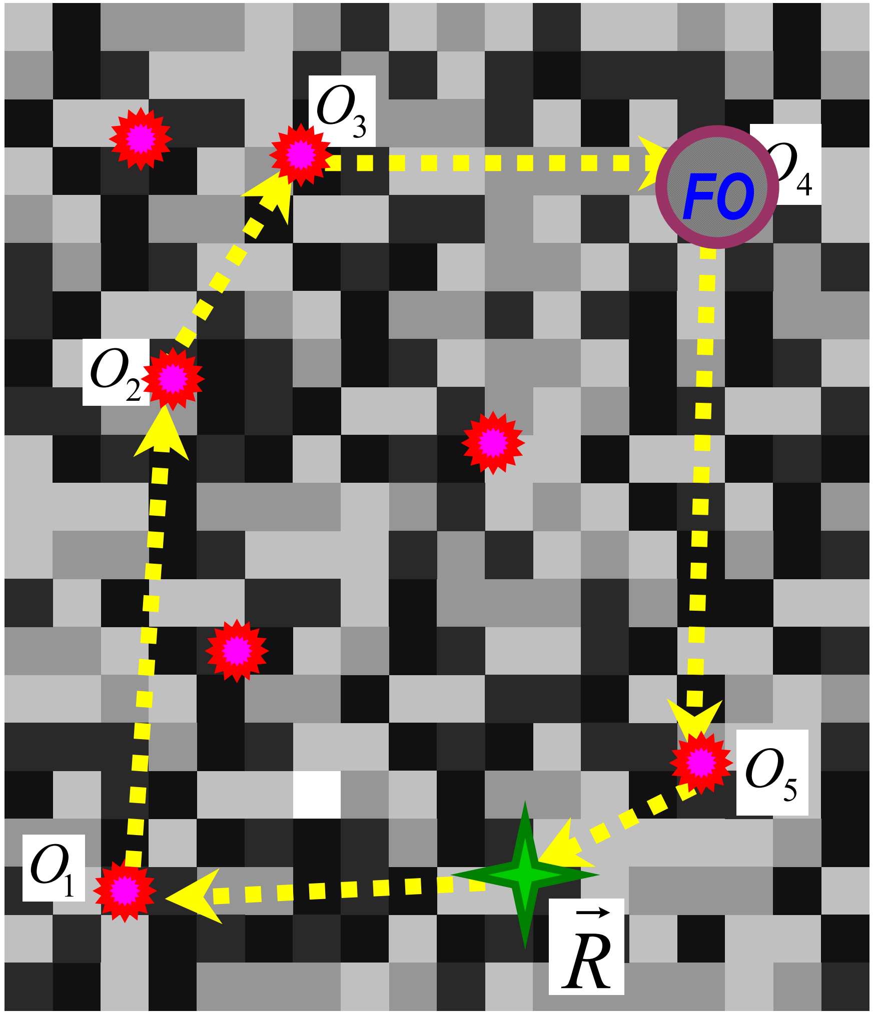
"Messy" Friedel oscillations - combination of the Friedel oscillations from different scatterers

