LECTURE 4A

Superconducting qubits

We recall the basic requirements (lecture 1B) for a quantum computer: we need a set of systems (« qubits »)each with a well-defined 2D Hilbert space,with the properties:

(1)easy initialization

(2)easy readout

(3)possibility of implementing 1-and 2-qubit operations,in particular arbitrary 1-bit gates and (e.g.) CNOT

(4)scalability

(5)(reasonable degree of) freedom from decoherence.

It is convenient to define for any given type of qubit a computational basis (CB)(usually,the basis in which initialization and readout occurs) and to represent the qubit as a « spin ½ » with the CB given by the eigenstates (0,1) of the Pauli matrix σ_z

Rather generally,the basic variables used in superconducting qubits are the phase difference $\Delta\phi$ across a Josephson junction (in the case of a SQUID ring expressed in terms of the flux Φ trapped through the ring) and the canonically conjugate variable ΔN .The external parameters used to control these variables include

gate voltage V_g

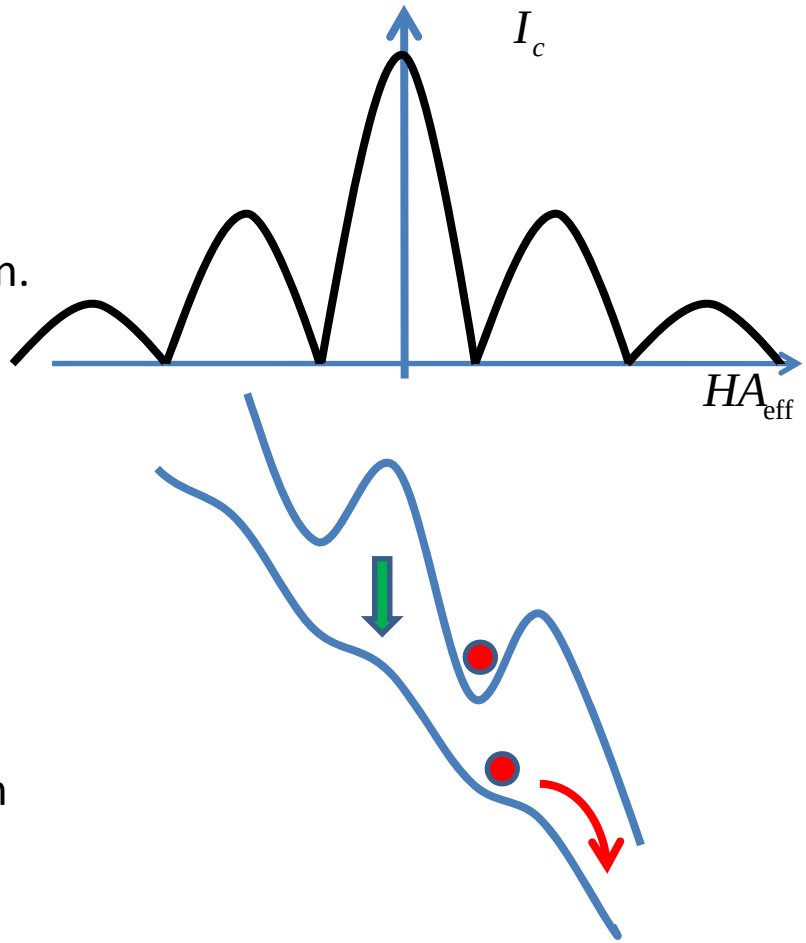
external current (of CBJ) I_{ext}

external flux Φ_{ext} through SQUID ring

In describing the use of Josephson systems as qubits, there are three further characteristics of Josephson junctions,not so far introduced,which we shall find useful:

