Disordered Elastic Media

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Contents

1 Glossary

Pinning: action exerted by impurities on an object. The object has a preferential position in space, and will only move in response to an external force if this force is large enough.

Scaling: the fact that two quantities vary as a powerlaw of one another.

Random manifold: a single elastic structure (line, sheet) embedded in a random environment.

Bragg glass: a periodic elastic structure embedded in a weakly disordered environment, nearly as ordered as a solid but exhibiting some characteristics normally associated with glasses.

Creep: very slow response at finite temperature of a pinned structure in response to an external force.

2 Definition of the subject and its importance

Many seemingly different systems ranging from magnets to superconductors, with extremely different microscopic physics, share the same essential ingredients, and can be described under the unifying concept of disordered elastic media. In all these systems an internal elastic structure, such as an interface between regions of opposite magnetization in the magnetic systems, is subjected to the effects of disorder existing in the material. A specially interesting feature of all these systems is that these disordered elastic structures can be set in motion by applying an external force on them (e.g. a magnetic field sets in motion a magnetic interface), and that the motion will be drastically affected by the presence of the disorder. What properties results from this competition between elasticity and disorder is an extremely complicated problem which constitutes the essence of the physics of disordered elastic media. The resulting physics present characteristics that are the ones of glasses. This poses extremely challenging fundamental questions to determine the static and dynamic properties of these systems. Understanding both the static and dynamic properties of these objects is not only an important question from a fundamental point of view but has also strong practical applications. Indeed, being able to write an interface between two regions of magnetization or polarization and the speed of writing and stability of such regions is what conditions, in particular, our ability to store information in such systems, as for example recordings on a magnetic hard drive. The physics pertaining to the disordered elastic media directly condition how we can use in practise these systems for applications.

3 Introduction

Understanding the statics and dynamics of elastic systems in a random environment is a longstanding problem with important applications for a host of experimental systems. Such problems can be split into two broad categories: (i) propagating interfaces such as magnetic [1, 2, 3, 4], spintronic [5, 6], or ferroelectric [7, 8] domain walls, fluid invasion in porous media [9], contact line in wetting [10], epitaxial growth [11] or crack propagation [12,

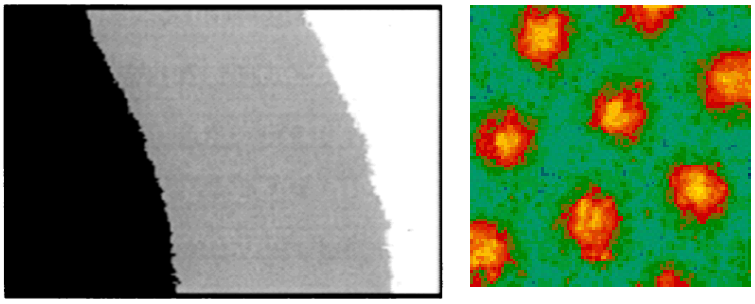

Figure 1: Left: an interface in a magnetic system separating two different polarization of the magnetization (dark and white). The image is $90 \times 72\mu m^2$. The roughness of the domain wall due to the presence of disorder in the system is obvious on the image. Two positions of the interface are shown. Dark and grey correspond to two consecutive images after the interface has been pulled to the right by applying a magnetic field to the sample favoring the magnetization direction on the left of the interface. Such a magnetic field acts as a force pulling the domain wall [From [1] (Copyright 1998 by the American Physical Society)]. Right: A vortex lattice image, from Scanning Tunnelling Microscope, in the superconductor MgB_2. The tip of the vortices on the surface of the sample are shown in the image, and correspond to the red parts. The image is about $250nm^2$. In a perfectly pure system, the vortex lattice is a periodic arrangement (here in a triangular lattice) of objects of a given size (here the core size of the vortex). Disorder affects over large distance this perfectly periodic arrangement [From [24] (Copyright 2002 by the American Physical Society)].

13]; (ii) periodic systems such as vortex lattices in type II superconductors [14, 15, 16], charge density waves [17, 18], magnetic bubbles [19], colloids [20], Wigner crystals of classical particles [21] or of electrons [22, 23].

Although all these systems have very different microscopic descriptions, one aspect of their physics is identical at a more macroscopic scale. An object exists that obeys a macroscopic elastic description. For the case a) this is an interface separating two different regions of the system, for example in a magnetic material a domain wall separating regions of opposite magnetization. An example of such an interface is shown in Fig. 1. Since creating an interface costs energy, the interface left to itself would like to be flat, and there is an elastic cost to its deformations. Since this object lives inside a microscopic crystal with disorder, it is also subjected to potentials that tend to roughen it and pin it in specific regions of space. This interface can be set in motion by applying an external force on it, caused for example by a magnetic field for the magnetic domain wall or an electric field for a ferroelectric. For the case b) of periodic systems a similar physics exists.

A "crystal" of objects (lines for vortices, points for magnetic bubbles and colloids, or sheets for charge-density waves) exists inside the system. Since these objects repel each other, in the absence of disorder they would tend to form a perfect periodic crystal. In a way similar to the interfaces, such crystals can be set in motion by applying an external force (for example a current for the case of vortices). The disorder present at the microscopic level tends to pin this crystal. It is important to note that one is dealing here with the physics of a crystal embedded in an external medium, containing impurities. The disorder can thus vary at a lengthscale much smaller than the lattice spacing of the moving crystal, which leads to a physics radically novel compared to the one of chemical impurities in a regular solid. Understanding the physics, both static and dynamics, of these objects has thus been a considerable challenge in the last 50 years or so. There are several reasons to this interest, and for the challenges posed by this field of disordered elastic media.

First at the fundamental level these systems pose extremely difficult and important questions. It is known since the 1970's that the presence of disorder is crucial [25] and changes the physics completely. Unfortunately the resulting models are very difficult to solve, and have contributed to pushing the limits of our understanding of disordered systems, and to develop new techniques of statistical physics to deal with such issues. In particular it is clear that from the competition between disorder and elasticity emerges a complicated energy landscape with many metastable states. This results in glassy properties [26] such as hysteresis and history dependence of the static configuration. Initially viewed as some toy models of glasses these systems have acquired their own importance and posed their own challenging questions. Understanding the static properties of such system has stimulated the development of sophisticated approaches such as replica theory [27], functional renormalization group [28] or numerical methods. Much progress was recently accomplished both due to these analytical and numerical advances. If the static allows us to improve our techniques of statistical physics, the dynamics is even more complicated since most of our theoretical tools fail. These system thus provides wonderful motivations to develop new techniques to tackle the out of equilibrium dynamics of disordered systems and to understand and unify the concepts of out of equilibrium physics of glasses.

Second, in addition to this theoretical motivation, the possibility of realizing such system in so many different physical systems is a tremendous motivation and challenge. The various realizations allow to put stringent tests on the proposed theories and have, as we will see, very often served to kill wrong proposals or to put the theory on the right track in these quite complicated systems. Experiments in these systems can be remarkable by the range they offer. For example for vortex systems one can vary the vortex lattice spacing by several orders of magnitude, just by changing the mag-

netic field applied to the sample, something impossible to do on a simple crystal. Similarly, for magnetic domain walls, measurements of the velocity in response to an external force are spanning about ten orders of magnitude. This interplay and exchange between theory and experiments has fuelled the field and contributed greatly to its progress.

Last but not least, the phenomena studied for disordered elastic media have a potential impact for applications. Creating interfaced in magnetic or more recently ferroelectric materials is a way to store information (with the "0" being one direction of magnetization or polarization and "1" being the reversed one). This idea is at the root of storage of information in a magnetic hard drive or in a ferroelectric memory, or a magnetic bubble one. How well one can store the information is thus directly related to the properties both static and dynamics of such interfaces. In particular the stability of the written information is only ensured if the interface is pinned and will not meander, for example by thermal agitation. In a similar way there has been much interest recently on spintronic materials where the magnetic properties can be manipulated by applying electrical currents [5, 6]. In the same vein, multiferroic materials [29] allows to manipulate ferroelectric properties by the application of magnetic fields. How the disordered interfaces behave in such materials will certainly condition their possible use for information technology. In a similar way the vortex lattice in a superconductor is set in motion by the application of a current, while its motion generates a voltage. The dynamic properties of the vortex lattice, and how well it is pinned, thus directly condition the absence of resistance of a superconductor [14, 15, 16], hence its potential uses. Other examples, such propagation of fractures, clearly show the potential importance of such phenomena for applications.

This chapter presents the basic concepts and results in this very active field. Sec. 4.1 presents the basic concepts and discusses the static properties of interfaces and domain walls. Sec. 4.2 deals with the periodic systems and their differences compared to the interfaces. Sec. 5 presents the concepts and important questions for the dynamics of disordered elastic media, with focus on the depinning in Sec. 5.2, on the large velocity behavior in Sec. 5.3 and response to a small external force in Sec. 5.4. Finally Sec. 6 discusses the future directions and perspectives of the field.

4 Static properties of disordered elastic media

4.1 Interfaces and basic concepts

Let me introduce in this section the basic ingredients of the systems under study, and discuss the specific case of interfaces. An interface is a sheet of dimension d living in a space of dimensions D. For realistic interfaces $D = d + 1$ but generalization are of course possible. Calling r the internal coordinate of the interface and z all its transverse directions, the interface

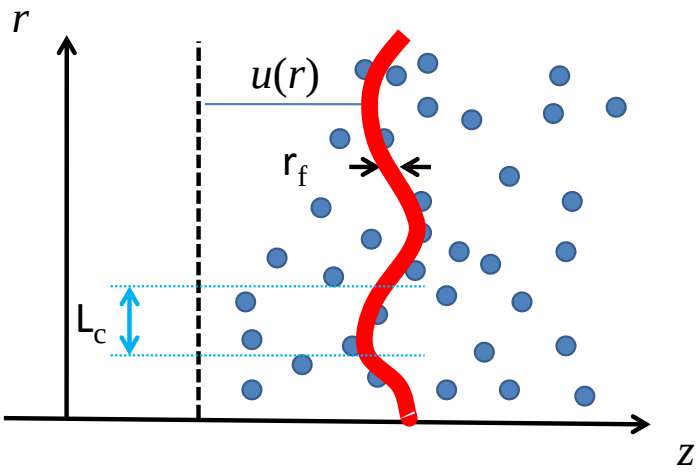

Figure 2: A one dimensional interface (such as a magnetic domain wall), shown in red, living in a two dimensional space (film). The position of the interface is determined (provided there are no overhangs or bubbles) by the displacement $u(r)$ from a flat configuration, indicated by the dashed line. In the absence of disorder, denoted by the blue dots, which pin the line in preferred positions in space, the line would be flat. The competition between elasticity and disorder leads to the physics of disorder elastic media and to glassy properties. The thickness of the line, denoted r_f or the correlation length of the disorder define the Larkin length L_c for which the relative displacements are of the order of r_f, namely $u(L_c) - u(0) \sim r_f$.

position is labelled by a displacement $u(r)$ from a flat configuration. This determines totally the shape of the interface provided that u is univalued, i.e. that there are no overhangs or bubbles. The modelization of a one dimensional interface ($d = 1$) in a two dimensional film is shown in Fig. 2.