$$\delta\rho(\vec{r}) \propto \sum_{paths\ \alpha} A_\alpha \sin(k_F L_\alpha)$$

$\alpha = \{O_1, O_2, O_3, \ldots, O_n,\}$ a path

L_α total length of this path

Multiple impurity scattering - diffusive case.
Compensation of Phases



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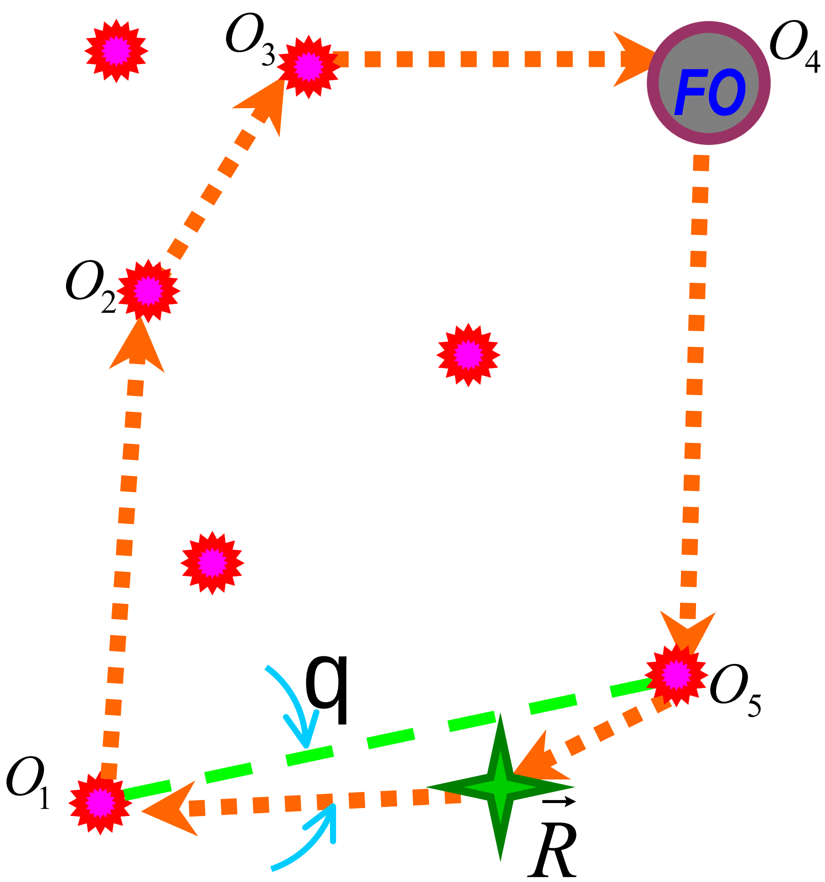
Multiple impurity scattering - diffusive case Compensation of Phases



"Messy" Friedel oscillations - combination of the Friedel oscillations from different scatterers

$$\mathrm{d\,r}(\vec{r}) \propto \sum_{paths\ \mathrm{a}} A_{\mathrm{a}} \sin\left(k_F L_{\mathrm{a}}\right)$$

$$\mathrm{a} = \{O_1, O_2, O_3, \ldots, O_n,\}$$ a path

L_{a} total length of this path

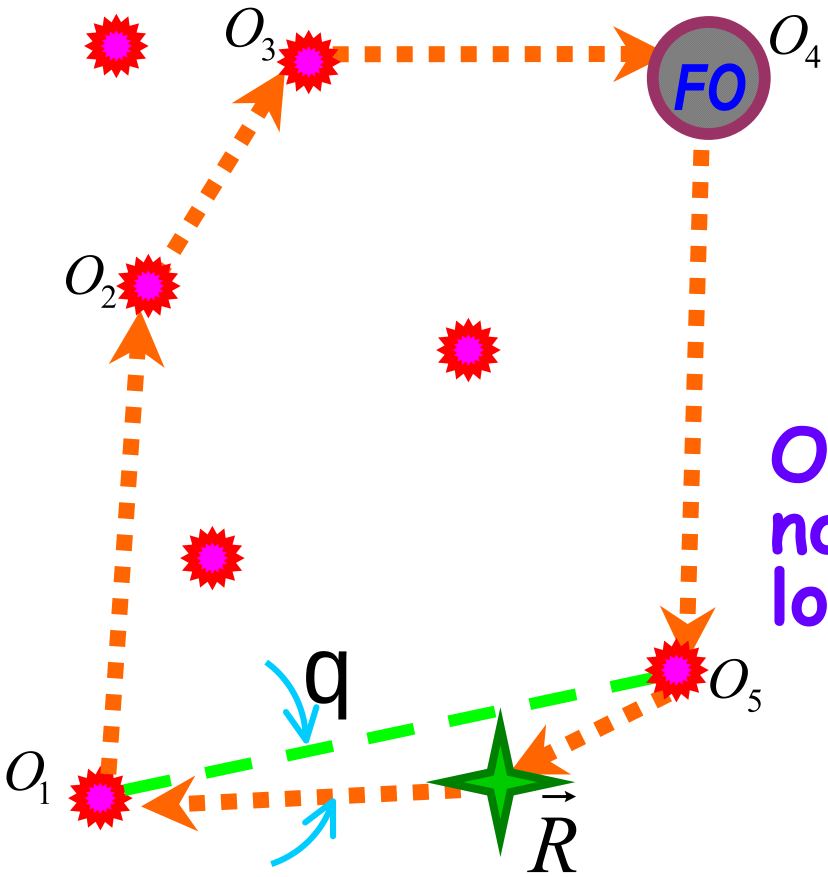
phase factor at small angle q :