(1)The critical current of a junction can be modulated by applying a magnetic field in the plane of the junction. Crudely speaking,this is because in the presence of the field the phase difference varies over the dimension perpendicular to the field;the total critical current (or equivalently the coupling energy)is then the (algebraic) average over this dimension, leading in general to destructive interference.The general formula is of Fraunhofer type:

$$I_C(H) = I_C(0)\,|\sin x / x\,|, x \equiv \pi H A_{eff} / \Phi_0$$

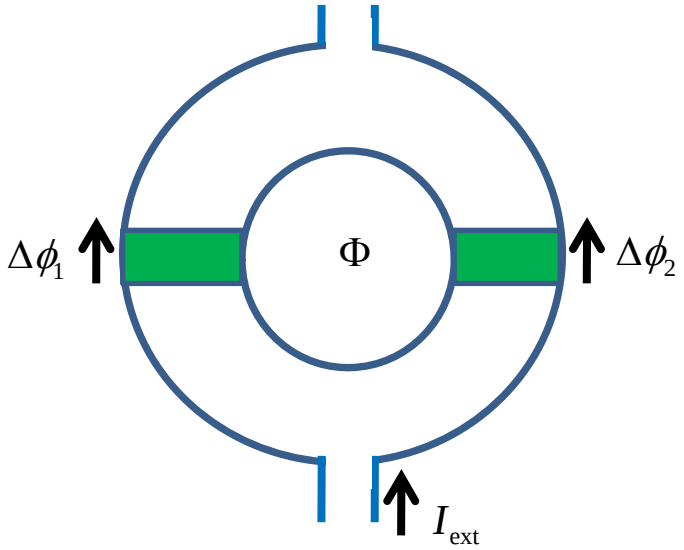


where the « effective » area of the junction is its length times the sum of its actual width plus the bulk London penetration depths of the two metals.

(2) If a junction has to carry a current greater than its critcal current,then it inevitably develops a voltage across it.This is easy to visualize in the « washboard potential » picture:if the slope exceeds I_C ,then there are no longer any stable or even metastable minima,and the relative phase $\Delta\phi$ accelerates down the slope until brought to a steady-state nonzero value by the shunting conductance 1/R.In this state the voltage (which we recall from the second Josephson relation is proportional to the « velocity » of $\Delta\phi$) is nonzero.Thus, the presence or absence of a finite voltage across the junction can be used as an indicator of whether or not $I > I_C$

Junctions in parallel (dc SQUID)

This arrangement in some sense combines a CBJ with an rf SQUID (ring with single junction).For simplicity consider identical junctions with the signs of the phase drops $\Delta\phi_1, \Delta\phi_2$ defined as shown,then the total Josephson energy is



$$E_J^{tot} = -E_J(\cos\Delta\phi_1 + \cos\Delta\phi_2)$$

there is a term associated with the external current, which by invoking Kirchhoff's law in equilibrium is

$$-I_{ext}(\Delta\phi_1 + \Delta\phi_2)/2$$

and finally a term arising from the self-inductance of the loop,which as in the simple rf SQUID case is given by

$$E_{ind} = (\Phi - \Phi_{ext})^2 / 2L$$

If we define $\Delta\phi \equiv (\Delta\phi_1 + \Delta\phi_2)/2, \xi \equiv \Delta\phi_1 - \Delta\phi_2$,then $E_J^{tot} = -2E_J\cos(\xi/2)\cos\Delta\phi$ .However,by the same argument as used in the single-junction ring, $\xi = 2\pi\Phi/\Phi_0$ .Substituting this in the expression for the total Josephson energy and adding the external-current term,we find for the total potential energy of the dc SQUID the result

$$V(\Phi:\Delta\phi) = (\Phi - \Phi_{ext})^2/2L - 2E_J\cos(\pi\Phi/\Phi_0)\cos\Delta\phi - I_{ext}\Delta\phi$$

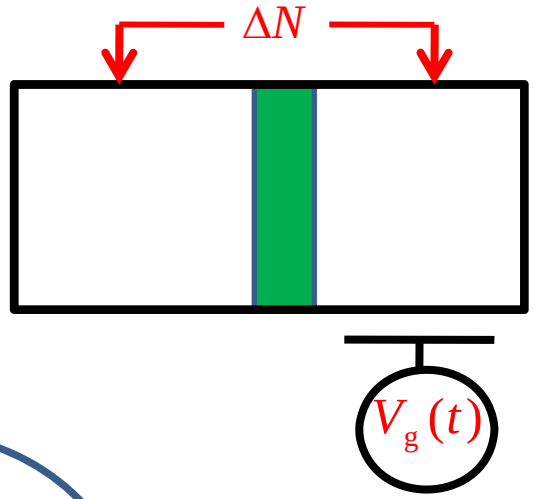
Thus for fixed total trapped flux Φ the dc SQUID behaves like a simple CBJ with critical current