Since the interface distortions cost elastic energy, its zero temperature equilibrium configuration in the absence of disorder is the flat one. Deviation from this equilibrium position are described by an Hamiltonian $H[u]$ which is a function of the displacements u. For small displacements one can make the usual elastic approximation

$$H[u] = \frac{1}{2} \int \frac{d^d q}{(2\pi)^d} c(q) u_q^* u_q \tag{1}$$

where u_q is the Fourier transform of $u(r)$ and $c(q)$ are the so called elastic coefficients. If the elastic forces acting on the interface are short ranged then one has $c(q) = cq^2$ which corresponds to

$$H[u] = \frac{c}{2} \int d^d r (\nabla u(r))^2 \tag{2}$$

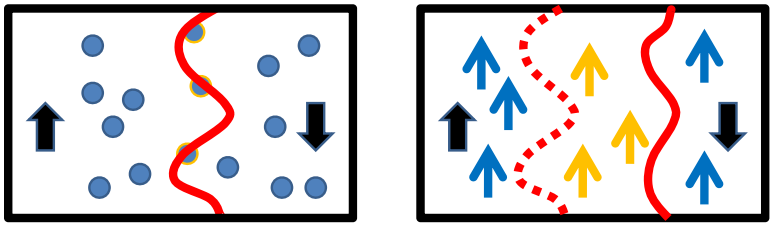

Figure 3: The two universality classes of disorder (the names are coming from the magnetic realization of such systems). In both figures the domain wall in red, separates to regions with different order parameters, denoted by the two thick black arrows. Left: random bond disorder. The impurities, denoted in blue couple symmetrically to the two sides of the domain wall, thus only the impurities on the domain wall (denoted with the orange circle) contribute to the energy. Right: random field disorder. The impurities, denoted by the blue arrows, favor one of the two sides. Thus all the impurities (denotes in orange) between two configurations of the domain wall contribute to the energy. This leads to a disorder seen by the domain wall which has long range correlations even if the microscopic disorder is short range correlated.

For some interfaces where long range interactions play a role different forms for the elasticity are possible. This is in particular the case when dipolar forces [30] are taken into account [8] or for the contact line in wetting [31] and crack propagation [32].

In addition to the elastic energy the interface gains some energy by coupling to the disorder. Two universality classes for the disorder exist (see Fig. 3). The so called random bond disorder corresponds to impurities that directly attract or repel the interface. On the contrary, for the so called random field disorder the pinning energy is affected by all the randomness that the interface has encountered in its previous motion. On a more technical level, the random bond disorder couples in a symmetric way to the two order parameters on each side of the domain wall, while the random field introduces an asymmetry between these two inequivalent order parameters. If $V(r,z)$ denotes the random potential generated by the impurities the pinning energy writes:

$$H_{\rm dis}[u] = \int d^d r \begin{cases} V(r,u(r)) & \text{random bond} \\ \int_0^{u(r)} dz V(r,z) & \text{random field.} \end{cases} \tag{3}$$

As is obvious from Fig. 3, even if the microscopic disorder is short range correlated, in the case of the random field disorder, the fact that the energy of the system integrates between two positions of the interfaces, means that long range correlations exist if one considers only the description in terms of the interface. The full Hamiltonian given by (1) and (3) determines the

properties of disordered elastic media, and despite its apparent simplicity hides an extremely rich physics.

The competition between disorder and elasticity manifests itself in several properties of the interface. From the energetic point of view this competition leads to a complicated energy landscape for the configurations of the system, with many metastable states leading to glassy properties. The competition manifests itself also in the shape of the interface. In particular, it deviates from the flat configuration and becomes rough. From the scaling of the relative displacements correlation function, a roughness exponent ζ can be defined from the correlation function of the displacements

$$B(r) = \overline{\langle [u(r) - u(0)]^2 \rangle} \propto r^{2\zeta} \tag{4}$$

where $\langle \ \ \rangle$ denotes thermodynamic average and $\overline{\cdots}$ denotes disorder average. There are relations between the shape of the line and the energetic properties. In particular (4) suggests that displacements would scale with distance as $u(L) \sim L^{\zeta}$. Using (2) suggests that the energy of a sample of size L fluctuates from sample to sample as

$$\Delta F(L) \sim L^{d-2+2\zeta} \tag{5}$$

Given the complexity of the problem, several approximate methods have been put forward to guess the role of the disorder. A remarkable model to probe the physics of such systems was introduced by Larkin [25], and goes by the name of the Larkin model. The idea is to focus on short length scale properties. In that case the displacements are small and one can expand the disorder term in power of the displacements

$$H_{\rm dis} = \int d^d r V(r, u(r)) \simeq \int d^d r [V(r,0) + \nabla_r V(r,0)|_{z=0} \, u(r)] \tag{6}$$

The first term is a trivial constant and the second one indicates that the interface is subjected to a random force

$$H_L = \int d^d r f(r) u(r) \tag{7}$$

Although this model has several pathologies, it has the advantage of being quadratic in the displacement field u and thus of being exactly solvable. It shows that below $d = 4$, the disorder plays a major role. The displacements grow as a function of distance and

$$B(r) = r^{4-d} \tag{8}$$

This confirms that there is algebraic roughening of the interface with the displacements growing as a power law of distance. One can define the scaling $u(L) \sim L^{\zeta}$, with the Larkin model giving $\zeta = (4-d)/2$. Below $d = 4$ the

disorder is relevant and drastically modifies the physical properties of the interface compared to those of the non disordered one. In addition to the exponent itself, since the displacements grow unboundedly, there exists a lengthscale, L_c, called the Larkin length at which the displacements become of the order of the only characteristic scale available, namely either the correlation length of the random potential or the size of the interface r_f, as shown in Fig. 2. Clearly this is also the length where the applicability of the Larkin model breaks down since beyond that length the potential $V(r,z)$ is not smooth anymore and thus the expansion in powers of u is not justified. Beyond this length the system will thus truly feel the effects of the random potential. One can thus expect metastability, glassy effects and pinning to appear above that length. One has to determine the physics of this regime appearing above the Larkin length, which we will call the random manifold regime. The Larkin length is thus an important lengthscale for the static properties since it separates two different regimes for the interface. As I will discuss in Sec. 5, the Larkin length has also considerable consequences for the dynamics.

To solve the problem in the random manifold regime is not easy and requires greatly sophisticated techniques of statistical physics. To get a rough idea, one can simply use a scaling argument, known as Flory argument [14]. At the scale L the elastic energy scales as $cL^{d-2}u(L)^2$. To estimate the disorder term is more complicated but one can assume that if L is large enough one sums random variables $V(r,z)$. If one considers for example the random bond disorder, because the disorder is short range correlated one has

$$\overline{V(r_1,u_1)V(r_2,u_2)} = D\delta^m(u_1-u_2)\delta^d(r_1-r_2) \tag{9}$$

Since a $\delta(r)$ function has the dimension of $1/r^d$, this leads to the scaling

$$V(r,u) \sim D^{1/2}u(L)^{-m/2}L^{-d/2} \tag{10}$$

where m is the number of components of u (for an interface $m=1$). The disorder term thus scales as

$$H_{\rm dis} = D^{1/2}u(L)^{-m/2}L^{d/2} \tag{11}$$

Balancing the elastic and disorder term leads to a scaling $u(L) \sim L^{\zeta}$ with $\zeta = \frac{4-d}{4+m}$ for the random bond case and $\zeta = \frac{4-d}{4-m}$ for the random field one. This argument suggests that even in the random manifold, the interface remains rough, with unbounded displacements growing with an *algebraic* roughness. The value of the exponent is characteristic of the universality class of the disorder, and different from the one occurring below $L < L_c$ where the Larkin model applies.

Clearly this simple argument needs to be substantiated by more rigorous calculations. Because the system is subjected to a random potential

and the metastability and glassy effects matter, one can use the techniques traditionally used for disordered systems and spin glasses. For a one dimensional interface, this problem can be solved exactly and the roughness exponent $\zeta = 2/3$ has been obtained [33, 34]. Note that this exact value is of course slightly different from the mean field estimate. In higher dimensions three main methods have been used to tackle this problem. The first one used the so called replica trick [26] to average over the disorder and then a variational approach to solve the corresponding field theory [27]. In this method the initial symmetry between replicas is broken, something familiar in spin-glasses and characteristics of glasses with many metastable minima in the energy. This approximate method gives back the Flory exponent. The second method applies the traditional renormalization technique (RG), so successful for standard critical phenomena. This consists in looking at the problem at larger and larger lengthscales, eliminating degrees of freedom, while changing the Hamiltonian to ensure that the large lengthscale physics remains invariant. Usually one can expand the interaction potential, and only a few terms are relevant, which means that the RG consists in the flow of a few numbers of coupling constants. In the case of disordered elastic systems, the task is considerably more complex since all powers in the expansion of the correlator of the disorder have the same scaling dimension. During the flow the *whole correlator of the disorder* is modified. One must thus follow the renormalization of a whole function, hence the name of functional renormalization group (FRG) [28]. This leads to a remarkable property. Beyond a lengthscale that coincides with the Larkin length, the disorder correlator, initially a smooth analytic function, becomes non-analytic and develops a cusp. The appearance of this non analyticity is, in this method, the signal of glassy physics. The FRG allows to obtain the roughening exponent in a systematic expansion in $\epsilon = 4-d$. This has been worked out for the moment up to second order in $\epsilon = 4 - d$, leading to $\zeta = 0.20829804\epsilon + 0.0068582\epsilon^2$ and $\zeta = \epsilon/3$ for the random bond and random field disorder respectively [35]. Note that for the random field the mean-field (Flory) exponent was exact due to the long range nature of the disorder. In addition to these analytical approaches, a very useful approach is provided by numerical studies of such systems, using either molecular dynamics simulations [36], Monte Carlo techniques [37, 38], or specially designed algorithms [39, 40]. Numerical approaches are of course quite challenging due to the glassy nature of the system with many metastable minima close to the ground state. However they have proven quite useful in obtaining not only the asymptotic regime but also the full crossover between the Larkin and random manifold regimes, as well as incorporating the effects of finite temperature.