$$\sin\left(k_F L_{\mathrm{a}}\right) e^{ikL_{\mathrm{a}}} \approx \exp\left({}^{ieL_{\mathrm{a}}}\!\big/_{v_F} \right)$$

Again, oscillations are not important as long as

$$L_{\mathrm{a}} < r_{\mathrm{e}} \approx {}^{v_F}\!\big/_{\mathrm{e}} \to \infty$$

Multiple impurity scattering - diffusive case Compensation of Phases



phase factor at small angle q :

$$\sin(k_F L_a) e^{ikL_a} \approx \exp\left(\frac{ieL_a}{v_F}\right)$$

Oscillations are not important as long as

$$L_a < r_e \approx \frac{v_F}{e} \to \infty$$

Magnitude of the correction to the DoS is determined by the return probability

If the interaction is not weak, the relative corrections to the DoS are the same as the weak localization corrections to the conductivity

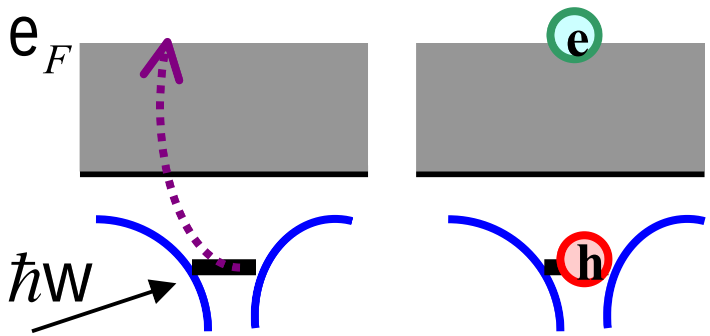
Tunneling Density of States Two singular contributions

Anderson Orthogonality Catastrophe:

Creation of virtual soft electron-hole pairs
Second order in the e-e interaction

This effect exists in disordered systems as well !

Analogy with the X-ray edge singularity problem:
(no translation invariance)



Two distinct singular terms in the ionization probability:

1. *Mahan term - interaction between the "new born" electron and the localized hole*

2. *Anderson term - interaction between the hole and the rest of the Fermi sea*

Tunneling Density of States Role of the translation invariance

In the presence of the disorder the anomaly is due to the simultaneous scattering of the electrons off the disorder and off the Friedel oscillations.

Role of the disorder:

1. It preforms Friedel oscillations
2. It increases the return probability

1. It is only due to the disorder the nontrivial correction to the density of states appears already in the first order in the interaction constant