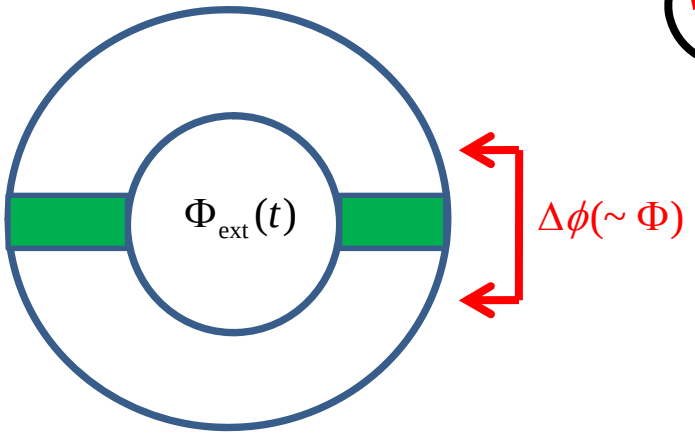
$$2I_C\cos(\pi\Phi/\Phi_0) \equiv I_C^{eff}(\Phi)$$

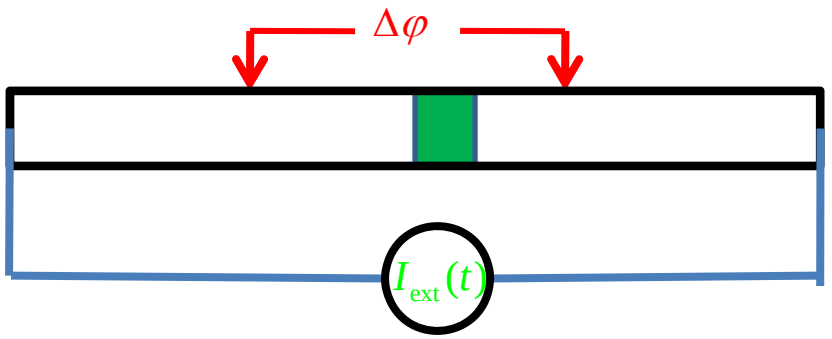
In particular,when the external current exceeds $I_C^{eff}(\Phi)$ the junction switches from the zero-voltage to the voltage state.Now Φ is of course a dynamical variable,but in equilibrium it is determined by minimizing $V(\Phi)$,and hence is a function of Φ_{ext} ;in particular,for $\beta_L << 1$ it is approximately equal to Φ_{ext} .Hence,the switching behavior can be controlled by Φ_{ext} .

Types of superconducting qubit-overview

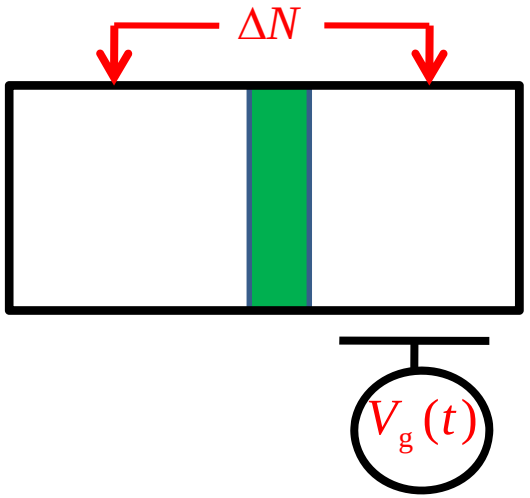
Name	Computational basis (approximate) eigenstates of	Control parameter
1.Charge qubit	ΔN	V_g
2.Flux qubit	$\Phi, i.e. \Delta\phi$	Φ_{ext}
3.Phase qubit	$\Delta\phi$	I_{ext}
4.Hybrids,e.g. dc SQUID, ''quantronium »		