These predictions can be verified experimentally. I show in Fig. 4 the roughness exponent as measured in a magnetic and a ferroelectric film. The algebraic growth of the correlation function $B(r)$ is clearly seen. These two experimental situations correspond to two different dimensionalities for the

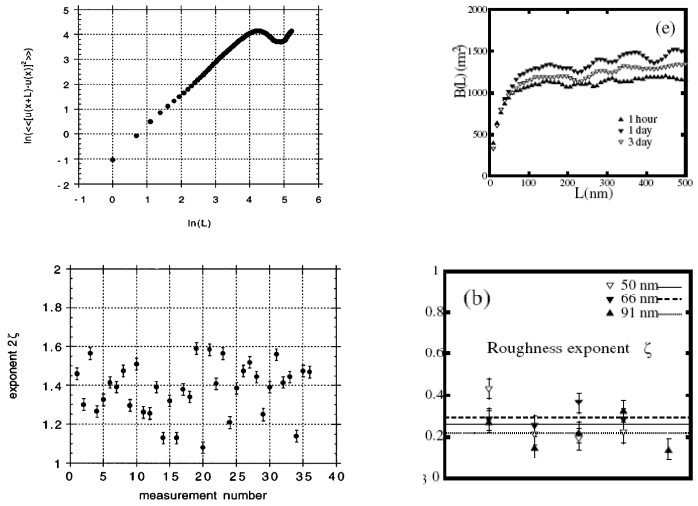

Figure 4: Measurements of the roughness exponent ζ in two experimental system. These two examples show the algebraic roughness of the domain walls. The top figures are the correlation function (4) of the displacements $B(r)$, and the bottom ones the measured value of the roughness exponent ζ. Left: Magnetic domain walls in thin magnetic films. An exponent of $\zeta \sim 0.6$ is measured compatible with the value $\zeta = 2/3$ expected for a one dimensional wall in a two dimensional space [From [1] (Copyright 1998 by the American Physical Society)]. Right: Ferroelectric domain wall in a ferroelectric film. An exponent of $\zeta = 0.26$ is measured. This value is compatible with the value expected for a two dimensional wall in a three dimensional space in presence of long range dipolar interactions [From [8] (Copyright 2005 by the American Physical Society)].

domain walls, due to the different thickness of the material and different characteristics of the domain wall.

We can thus say that now we have a rather good understanding of the static properties of the interfaces at least for the simple case of local elasticity show here. Of course even for the statics this is not the end of the story since several microscopic systems such as the contact line of a fluid, or ferroelectric systems have long range interactions (dipolar interactions) making even the static properties quite challenging to determine. Other open questions will be discussed in Sec. 6.

4.2 Periodic systems and Bragg glass

Similar concepts apply directly to the case of periodic systems. In all these systems the constituent elements (lines for vortices, points for colloids and magnetic bubbles, sheets for phase maxima in the charge density wave sys-

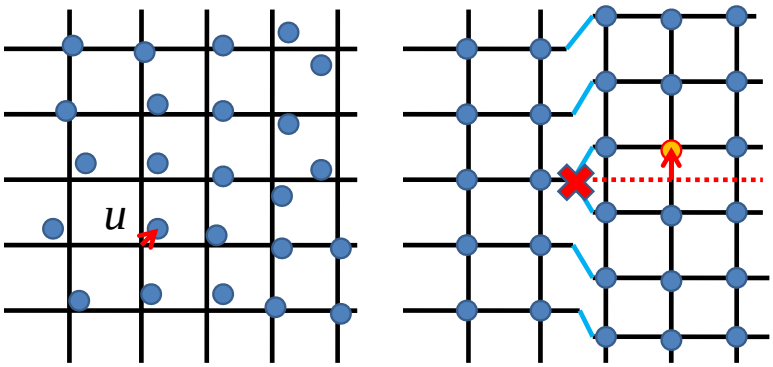

Figure 5: For a periodic system the possibility to define a displacement $u(r)$ for the objects (blue dots) compared to the perfect lattice (here a square lattice corresponding to the intersections of the black lines), necessitates the absence of topological defects. Left: if there are no topological defect one can associate for each site R_i^0 of the perfect lattice a displacement u_i, as indicated by the red arrow. Right: here there is a topological defect, denoted by the red cross, corresponding to the addition of one line of particles. In that case the displacement u_i is not univalued. From the point of view of the particles on the left of the topological defect, the orange particle has a displacement u of half a lattice spacing, while one could take $u = 0$ looking only at particles on the right of the topological defect.

tems) form a solid, that is embedded into the microscopic system but can have widely different characteristics and in particular widely different lattice spacing. For example in the case of vortices the lattice spacing is controlled by the magnetic field and can easily be varied. An important characteristic is thus that in a similar fashion to the interface, this crystal can be embedded in the "external" disorder, that corresponds to the imperfections of the real microscopic lattice in which this artificial crystal lives. It is important to note that the variation of the disorder potential can thus occur at lengthscales much smaller than the lattice spacing.

Each point of the system can be described by an equilibrium position R_i^0 forming a perfect lattice (usually triangular for the vortex lattice), and a displacement u_i relative to this equilibrium position, as shown on Fig. 5. As for the interfaces the interactions between the objects forming the crystal favor a perfectly ordered crystal. The energy of the system can be expanded for small deviations and lead to a quadratic expansion in u characteristics of an elastic energy. It is important to note that for such an expansion to be valid it is only necessary for the relative displacements $u_i - u_j$ between two neighbors to be small but the displacements themselves can be arbitrary (for example translating the whole crystal by a uniform displacement does

not change the energy).

$$H = \frac{1}{2} \sum_{ij} C_{ij} (u_i - u_j)^2 \tag{12}$$

where the C_{ij} are the elastic coefficients of the system. Since the interactions can be long range, the elastic coefficients are not necessarily limited to nearest neighbor only. Note that for such an expansion to be meaningful, it is necessary for the displacements to be uniquely defined. This assumes that there are no topological defects (dislocations, disclinations) in the crystal. Indeed in the presence of such defects, as shown in Fig. 5 the displacements have two different values when circling around the defect. In order to use the elastic approximation it is thus important to ascertain that topological defects are not generated. I will come back to this crucial point below.

As for the interfaces the minimum energy configuration is the perfect crystal with all $u_i = 0$. In the absence of disorder this perfect crystal can only be affected by the thermal fluctuations. If temperature becomes too large the crystal will melt. A rule of thumb for the melting is when the relative displacements between two neighbors become a sizeable fraction of the lattice spacing

$$\langle (u_i - u_{i+1})^2 \rangle = C_L^2 a^2 \tag{13}$$

where $\langle \cdots \rangle$ denotes the thermal average and C_L is a phenomenological constant which turns out to be of the order of $C_L \sim 0.1$ to reproduce reasonable values for the observed melting of solids. This rule of thumb, called the Lindemann criterion for melting, gives in fact quite decent results.

In the presence of disorder, one must add to the elastic energy term (12) the energy coming from the random potential created by the disorder. This takes the form

$$H_{\rm dis} = \int d^d r V(r) \rho(r) = \int d^d r V(r) \sum_i \delta(r - R_i^0 - u_i) \tag{14}$$

and this term will clearly tend to disorder the crystal.

The case of a periodic system constitutes a specially important and interesting situation. Indeed the nature of order and the possible phases are more complex than for the case of the interfaces. An important question is thus whether these two systems are in the same universality class or not. In a general way the order in a periodic system is characterized by a positional order, indicating if one can find a particular particle of the solid at a given position, knowing the position of a reference particle. This positional order can be measured by the structure factor, which is the correlation function of the Fourier transform of the density $S(q) = \langle |\rho_q|^2 \rangle$. In a perfect crystal, the structure factor has *divergent* peaks at the position of the reciprocal vectors K_0 of the perfect lattice. The presence of such divergent peaks indicates

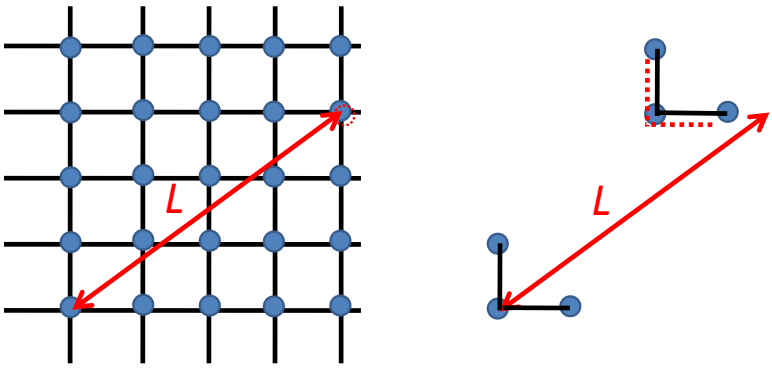

Figure 6: A periodic system possesses both positional, orientational and topological order. Left: positional order. A solid has perfect positional order if by knowing the position of a reference particle one can predict the position of a particle at distance L as indicated by the dashed circle and the red arrow. Right: orientational order. A solid has orientational order if by knowing the orientation of the bonds in a region of space, one can predict the orientation at a distance L, as indicated by the red dashed lines. Note that the system need not possess positional order for the orientational order to exist. Topological order: topological order exist, if after a triangulation the topology (i.e. the number of neighbors) of each point of the solid is fixed and the displacements can be defined in a univalued manner. The picture on the left possess perfect topological order.