2. DoS singularity gets stronger due to the disorder (e.g, $\epsilon^{-1/2}$ instead of $\log \epsilon$ in 1D)

Tunneling Density of States. Leading correction

t- mean free time
n- density of states
E_F- Fermi energy

$V(q)$ - Fourier transform of the short range interaction potential

dn(e)/n	*d=3*	*d=2*	*d=1, N* channels
Diffusive $et \ll \hbar$	$\frac{l}{(E_F t)^2}\sqrt{et}$	$\frac{l}{E_F t}\log\left(\frac{e}{E_F}\right)$	$\frac{l}{N\sqrt{et}}$
Ballistic l ® *0*; $et \gg \hbar$	$\frac{l e^2}{E_F t}\log\left(\frac{e}{E_F}\right)$	$\frac{l}{E_F t}\left\lvert\frac{e}{E_F}\right\rvert$	$\frac{l}{N}\log\left(\frac{e}{E_F}\right)$
Clean t ® ¥; $l \ll 1$	$\frac{l^2 e^2}{E_F^2}\log\left(\frac{e}{E_F}\right)$	$l^2\frac{\lvert e\rvert}{E_F}$	$\frac{l^2}{N}\log\left(\frac{e}{E_F}\right)$

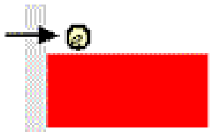
$$l \propto \left[V(0) - 2V(2p_F)\right]h$$

Fock

Hartree

“Carbon Nanoelectronics”
talk at ITP UCSB, Aug. 2001

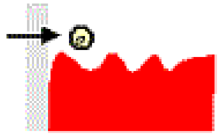
Tunneling into a Luttinger Liquid



Fermi Liquid

$dI/dV \sim const$

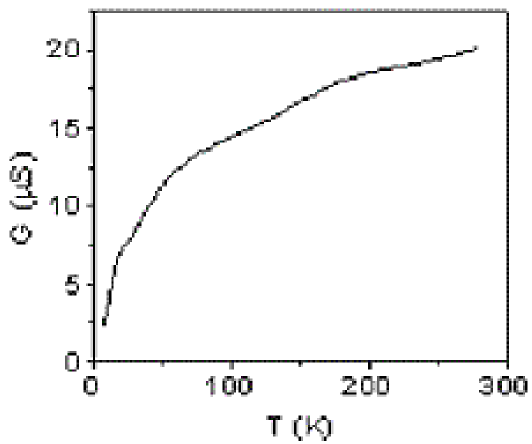
$G(T) \sim const$



Luttinger Liquid

$dI/dV \sim V^{\alpha}$

$G(T) \sim T^{\alpha}$



Expt:
Bockrath et al. (99)
Yao et al. (99)
Postma et al. (01)
Theory
Kane Balents and Fisher (97)
Egger and Gogolin (97)

Luttinger Liquid 1D

$$G_t(eV,T)\propto\left(\max\{eV,T\}\right)^{\alpha}$$

$$x^{\alpha}=\exp(\alpha\ln x)=1+\alpha\ln x+(\alpha\ln x)^2+\ldots$$

Tunneling to the bulk - translation invariance is **preserved**. Effect starts from the **second order** of the perturbation theory.

Tunneling to the end - translation invariance is **violated**. Effect exists already in the **first order** of the perturbation theory.

$$\alpha_{bulk}=\frac{(1-K)^2}{8K}\propto(1-K)^2 \qquad \alpha_{end}=\frac{1-K}{4K}\propto 1-K$$

1-K is the perturbative coupling const.

Theory

C.L.KANE & M.P.A.FISHER,
PRL, 68, 1220 (1992).

K.A.MATVEEV & L.I.GLAZMAN,
PRL, 70, 990-993 (1993).

Experiment:

MARC BOCKRATH,
DAVID H. COBDEN, JIA LU,
ANDREW G. RINZLER,
RICHARD E. SMALLEY,
LEON BALENTS & PAUL L. MCEUEN
Nature **397**, 598 - 601 (1999)

+Disorder — Beyond the first correction

Yu. V. Nazarov, Zh. Eksp. Teor. Fiz. 95, 975 (1989)[Sov. Phys. JETP 68, 561 (1990)].