Charge qubit



The Hamiltonian is expressed in terms of the phase difference across the junction, $\Delta\phi$, and the conjugate variable ΔN,which is the number of pairs transferred across the junction (relative to some (integer) reference value):

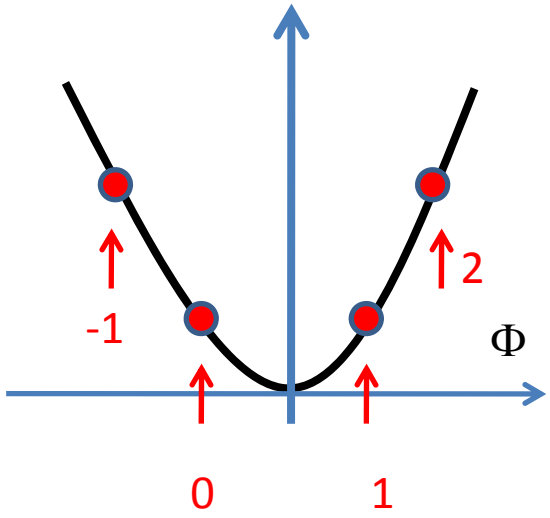
$$H(\Delta\phi,\Delta N) = -E_J \cos\Delta\phi + E_C(\Delta N)^2 - 2eV_g\Delta N$$

It is important to bear in mind that we can use this effective Hamiltonian only provided the characteristic energy scales E_J, E_C are small compared to the superconducting energy gap Δ , which is about 3K for Al at about 16K for Nb.Thus we need $E_J, E_C \leq$1K,which requires a capacitance of the order of 5 femtofarads or less,corresponding to a typical dimension of the CPB of order of a few microns .It is also necessary ,in order to avoid complications associated with the presence of quasiparticles,that the temperature satisfy the condition $T \ll \Delta / k_B \ln N$ ~80mK for Al,where N is the total number of electrons on the CPB (typically $\sim 10^9$).

The nature of the energy eigenstates depends strongly on the ratio E_J / E_C .If this is >>1,we are in the « Josephson » regime and the energy eigenstates are approximately eigenstates of $\Delta\phi$ (with mean-square fluctuations << 2π).The lowest states are therefore very harmonic-oscillator-like,and thus useless for building a qubit.We need therefore to go to the limit where $E_J \leq E_C$ .In this limit it is more natural to work in the eigenbasis of ΔN ; it is often convenient to view the system as the analog of an (in general driven) quantum pendulum, with the analog of $\Delta\phi$ being the angle made by the pendulum with the vertical; ΔN is then the analog of the angular momentum of the pendulum.

The Hamiltonian of the charge qubit in the charge basis

$E_c(\Delta N)^2 - 2eV_g\Delta N$



In the charge basis the charging energy has the form

$$E_C(\Delta N)^2 - 2eV_g\Delta N$$

so would be a (shifted) parabola if ΔN were a continuous variable.However, ΔN can take only discrete (integral) values,so the allowed states correspond to points on this curve,with positions determined by .In particular,if V_g is close to the value $E_C / 2e(\equiv e/C \sim 0.1-1mV)$ then the two lowest states are nearly degenerate,and are separated from next lowest states by a distance >> their splitting $\Delta E_C = e(V - E_C/2e)$;they therefore form to a good approximation a 2-state system,and we can take the computational basis as the eigenstates of charge,so that the charging part of the Hamiltonian has the form $E_{ch} = -\Delta E_C \sigma_z$ .What form does the Josephson energy take in this basis? From the « pendulum » analogy it is clear that in the $\Delta\phi$ -basis the eigenstates of have the representation $\exp i(\Delta N)\Delta\phi$,and thus the matrix elements of the expression $\cos\Delta\phi$ are ½ between neighboring values of ΔN and zero everywhere else :

$$< \Delta N | \cos\Delta\phi | \Delta N' >= (1/2)\delta_{\Delta N, \Delta N' \pm 1}$$

Now within the 2D Hilbert space of the lowest two states these matrix elements are identical (apart from the factor of ½) to those of the Pauli matrix σ_x ,so we can write the effective Hamiltonian in the form

$$H = -\Delta E_C(V_g)\sigma_z - (E_J/2)\sigma_x$$

thus demonstrating explicitly its potential as a qubit.