a perfect positional order. The fact that one sees peaks also indicates the existence of another type of order, namely the orientational order in a solid. This is illustrated in Fig. 6. The orientational order indicates that if the bonds are having a certain orientation in a region of space, this orientation is preserved in the other parts of the solid. Losing the orientational order replaces the peaks in the structure factor by a ring since the orientation of a given peak is not defined any more. In standard solids both order are lost usually at the same time, and the solid melts to a liquid, usually by a first order phase transition. But we also know that in some cases for pure systems, such as for example two dimensional solids, the melting may occur as a two step process where the positional order is lost first and then only the orientational order, leading to a so called hexatic phase [41]. A summary of the various cases is shown in Fig. 7. In addition to these standard order parameters, a periodic system is also characterized by a topological order corresponding to the fact that the connectivity of the perfect crystal is preserved by small displacements. Such order is determined by a triangulation of the solid and a determination of the topology. If the topology is identical to the one of the perfect lattice it means that the displacements can be defined in a univalued way across the solid. In the liquid, topological defects

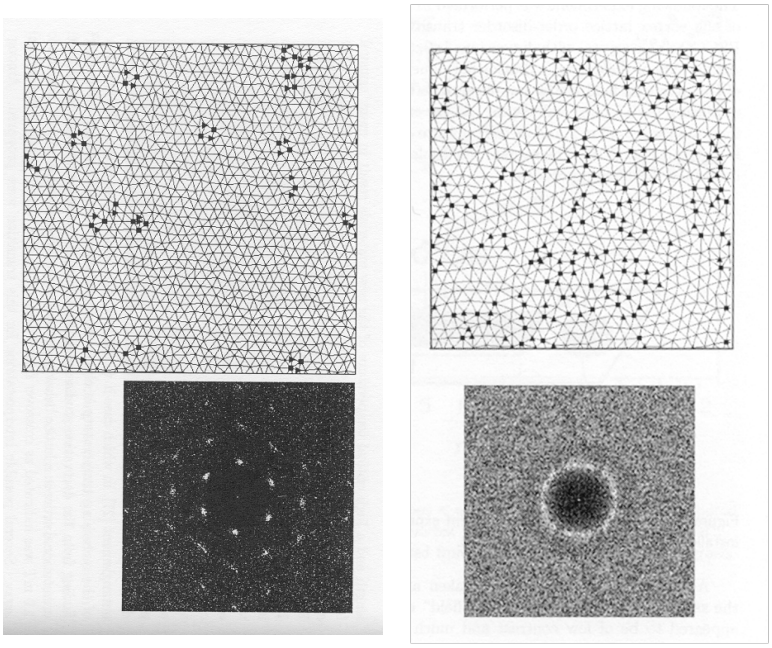

Figure 7: Decoration images of vortex lattices, illustrating the difference between solid and liquid phases. The top figures are the images in real space, while the bottom ones are the structure factor $S(q) = \langle|\rho(q)|\rangle$. Left: the system is in a solid like phase (in fact a Bragg glass phase (see text)). The system possesses good positional order and orientational order. This can be seen both from the pictures in real space and from the structure factor that shows Bragg peaks at the position of the reciprocal vectors K_0 of the perfect underlying lattice. As shown by the triangulation most sites have six neighbors. Topological defects where sites have five or seven neighbors (as indicated by the triangles and square black marks respectively) do exist, but are paired in $5-7$ pairs, making the system free of topological defects at large lengthscales. Right: the system is in a liquid like phase. Positional order and orientational order are lost. The Bragg peaks are gone and the structure factor has a ring like structure (indicating the loss of orientational order). The topological defects are proliferating and are unpaired contributing to the exponential decay of order in the system [Images from M. Marchevsky, J. Aarts, P.H. Kes (unpublished)].

such as dislocations and disclinations destroys this perfect topological order and the very concept of displacements around an equilibrium position becomes ill defined, since in that case the displacement field is not univalued any more.

As for interfaces the disorder changes the properties of the pure elastic system. In order to take into account the effect of disorder on periodic systems, it is thus important to address two different aspects of the problem: a) what is the effect of the disorder on an approximation of the real system given by the elastic theory; b) to determine whether the disorder is able to generate topological defects, in which case the very idea of an elastic approximation breaks down and another starting point should be found.

As was shown in the groundbreaking paper by Larkin [25], there exists always for $d \leq 4$, a characteristics lengthscale L_a for which the displacements become of the order of the lattice spacing a of the perfect crystal. The fact that displacements can become as large as the lattice spacing indicates that the *perfect* positional order is lost. The question of *how* this destruction is taking place and what is the resulting phase is a long standing problem. Given the complexity of the question no solution existed until recently, but it is interesting to see that the community converged, by inference on closely related models, to a consensus that was accepted for a long time but eventually proved wrong. The route followed was to learn as much as possible from the interfaces. At short distance one can make an expansion in powers of the displacements, and the system is described by the Larkin model. This ceases to be valid at the Larkin length L_c for which the displacements become of the order of the size r_f of the particle in the solid. Note that L_c and L_a are in general two different lengthscales since the size of the particle r_f and the lattice spacing a are usually different. Naturally one has $L_c < L_a$. For systems such as the vortex lattice or Wigner crystals the difference can be huge, while for charge-density waves where one expects to have $r_f \sim a$ and thus $L_c \sim L_a$. Below the Larkin length the system is described by the Larkin model and thus with an algebraic growth of the displacements with a roughness exponent of $\zeta = (4-d)/2$. Above the Larkin length L_c but for displacements smaller than a (i.e. for lengths smaller than L_a) one can consider that the various objects of the periodic system don't see each other except by their elastic forces. In particular they do not sample the same random potential given the smallness of the displacements. One has thus a regime very similar the random manifold regime of the interface. The displacement continues to grow algebraically $u \sim L^{\zeta}$, albeit with a different exponent. Above L_c, the connection between the growth of the displacements and the structure factor (the density-density correlations) is non-trivial since the model is non-gaussian. Indeed the structure factor is given by [42]

$$S(K_0 + q) = \int d^d q e^{iqr} C(r) \tag{15}$$

where

$$C(r) = \langle e^{iK_0 u(r) e^{-iK_0 u(0)}} \rangle \tag{16}$$

For Gaussian models such as the Larkin model one had $C(r) = \exp[-\frac{K_0^2}{2} B(r)] \sim \exp[-\frac{K_0^2}{2} r^{2\zeta}]$ indicating a stretched exponential decay of the positional order. Such an exponential decay of the positional order would lead to non divergent peaks in the structure factor. The constant finding of algebraic roughening, and the existence of the lengthscale L_a seemed to suggest that even beyond L_a the roughening was also algebraic, with perhaps another exponent. One would thus naively expect $C(r) \simeq \exp[-\frac{K_0^2}{2} a^2 (r/L_a)^{2\zeta}]$ giving Lorentzian like non divergent Bragg peaks with a width controlled by $1/L_a$. In addition to this exponential loss of positional order occurring even in the elastic theory one could question the very starting point of the analysis, namely the elastic limit and the single valueness of the displacements. One could indeed expect topological defects (dislocations, disclinations etc.) to be generated at the scale L_a where the displacements were of the order of the lattice spacing a. Indeed there were arguments [43] (as we will see incorrect) "showing" that *an arbitrarily small disorder* would always generate topological defects at lengthscale L_a, leading definitely to an exponential loss of positional order beyond this length. All these elements thus seemed to click together to suggest the picture of a crystal broken in little crystallites of size L_a as shown in Fig. 8. A consensus was thus reached in the community that disordered periodic system would just loose translational order and some theories for the vortex lattices were built on this incorrect premises . However this picture crumbled on two fronts. On the experimental side, it was in direct contradictions with experiments showing for example extremely large regions free of defects [44, 45] or a first order melting [46], which was hardly compatible with a very disordered solid in which all positional order would have been lost from the start. On the theory side, our understanding of glassy systems had reached a point where better solutions of this problem could be reached. The displacements were found to grow in fact only logarithmically [47, 48, 49, 42] with distance $B(r) = A'_d \log(r)$ (or $u(L) \sim \log(L)^{1/2}$). The prefactor A'_d was computed using either a variational approach [48, 49, 42] or an FRG one [49, 42]. The elastic disordered systems have thus a completely different rougheness than the interface. This a priori surprising behavior can be explained in a qualitative way as shown in Fig. 9. Quite interestingly the structure factor and positional order could still be computed for the full model [49, 42], and it was shown that in a quite non trivial way, the relation $C(r) = \exp[-\frac{K_0^2}{2} B(r)$ remains essentially applicable, leading to a *powerlaw* decay of the positional order $C(r) \propto (1/r)^\eta$, where the exponent η, is for all practical purposes a number determined only by the dimension [50]. For example $\eta = 1 - 1.2$ for a three dimensional vortex lattice. The algebraic decay of the positional order as well as the value of

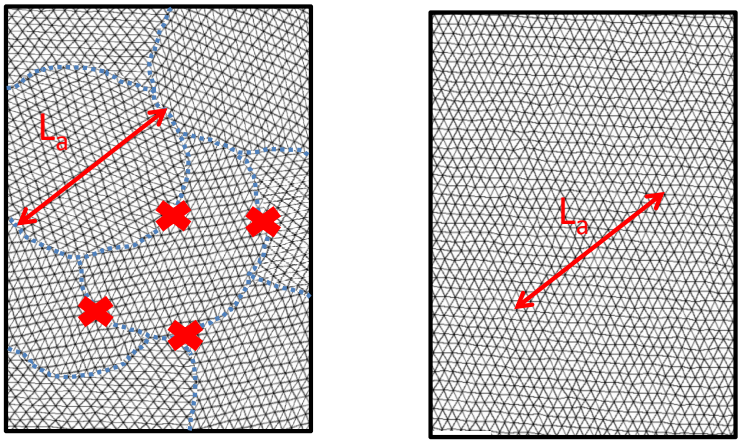

Figure 8: Left: the (incorrect) image of an elastic medium in presence of disorder. The system would be broken into "crystallites" of size L_a characteristic size for which the displacements become of the order of the lattice spacing a ($u(L_a) \sim a$). At the same lengthscale to release part of the elastic energy due to the disorder the system would prefer to create topological defects (schematically indicated by the red crosses). Beyond the size L_a indicated by the blue dashed line, the positional order would be lost exponentially fast. Right: the Bragg glass, describing the properties of a disordered periodic system in the presence of weak disorder. Although the positional order is destroyed at large lengthscale and the lengthscale L_a for which displacements are of order a exists, the system preserves quasi-long range positional order, and perfect topological order. No "crystallite" is thus associated with the lengthscale L_a, and no topological defects are generated by the disorder.