L.S.Levitov & A.V.Shytov, JETP Lett. 66, 215 (1997)

A.Kamenev & A.Andreev, Phys. Rev. B60, 2218 (1999)

$d > 1$

$d = 1$ *Luttinger Liquid*

E.Mishchenko, A.Andreev, & L.Glazman, PRL, 87, #24 (2001)

$$\frac{\nu(\varepsilon,T)}{\nu_0} = T\cosh\frac{\varepsilon}{2T}\int_{-\infty}^{\infty}\frac{\cos\varepsilon t\, dt}{\cosh\pi T t} * \exp\left\{\int_{o}^{\infty} d\omega\, V(\omega)\frac{\cosh(\omega/2T) - \cos\omega t}{\sinh(\omega/2T)}\right\}$$

$$V(\omega) = -\sqrt{\frac{2K}{\pi^2 N}}\,\Re\left(\frac{\sqrt{\omega + i/\tau}}{\omega^{3/2}}\right)$$

$$K = \frac{\pi e^2}{4v_F}\ln\left(\frac{L}{R}\right)$$

Gives both Clean(Luttinger Liquid) and disordered limits

What about zero dimensions, i.e., quantum dots ?

Zero Bias Anomaly ⟹ **Coulomb Blockade**

***d*=1,2,3** **Interaction channels**: spin singlet;
spin triplet &
Cooper

Universal Hamiltonian

$$\hat{H} = \hat{H}_0 + E_c\hat{n}^2 + J\hat{S}^2 + \lambda_{BCS}\hat{T}^+\hat{T}.$$

$\hat{n}$ total number of electrons

$\hat{S}$ total spin of the electrons

$$\hat{T}^+ = \sum_{\alpha} a^+_{\alpha,\uparrow} a^+_{\alpha,\downarrow}$$

Environment Theory

Yu. V. Nazarov, Zh. Eksp. Teor. Fiz. 95, 975 (1989)[Sov. Phys. JETP 68, 561 (1990)].

M. H. Devoret, D.Esteve, H. Grabert, G.-L. Ingold, H. Pothier & C. Urbina, PRL, 64, 1824 (1990)

S.M. Girvin, L.I. Glazman, M. Jonson, D.R. Penn & M.D. Stiles, PRL, 64, 3183 (1990)

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F.Pierre, H.Pothier, P.Joyez, Norman O.Birge, D.Esteve, & M. H. Devoret, PRL, 86, #8, 1590 (2001)

Electrodynamic Dip in the Local Density of States of a Metallic Wire

F. Pierre, H. Pothier, P. Joyez, Norman O. Birge,* D. Esteve, and M. H. Devoret
Service de Physique de l'Etat Condensé, Commissariat à l'Energie Atomique, Saclay, 91191 Gif-sur-Yvette, France
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We have measured the differential conductance of a tunnel junction between a thin metallic wire and a thick ground plane, as a function of the applied voltage. We find that near zero voltage, the differential conductance exhibits a dip, which scales as $1/\sqrt{V}$ down to voltages $V \sim 10k_BT/e$. The precise voltage and temperature dependence of the differential conductance is accounted for by the effect on the tunneling density of states of the macroscopic electrodynamics contribution to electron-electron interaction, and not by the short-ranged screened-Coulomb repulsion at microscopic scales.