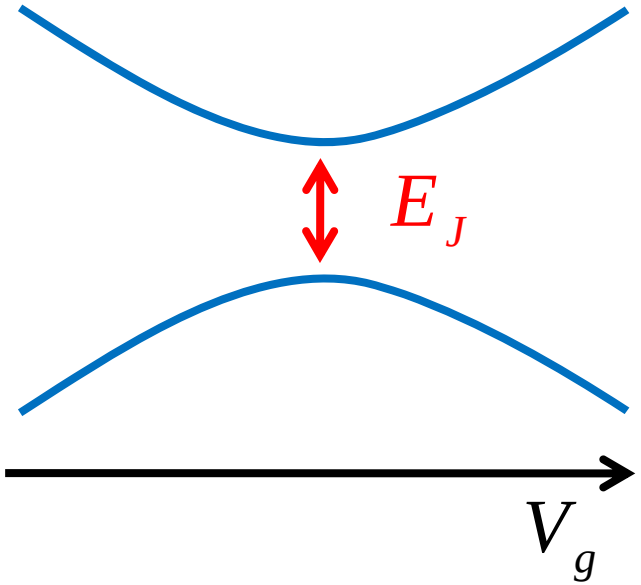
Recap:the « 2-state Hamiltonian » of a charge qubit has the form

$$H = -\Delta E_C(V_g)\sigma_z - (E_J/2)\sigma_x$$

where the charging energy ΔE_C can be manipulated by the experimenter as a function of time through the externally applied gate voltage V_g .Evidently the eigenvalues of this Hamiltonian are

$$E = \pm[(E_J/2)^2 + (\Delta E_C(V_g))^2]^{1/2}$$

and as a function of V_g behave as shown in the figure.Thus by performing spectroscopy on the qubit we can determine the values of the two parameters E_J and $\Delta E_C(V_g)$

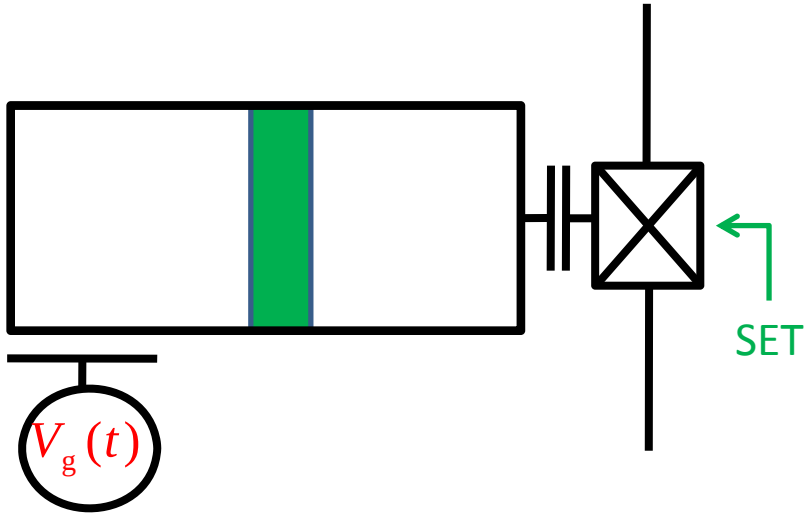


An interesting point concerning the charge qubit is the extent to which it can really be described,as we have done,entirely in terms of the behavior of an individual Cooper pair.One might at first sight think that when a pair tunnels from (say) L to R of the CPB,all the electrons on the individual electrodes L and R « recoil » in response to the tunnelling.If that were really so,the tunnelling matrix element would be reduced by the tiny FC (Franck-Condon)factor corresponding to the (very small) overlap of the initial and final states of the « other » electrons. Fortunately,it turns out that provided the dimension of the box is >>the Debye screening length (almost always true in practice) there is little recoil and the FC factor is very close to 1.Thus, contrary to the case of the « flux qubit » to be discussed,the charge qubit does not involve a high degree of « disconnectivity » (crudely speaking,a quantum superposition of states in which a large number of microscopic particles are behaving differently),and thus while of great interest In the context of quantum computing is not particluarly relevant to the « realization » problem.