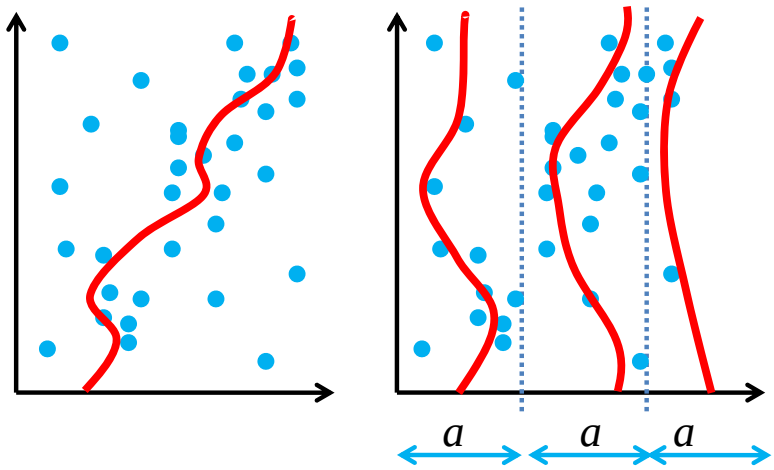

Figure 9: Schematic explanation of the difference of roughness between the interfaces and the periodic systems. Left: In an interface the large roughness is produced by the fact that there are always regions where it is energetically favorable for the line to go, and from there further to another region, increasing endlessly the displacements. Right: For a periodic system (here a periodic system of lines of period a), since what counts it the *total* energy of the system there is no interest for one line to make displacements much larger than the interline distance since it would just steal the disorder from the neighbor. Thus even with the same disorder, displacements would thus "saturate" (in fact still grow but very slowly with distance) when they reach the interparticle distance.

the exponent indicated that the system still retained *divergent* Bragg peaks in its structure factor and thus, although losing indeed the positional order, the loss was very slow and the system was nearly as ordered as a perfect solid. Furthermore is has been shown [42] that the argument claiming that disorder would always generate dislocations was incorrect and that on the contrary, due to the slow algebraic decay of the positional order, a three dimensional system is *stable* to the generation of dislocations, at least when the disorder is below a certain threshold. This has led to a radically different physics for a periodic disordered system, than the generally accepted consensus. Namely the two following facts: (a) algebraic (quasi-long range) decay of the positional order, and divergent Bragg peaks; (ii) absence of topological defect has led to the prediction [42] that the disordered periodic system were in fact in a new state of matter, the Bragg glass. Such a system is a disordered system with glassy properties: energy landscape with many metastable states, the dynamics of a glass; but which would "look" nearly as ordered as a perfect solid. After the Bragg glass was first predicted its existence has been supported by further analytical [51, 52, 53] and numerical [54, 55] calculations.

The existence of the Bragg glass phase has important consequences and made it possible to reconcile several apparently contradictory results on the phase diagram of the vortices. It allows to explain that very large regions free of dislocations could be observed [44, 45] while the system is obviously pinned by the disorder. It explains [56] also the narrow peaks observed in neutron scattering experiment [57, 58], with a width given by the experimental resolution, which were indicating an excellent degree of positional order. The powerlaw nature of the peaks was directly tested in neutron scattering experiments proving directly the existence of the Bragg glass phase [59]. In addition to its intrinsic properties the presence of the Bragg glass phase puts strong constraints on the phase diagram. Indeed, since it is a phase without free topological defects, this phase has to "melt" either when the temperature becomes too high or the disorder too strong, since topological defects have to appear. In vortex systems the latter can be done by changing the magnetic field. The Bragg glass thus provided a very natural explanation [42, 60, 61] for the existence of a "melting" phase transition as a function of the magnetic field [62, 63], such a transition being associated with the destruction of the Bragg glass phase [64, 65]. An example is shown in Fig. 10.

This section can only cover a fraction of the physics of periodic systems, and several other questions have been explored. I refer the reader to the above mentioned literature on the subject for more details.

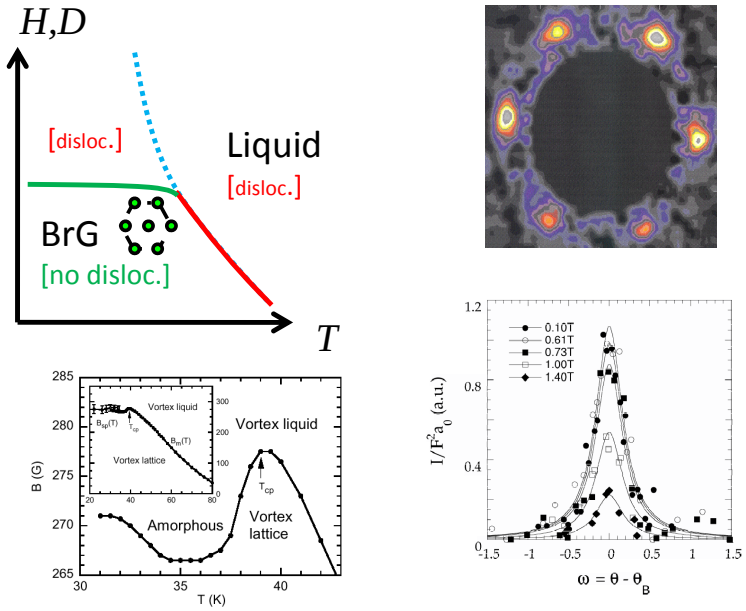

Figure 10: Left top: Schematic theoretical phase diagram for vortices as a function of the temperature T and the magnetic field H, or the disorder D. The Bragg glass (BrG) that has perfect topological order can melt either due to thermal fluctuations (red line) or because the disorder becomes too large (green line). The existence of the Bragg glass thus implies a single melting curve having a crossover between these two regimes. The melting of the Bragg glass thus explain the existence of a transition as a function of the magnetic field. The blue dashed line would be the melting line of the solid in the absence of disorder [After [61]] Left bottom: Measured phase diagram for high temperature superconductor BSCCO, showing that both melting transitions with temperature and with magnetic field are indeed the same melting curve [From [65] (Copyright 2001 by the Nature group)]. Right: Neutron diffraction on the superconductor BKBO. The structure factor shows clear Bragg peaks, indicating the good degree of both positional and orientational order despite the disorder present in the sample. As shown in the bottom diagram the width of the structure factor does not change when the magnetic field is changed, since it is controlled by the experimental resolution, while the height decreases. This is in agreement with the consequences of a powerlaw divergent Bragg peak, and is thus a test of the existence of the Bragg glass [From [59] (Copyright 2001 by the Nature group)].

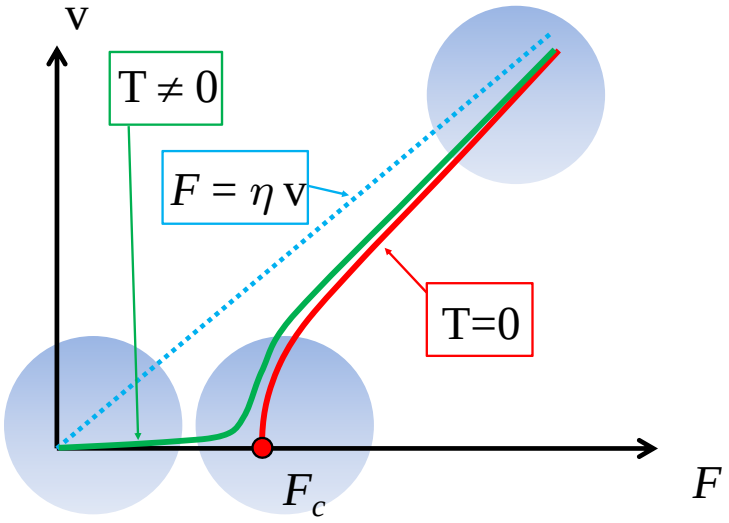

Figure 11: The velocity v induced by an external force F of a disordered elastic system. In the absence of pinning and with a damping coefficient η the steady state velocity $v = F/\eta$ is reached. At zero temperature $T = 0$ the system stays pinned until a critical force F_c is reached. At finite temperature a motion can occur even for forces below the threshold $F < F_c$ since the barriers to motion can always be passed by thermal activation. One can distinguish three very different regimes in this curve: the large velocity, the depinning and the small force response (creep).

5 Pinning and dynamics

Let us now turn to the dynamic properties. One of the main interests of such systems is the fact that their dynamics can easily be probed. Indeed most of these systems can be set in motion by an external force acting directly on the interface or on the crystal, and the velocity v versus force F characteristics is directly measurable. As mentioned before, this is of special importance since this characteristics is linked to paramount properties of the systems (voltage-current for vortices, current-voltage for CDW and Wigner crystals, velocity-applied magnetic field for magnetic domain walls). In addition to this practical importance, the dynamics will reflect, even in a more dramatic way than the statics, the competition between disorder and elasticity. In particular one can expect the dynamics to be dramatically sensitive to the glassy properties and the energy landscape.

The main issues relating to the application of an external force are shown in Fig. 11. In the presence of disorder it is natural to expect that, at zero temperature, the system remains pinned and only polarizes under the action of a small applied force, i.e. moves until it locks on a local minimum of the

tilted energy landscape. At a larger drive, the system follows the force F and acquires a non-zero asymptotic velocity v. So a first set of questions is prompted by the zero temperature properties: what is F_c and how can it be computed ? In addition, the $v - F$ curve at $T = 0$ is reminiscent of the curve of an order parameter in a second order phase transition. Here the system being out of equilibrium, no direct analogy is possible but this suggests that one could expect $v \sim (F - F_c)^\beta$ with a dynamical critical exponent β. Whether such an analogy with critical phenomena is true and what are the physical consequences and calculation of such exponents are of course important questions.