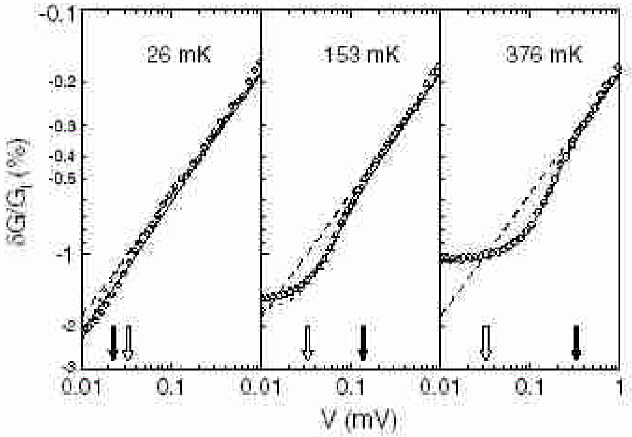

FIG. 2. Symbols: measured variations of the differential conductance of the tunnel junction, normalized to $G_t = 26.392\ \mu S$, as a function of voltage V, for 3 values of the temperature. Each curve corresponds to an average of ten to fifteen voltage sweeps. Solid lines: prediction of the full theory including the effect of temperature and of the finite length of the wire [equivalent circuit shown in Fig. 1(b)]. Dotted lines: predictions for an infinite wire with the same parameters, including the temperature. Dashed lines: predictions for the infinite wire at $T = 0$ showing the $V^{-1/2}$ dependence. Black arrows indicate the position of the crossover voltage $V = 10k_BT/e$. White arrows indicate the energy $\hbar D^*/(L/2)^2$.

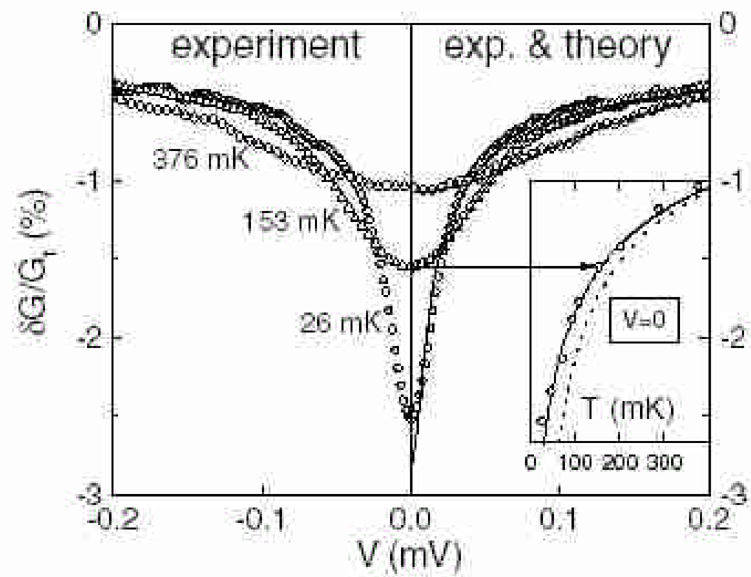

FIG. 3. Symbols in main panel: same experiment as in Fig. 2, but with data near $V = 0$ plotted on linear scale. Solid lines: Predictions for our finite length wire. Inset: $V = 0$ differential conductance. Solid line: Prediction for our finite length wire. Dotted line: $T^{-1/2}$ dependence expected for an infinite wire.

Environment Theory

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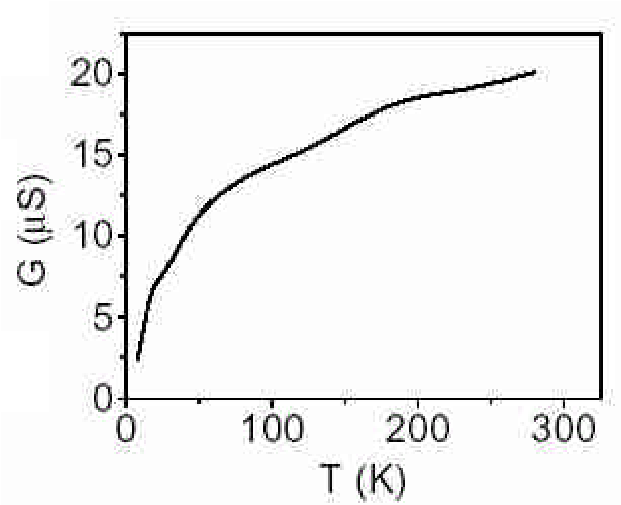
Environment theory is a bit incomplete