Another important set of questions pertains to the nature of the moving phase itself, and in particular to the behavior at large velocity: to what extent this moving system does or does not ressemble the static one. This concerns both the positional order properties and the fluctuations in velocity such as the ones measured in noise experiments. Can we expect novel physics there, or is the system simply "surfing" over the disorder ?

Finally, how does the system respond to a very small applied force. We are accustomed to the fact that a normal system when perturbed usually responds linearly to the perturbation. We could thus expect here naively $v \propto F$, with a coefficient that would define the "mobility" of the interface. Is this true, or due to the glassy nature of the system, do we have non linear response and more complicated physics ?

5.1 General description of the dynamics

Computing the dynamics is not an easy task. Let me illustrate the method on the case of the interfaces. The displacement field in each point is now a function of the time $u(r, t)$ and has to obey the equation of motion The starting point is the equation of motion

$$m\frac{du(r,t)}{dt^2} + \eta\frac{du(r,t)}{dt} = \sum F \tag{17}$$

where η is a friction coefficient that phenomenologically describes the dissipation processes that take place inside the object (interface, etc.) when there is motion. Usually one is interested in the steady state motion of the system in which case in the long time limit, the second order derivative becomes smaller than the first order one and a good approximation is to take $m = 0$ in the above equation.

The forces are of two types. There are the forces deriving from an Hamiltonian

$$F[u(r,t)] = -\frac{\partial H[u]}{\partial u(r,t)} \tag{18}$$

The two main contribution (elastic and disorder) lead in the equation of motion to the elastic forces, trying to keep the interface flat and to the pinning

forces. In addition to these forces that would be present in equilibrium one must add two other forces: the first one is the external force. I consider here only the simple case of a constant external force F. In presence of this force and in the absence of pinning it is natural to expect the system to reache a steady state velocity $v = F/\eta$. Note that such a state, although time independent, cannot be described by an equilibrium theory. In particular the fluctuation dissipation theorem, relating in equilibrium the fluctuations in the absence of a perturbation and the response of the system to an external perturbation is not obeyed in general any more. The second force is needed if we want to describe the system at a finite temperature. In that case one must add [66] a Langevin force $\zeta(z,t)$ which is a noise with correlations

$$\langle \zeta(r,t)\zeta(r',t')\rangle = \eta T\delta(r-r')\delta(t-t') \tag{19}$$

The equation of motion thus becomes in its simplest incarnation

$$\eta\frac{du(r,t)}{dt} = -\frac{\partial H[u]}{\partial u(r,t)} + F + \zeta(r,t) \tag{20}$$

As is well known the presence of the Langevin noise ensures that in the absence of an external force F the time evolution of the system reproduces the thermodynamic ensemble average. In other words the equal time correlations $\langle u(z,t)u(z',t)\rangle$ obtained by averaging over the thermal noises, are identical to the equilibrium correlation function $\langle u(z)u(z')\rangle_H$ that one would have obtained for a system with the Hamiltonian H at the temperature T.

In the absence of disorder the equation becomes quite simple and is known as the Edwards-Wilkinson equation

$$\eta\frac{du(r,t)}{dt} = -\nabla_r^2 u(r,t) + F + \zeta(r,t) \tag{21}$$

The system thus slides at a constant velocity $v = F/\eta$ and one can see that in the moving frame the interface is at equilibrium since the change of variable $u(r,t) = \frac{F}{\eta}t + \delta u(r,t)$ gives for the relative displacements $\delta u(r,t)$ exactly the same equation than in equilibrium in the absence of any external force. Even in this simple case, there are several effects of the motion that need to be taken into account, in particular the presence of a cutoff in the system generates terms that would not normally have been incorporated in the original equation of motion and that can modify the behavior of the system. The most well know is the so called Kardar-Parisi-Zhang (KPZ) term [67].

In the presence of disorder the equation of motion becomes extremely complicated to solve since the pinning force is a random variable depending on the particular realization of the disorder, and a double averaging must be done, both on the thermal noise and on the disorder. No perfect method exists to treat such an equation. Since we are usually more equipped to

deal with integrals than with differential equations, specially with stochastic terms, a convenient rewriting of this equation exists, which formally gives back the equivalent of a path integral and an action. This is the so-called Martin-Siggia-Rose (MSR) formalism [68, 69]. I refer the reader to the literature on this relatively specialized method [66]. The advantage is to allow the averaging over the disorder from the start. This in particular paves the way for an FRG treatment of the problem.

5.2 Depinning

The first set of questions arises close the depinning. Indeed, in the presence of disorder the naive expectation is that the interface is unable to move, at zero temperature, below a certain threshold of force F_c called the pinning force. Computing this pinning force is not easy. In a remarkable feat of physical intuition Larkin has shown that the pinning force can be directly obtained from the static behavior of the system [70]. Indeed the idea is that the pinning force is related to the appearance of many metastable states and the presence of the random potential. Because it is quadratic in the displacements, the Larkin model does not exhibit a pinning force. The idea would thus be to relate the pinning force to the lengthscale at which the Larkin model stops to apply. As we discussed before this is the length L_c for which the displacements are of the order of the correlation length of the random potential or the width of the elastic object. At that scale the elastic plus disorder energy is scaling as $cL_c^{d-2}r_f^2$ while the additional energy due to the force scales as

$$H_F = \int d^d r F u(r) \sim F L_c^d r_f \tag{22}$$

Balancing the two terms leads to the famous Larkin collective pinning force

$$F_c = \frac{c r_f}{L_c^2} \tag{23}$$

This is a remarkable relation since it relates a dynamic property to purely static quantities. This intuitive result can be substantiated by considerably more complicated calculations. In the next section we will see another rough estimate based on a large velocity expansion. Finally, starting from the equation of motion (17), it is possible to obtain F_c from an FRG calculation [71, 72], confirming from this microscopic calculation Larkin's result. Numerical methods have allowed an extremely precise calculation of F_c [39].

Besides the existence of the pinning force itself, the description of the depinning is a considerable challenge. A very fruitful line of approach for this problem was suggested by D.S. Fisher [73]. Indeed looking at the $v - F$ characteristics is strongly reminiscent of the curve of an order parameter as a function of temperature in a second order phase transition (zero for $T > T_c$

and non-zero for $T < T_c$). This strongly suggests to use an analogy with a standard critical phenomenon to analyze the depinning. In particular, one can infer from this analogy that a divergent lengthscale exists at the transition, and that one can define scaling behavior and critical exponents as a function of this lengthscale. One can define a critical exponent for the velocity $v \sim (F - F_c)^{\beta}$, for the correlation length $\xi \sim (F - F_c)^{-\nu}$ and a dynamical exponent relating space and time divergences $\tau \sim \xi^z$. These exponents are related by scaling relations, analogous to the ones of standard critical phenomena and that can be computed by looking at the scaling of the equation of motion. The scaling are

$$\nu = \frac{\beta}{2-\zeta} = \frac{1}{z-\zeta} \tag{24}$$

Such scaling behavior is directly confirmed from solutions of the equation of motion, either from FRG or from numerical simulations. The lengthscale ξ can be identified as the lengthscale of avalanches. Computing and measuring these exponents is a considerable challenge and sophisticated FRG [71, 74, 72, 35] or numerical [39, 75] techniques have been developed for this goal .

In addition to these quantities characterizing the motion of the line, other important physical observables are modified by the application of the external force. This is in particular the case of the roughness of the line. Right at depinning $F = F_c$ the line is much more rough than when in equilibrium, since it is one the verge of depinning. There is thus a new roughness exponent $\zeta_{\rm dep}$ which can be computed and is $\zeta_{\rm dep} \sim 1.2$. for a one dimensional interface. This result has two very important consequences. The first one comes from the value of the roughness exponent itself. Since, at least for a line, this value is larger than one, this immediately suggests that close to depinning the elastic model will run into trouble. Indeed when u scales more than linearly with distance, the vary basic of the elastic approximation $\nabla u \ll 1$ is violated at large lengthscales. The line will thus have to generate defects (overhangs) to heal this fact. What is the resulting physics when this is taken into account is a challenging and yet open question. The second observation concerns the steady state aspect of the line. At large lengthscales, because of the finite velocity, the system will average over the disorder. We will come back in more details on this point in the next section, but for the moment, stick with this simple vision. In that case, beyond the length ξ one can expect the disorder to be irrelevant and thus to recover the pure thermal roughness exponent. The system will thus have the depinning roughness exponent $\zeta_{\rm dep}$ for lengthscales below ξ and the thermal one $\zeta_{\rm th}$ for lengthscales above ξ. This is summarized in Fig. 12. One important question is now what happens at a small but finite temperature. The first effect is of course to smoothen the $v - F$ characteristics. This leads to the important question of whether one can define a scaling with the temperature of this thermal rounding of the depinning (see e.g. [77] and references

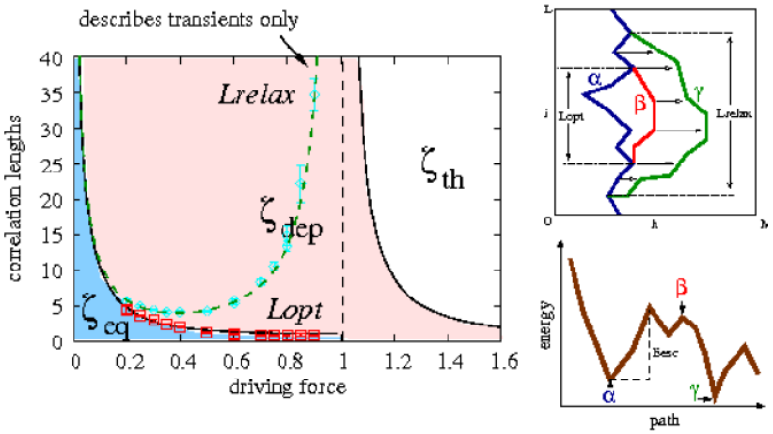

Figure 12: Close to depinning the motion proceeds by avalanches between two configurations α and γ. Above F_c, there exists a divergent lengthscale ($L_{\rm opt}$ on the figure) below which the line is characterized by the roughness exponent $\zeta_{\rm dep}$ and above which the line shows the thermal roughness $\zeta_{\rm th}$. A normal critical phenomenon would have had a similar divergent lengthscale for $F < F_c$. This is not the case for the depinning. A transient divergent lengthscale $L_{\rm relax}$ does exist, but does not show up in the steady state properties of the line. Contrarily to naive expectations from a "standard" critical phenomenon, one observes the equilibrium roughness exponent $\zeta_{\rm eq}$ at *short* distances. This shows that the analogy between depinning and "standard" critical phenomena although very fruitful, must be taken with a grain of salt. On the right the schematic shape of the line and energy profiles are shown. [After [76]]

therein). Even more interesting is the question of the roughness of the line. The analogy with a critical phenomenon would simply suggest that a similar divergent lengthscale should exist for $F < F_c$, leading to the standard pattern of "critical regime". However, as indicated in Fig. 12, such a divergent lengthscale does not exist [76]. This leads to a very puzzling behavior and shows that although the analogy with standard critical phenomena can be extremely fruitful, it must be taken with a grain of salt. The depinning, which is by essence a dynamical transition possesses its own physics.

5.3 High velocity phase

Anther important set of questions and physics occurs when the interface is moving at large velocity, i.e. for $F \gg F_c$. This is apparently the simplest regime since one could expect that at large velocity one has a control parameter on the theory and that an expansion in $1/v$ is possible. This is indeed the case for the $v - F$ characteristics. A large velocity expansion of the disorder term can be made by going into the moving frame of the elastic media. One can indeed write $u(r,t) = vt + \delta u(r,t)$, where the $\delta u(r,t)$ describe the displacements in the moving frame. Because the system surfs over the disorder at very large velocity one can expect the effects of the disorder, and hence $\delta u(r,t)$ to be small. This is confirmed by a well controlled large velocity expansion [78, 79]. In particular, the correction due to the disorder to the velocity can be computed and behave as

$$\frac{F - \eta v}{\eta v} \propto D \left(\frac{1}{\eta v} \right)^{\frac{4-d}{2}} \tag{25}$$

This shows clearly that the effects of disorder are kept small at large velocity or large force and become increasingly important as the force/velociy is getting smaller, in agreement with Fig. 11. The relative correction to the velocity is growing in dimensions smaller than $d = 4$, confirming that disorder is relevant below this dimension. Although one cannot extrapolate the perturbative expressions, a crude way to estimate the critical force F_c is when the deviations (25) become of order one. This method gives back the estimate (23) for F_c, obtained from totally different considerations.

The large velocity expansion allows also to address the physics of the moving phase, namely what is the shape and properties of the elastic system in the moving frame. A calculation of these effects was performed [80], by computing the displacements δu from the large velocity expansion. This leads to the striking result that at large velocity the effect of disorder disappears and can be absorbed in a simple modification of the temperature of the system, leading to an effective temperature T_{eff}. This has important consequences on the properties of the system in the moving frame. For an interface, this is consistent with the idea, exposed in the previous section, that at large distance one recovers the thermal roughening exponent. For

periodic systems, since there is the possibility of melting, this has the more drastic consequences that driving the system could induce a velocity controlled melting. Indeed for large velocities, the effective temperature is small while it would be large for smaller velocities. The system would thus be a disordered system while static, then close to depinning where the effective temperature would be large it would be in a melted (liquid) state, and then recrystallize when moving at larger velocities.

Although the concept of effective temperature is extremely fruitful, in particular for this dynamic recrystallization, the properties of the moving periodic system are richer [81] than those a simple solid subjected to a temperature. Indeed periodic systems have a structure in the direction *transverse* to the direction of motion. This structure and the corresponding disorder structure cannot be averaged by the motion, however large the velocity remains. This leads to the fact that disorder remains even when the system is in motion. In other words a moving periodic system remains a glass. The way the motion takes place is quite peculiar. The system finds optimal paths [81] which are a compromise between the elastic energy, disorder and the motion. These paths are rough, in a similar way than a static system in a disordered environment is rough. This is shown in Fig. 13. For periodic systems pinning effects still manifest themselves in the moving system. The glassy nature and the channel motion have been confirmed both numerically [82, 83, 84] and experimentally [85]. The channel motion leads to an interesting consequence. Since the effects of disorder are weakening as the velocity increases, the channels undergo a transition between a regime for which the particles in different channels are coupled or decoupled [86, 87]. In the first case, the system is essentially moving as a "solid" (in fact a moving Bragg glass) since the topological order is perfect even if the system is still distorted by disorder. In the second case the channels are decoupled, which means that a smectic like structure of the particles inside the channels is expected. These transitions as described in Fig. 13 have also been observed both numerically and experimentally as shown in Fig. 14. An additional consequence of the existence of such channels is the existence of a *transverse* pinning force [81, 87]. Indeed even if the particles themselves are moving along the channels, the channels themselves are pinned if one applies an additional force transverse to the direction of motion. This surprising phenomenon has been numerically confirmed [82, 83, 84], but observing it in a classical periodic systems it is still an experimental challenge. Experiments showing the absence of Hall effect in Wigner crystal systems [88] could constitute an experimental proof of such a transverse critical force, but clearly further experimental data would be needed to unambiguously decide on that point.

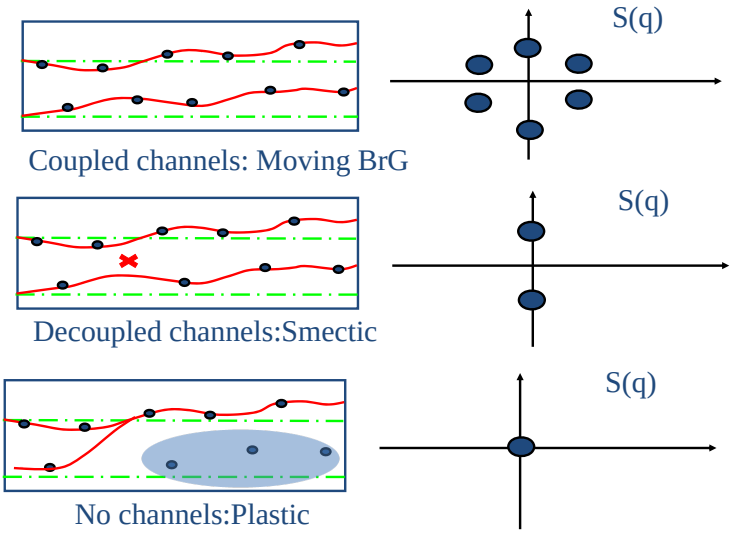

Figure 13: Motion of a periodic system. The system develop rough channels which compromise between elastic energy and the transverse component of the disorder that are poorly averaged by the motion. All the particles follow on these channels like cars on highways. Depending on the velocity, the channels are increasingly coupled: Bottom image: close to depinning motion can proceed through a plastic regime where unpinned and pinned regions (denoted by the blue circle) coexist; Middle image: topological defects can exist between the channels, so although the channels themselves are well formed the particle in them are essentially decoupled leading to a smectic like behavior; Top image: the channels are coupled and the system is a moving Bragg glass with both effects of disorder and elasticity and no topological defects. On the right the corresponding structure factors are indicated.

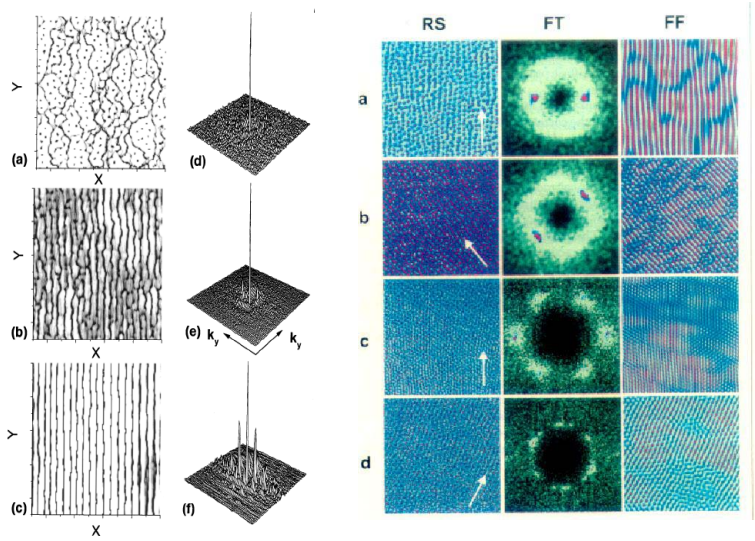

Figure 14: Left: numerical simulations confirming the presence of channels for moving periodic structures and the sequence of transitions depicted in Fig. 13. The left part of the image are the real space trajectories, while the right part is the structure factor. The force is increasing from the top to the bottom of the image [From [83] (Copyright 1999 by the American Physical Society)]. Right: A decoration image of moving vortices showing also the presence of channels. The direction of the applied force is indicated by an arrow. The left column is the raw decoration image, the center one is the Fourier transform giving the structure factor, the right column is a filtered version of the image showing the channels more clearly [From [85] (Copyright 1999 by the Nature group)].

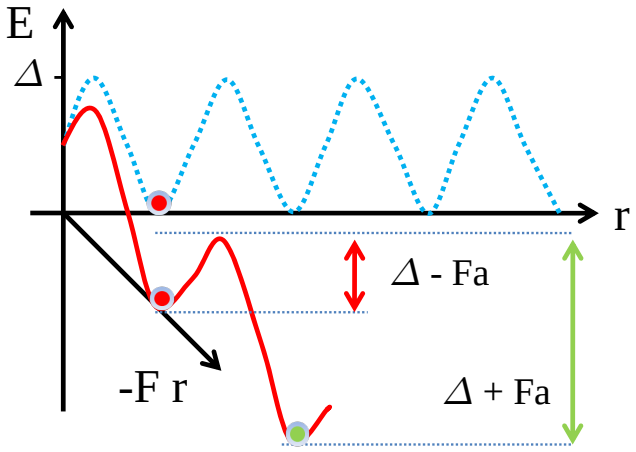

Figure 15: In the thermally assisted flux flow [89] a region of pinned material is considered as a particle moving in an energy landscape characterized by characteristic barriers Δ, schematized by the blue dashed periodic potential, of period a. Applying an external force tilts the energy landscape. The motion over barriers can always proceed by thermal activation. Due to the tilt the barrier to forward motion (in red) is smaller than the reverse barrier (in green). This results in an exponentially small but linear response when a small external force is applied to the system.