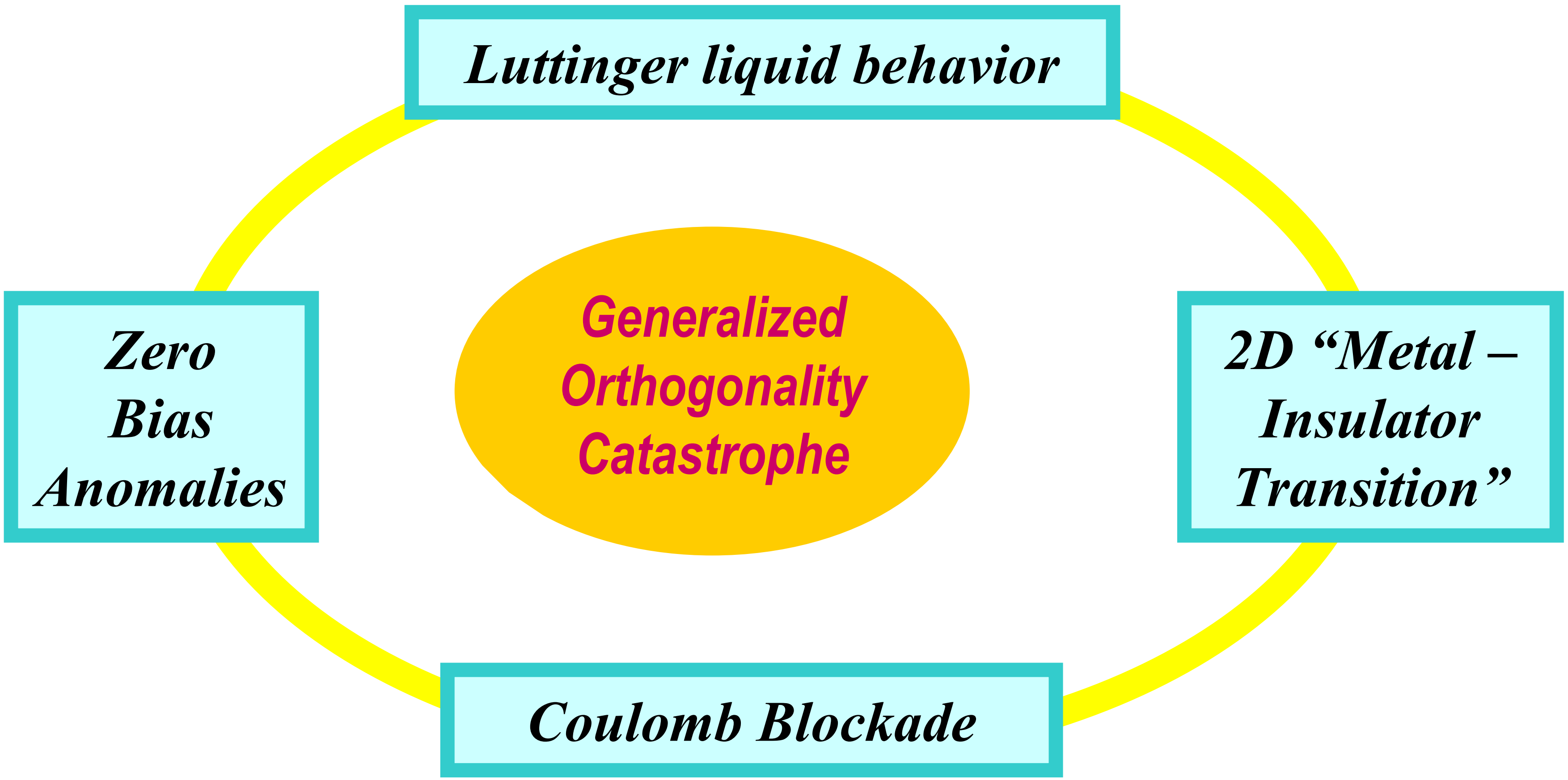
Most important point: it takes into account only **electromagnetic** fluctuations of the environment and **neglects** its **spin** fluctuations.

As a result – **only minimum** in the tunneling DoS (!)



Q: Is it a Luttinger liquid
or maybe it is environment
or it is electron-electron interaction ?

A: All the above is also electron-electron interaction in a form of generalized orthogonality catastrophe



! have nothing to do with any dephasing !

The End