5.4 Small applied force and creep motion

Finally let us look at the response of the system to a small external force. At zero temperature, one is below the pinning force, and thus except for a transient motion the system remains pinned. Motion can thus only take place due to thermal activation over the energy barriers. The response to a small external force is thus a method of choice to probe for the nature of the energy landscape of such systems. For usual systems one expects the response to be linear. Indeed earlier theories of such a motion have found a linear response. The idea is to consider that a blob of pinned material has to move in an energy landscape with characteristic barriers Δ as shown in Fig. 15. The external force F tilts the energy landscape, thus making forward motion possible. The barriers are overcome by thermal activation [89] (hence the name: Thermally Assisted Flux Flow (TAFF)) with an Arrhenius law. If the minima are separated by a distance a the velocity is

$$v \propto e^{-\beta(\Delta - Fa/2)} - e^{-\beta(\Delta + Fa/2)} \simeq e^{-\beta\Delta} F \tag{26}$$

The response is thus linear, but exponentially small.

However this argument is grossly inadequate for a glassy system. The reason is easy to understand if one remembers that the static system is in a

glassy state. In such a state a characteristic barrier Δ does not exist, since barriers are expected to diverge as one gets closer to the ground state of the system. The TAFF formula is thus valid in systems where the glassy aspect is somehow killed and the barriers do saturate. This could be the case for example for a finite size interface. When the glassy nature of the system persists up to arbitrarily large length scales the theory should be accommodated to take into account the divergent barriers. This can be done qualitatively within the framework of the elastic description using scaling arguments [90, 91, 92, 47]. The basic idea rests on two quite strong but reasonable assumptions: (i) the motion is so slow that one can consider at each stage the interface as motionless and use its *static* description; (ii) the scaling for barriers, which is quite difficult to determine, is the same as the scaling of the minimum of energy (metastable states) that can be extracted again from the static calculation. If the displacements scale as $u \sim L^{\zeta}$ then the energy of the metastable states (see (2)) scales as given by (5): $E(L) \sim L^{d-2+2\zeta}$. Since the motion is very slow, the effect of the external force is just to tilt the energy landscape

$$E(L) - F \int d^d r u(r) \sim L^{d-2+2\zeta} - F L^{d+\zeta} \tag{27}$$

Thus, in order to make the motion to the next metastable state, one needs to move a piece of the pinned system of size

$$L_{\rm opt} \sim \left(\frac{1}{F}\right)^{\frac{1}{2-\zeta}} \tag{28}$$

The size of the optimal nucleus able to move thus grows as the force decrease. Since the barriers to overcome grow with the size of the object, the minimum barrier to overcome (*assuming* that the scaling of the barriers is *also* given by (5))

$$U_b(F) \sim \left(\frac{1}{F}\right)^{\frac{d-2+2\zeta}{2-\zeta}} \tag{29}$$

leading to the creep formula for the velocity

$$v \propto \exp\left[-\beta U_c \left(\frac{F_c}{F}\right)^{\mu}\right] \tag{30}$$

where F_c is the depinning force and U_c a characteristic energy scale and the *creep exponent* μ is given by,

$$\mu = \frac{d-2+2\zeta}{2-\zeta} \tag{31}$$

Equations (30) and (31) are quite remarkable. They relate a dynamical property to *static* exponents, and show clearly the glassy nature of the system. The corresponding motion has been called creep since it is a sub-linear

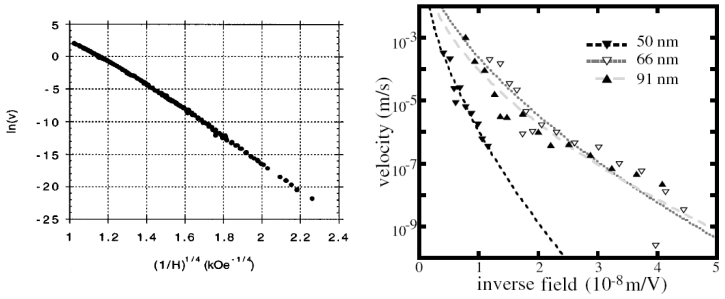

Figure 16: Experimental verification of the creep law for magnetic and ferroelectric domain walls. Left: Magnetic domain walls. The film is extremely thin thus the domain is a line in a two dimensional plane, leading to a creep exponent of $\mu = 1/4$. The creep law is observed on about ten orders of magnitude for the velocity [From [1] (Copyright 1998 by the American Physical Society)]. Right: Ferroelectric films. A creep behavior is observed over several orders of magnitude giving a creep exponent of $\mu \sim 0.58$. This value together with the measured roughness exponent $\zeta = 0.26$ leads to an effective dimension of $d = 2.5$ well in agreement with a two dimensional domain wall in presence of dipolar forces [From [8] (Copyright 2005 by the American Physical Society)].

response. It is a direct consequence of the divergent barriers in the pinned system.

Of course the derivation above is phenomenological, so it is important to ascertain by more microscopic methods whether the results hold. Although in principle one simply has to solve the equation of motion (20). In practice this is of course quite complicated. A natural framework for computing perturbation theory in off-equilibrium systems is the MSR formalism. Using this formalism and an FRG analysis, one can confirm the creep formula for the velocity [93, 94]. Numerical simulations also show the absence of linear response, and the existence of a creep response [95]. Creep has also been checked experimentally in various systems. Vortices show a creep behavior with an exponent $\mu = 1/2$ compatible with the existence of the Bragg glass ($d = 3$, $\zeta = 0$) [96]. However the range of velocities measurable makes it difficult to unambiguously check for this law. One spectacular determination of the creep law was performed in a magnetic film [1]. In such a situation the roughness exponent is known exactly ($\zeta = 2/3$) and thus the value of the creep exponent $\mu = 1/4$ is not an adjustable parameter making it a much more stringent test. As shown in Fig. 16 the velocity was measured over ten orders of magnitude, a spectacular feat, and a remarkable confirmation of the creep law. Measurements of the creep law have also been performed for domain walls in ferroelectrics where a simultaneous measurement of the

creep exponent and of the roughness exponent was performed [7, 8]. As shown in Fig. 16 the stretched exponential behavior for the velocity is well verified, and the formula (31) consistent with what is expected for a two dimensional domain wall in presence of dipolar forces, corresponding to the experimental situation.

The FRG derivation allows of course to probe deeper the physical understanding of the system. In particular it unravelled a new phenomenon. Although the velocity itself is dominated by events occurring at the thermal lengthscale (28), interesting physics takes place beyond this lengthscale. Indeed when a portion $L_{\rm opt}$ of the line has been able to move forward by thermal activation over the barriers, it serves as a nucleation center to trigger an avalanche [94] over a much larger lengthscale $L_{\rm av}$. The behavior between these two lengthscales is thus very similar to a depinning phenomenon where the temperature plays no role. Although of course the velocity is dominated by the first process, which is the slow one, the shape of the line reflects this much larger avalanche scale, in a way which is compatible with experiments [3]. The creep, being controlled by the time to overcome divergent barriers in the system has several other consequences, in particular on the issue of the out of equilibrium physics of such systems [97] and its connection to the aging of glasses [98].

6 Future directions

Both because of experimental drive (no pun intended) but also because of theoretical advances and the development of the proper tools, this field has thus known several breakthroughs in the last decade or so. There is now a good understanding of the static properties both for the interfaces and for the periodic systems and most of the misconceptions or folklore has been replaced by solid results. Novel phases have emerged such as the Bragg glass phase. The steady state dynamics has also made several progress, with the understanding of processes such as the creep motion. From the point of view of methods, these systems have made it possible to perfect methods to deal with glassy systems such as replica methods, functional renormalization group as well as special numerical methods. These results have found and continue to find applications in a large variety of experimental domains. Despite these advances, it is clear that many questions remain pending, making it still a very challenging field which is yet in constant progress. Experiments provide regularly new systems and new challenges and stimulate the theoretical analysis. Several lines of research are actually open and should carry the bulk of the research in that domain in the future.

From the point of view of the static, although the situation without defects is under control, we know next to nothing when elasticity, disorder and defects are included. For interfaces this means treating the overhangs

and bubbles, and for periodic systems all the topological defects such as the dislocations. Although we know now that the situation without the defects is robust below a certain threshold of disorder it is clear that being able to deal with the strong disorder situation is needed for many experimental systems. This is the case for the high field phase of vortices or strong disorder in the interfaces, both of which are dominated by defects.

In the dynamics, one of the very challenging questions that one has to face is the one of the out of equilibrium dynamics, when the system has not yet reached a steady state. A simple example of such a situation would be an interface relaxing slowly from a flat configuration or quenched at low temperatures from a high temperature configuration. Given that these systems are glasses the time evolution of such cases is highly non-trivial, and should show a generic phenomenon of glasses known as aging. This is directly a situation relevant to many experiments. From the conceptual point of view this is an extremely challenging question since most of the theoretical tools that we have fail to tackle such situations, and thus new tools or new concepts need to be invented.

Last but not least, we have dealt mainly with classical systems here. But disordered elastic systems can also be realized in the quantum worlds as briefly mentioned in the introduction. The question on how to extend the concepts of glasses to quantum systems is a largely open question. In particular one can expect the dynamics to be affected. Indeed classical systems can only pass barriers by thermal activation, while quantum systems are good at tunnelling through barriers. The extension of the above concepts to the quantum world is thus a very challenging direction.

The gold mine of disordered elastic media is thus far from being exhausted. It seems that each nugget we find is only the opening of a new vein, with even richer stones. The variety of experimental realization is ever growing, as is the depth of the questions that are now within our grasp.

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