We now discuss the various conditions necessary for this system to be a viable qubit

(1)Initialization is achieved simply by applying a sufficient bias and allowing sufficient time for the system to come to thermal equilibrium.Since the values of E_J and E_C are typically of order1 K,while to eliminate quasiparticles T must be <80 mK,in equilibrium only the lower energy eigenstate is occupied.

(2)Readout is not a trivial problem for the charge qubit.In the original experiment of Nakamura et al.,it was achieved by coupling one side of the CPB to a « probe » junction (presumably a tunnel oxide junction with a normal counterelectrode,though this is not stated) so that when the excited state of the qubit is realized,it decays to the groundstate with emission of two quasiparticles,producing a measurable current in the probe circuit.Most of the more recent experiments use an SET (single-electron transistor) for the detection:this is itself a « box »,but made of a normal metal and biassed with an external voltage:the current through this system,which is what is directly measured,is a sensitive function of the gate bias voltage.In the experiments,the charge qubit is capacitatively coupled to the SET,which can be used to read out the state of the qubit.This has the advantage that the « measurement » can be switched on or off by applying or removing the voltage bias on the SET (unlike the Nakamura setup, where the « detection » is on permanently,so that one has to perform one's qubit operations in a time small compared to the quasi-particle tunnelling (decay) time).



3.Qubit gates

a)Single-qubit gates

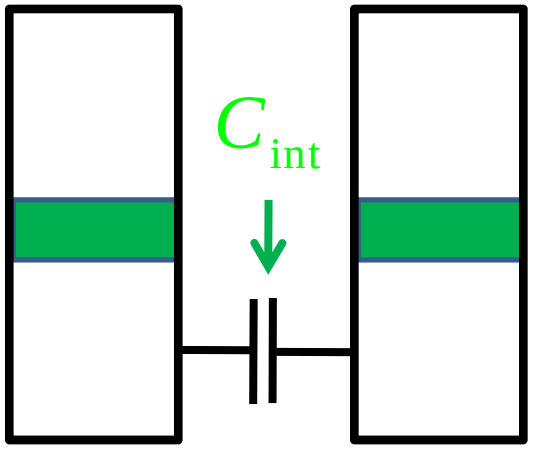
These are relatively simple to realize,by appropraite variation of the externally applied gate voltage in time.E.g.one can start with a large bias so as to realize the groundstate,then move adiabatically (i.e.over a time scale $>> \hbar / E_J$) to the « crossing » point ($\Delta E_C = 0$) and then apply a weak voltage oscillating at the frequency corresponding to the splitting at this point,namely $2E_J$;note that in the language of the NMR analogy the « external (dc) field » is now along x and the oscillating field along z,so if it has the right duration («$\pi/2$ pulse ») it has the usual effect of rotating the « spin » into the plane perpendicular to the fieldproducing a linear superpostion of the energy eigenstates , $2^{-1/2}(|0> + |1>)$ in the enrgy basis.More generally,one can rotate around the x-axis through an arbitrary angle and then allow a period of free precession so as to realize an arbitray single-qubit gate (cf.lecture 1B).

b) 2-qubit gates

The easiest way to achieve a 2-qubit gate is to couple one's two qubits capacitatively .In the setup shown in the figure the coupling provided by the capacitance C_{int} is of the form

$$H_{12} = (2e)^2(\Delta N \cdot_1 \Delta N_2) / 2C_{\text{int}} \equiv ((2e)^2 / 2C_{\text{int}})\sigma_{z1}\sigma_{z2}$$

where we choose the CB to be that of the number eigenstates for each qubit.Of course,it is not easy to vary H_{12} in time.However,once one has the above coupling one can perform 2-qubit operations such as CNOT by the spectroscopic technique described in Lecture 1B.



(also:stripline coupling)

4.Scaling

Quite generally with superconducting qubits (not just of « charge » type),because we are dealing with relatively macroscopic electrical circuits,the business of connecting them up is essentially classical electrical engineering,and so it is not usually anticipated that scaling up to many qubits will present any intrinsic difficulties.Of course,it is not necessarily trivial,once one has many qubits,to make sure that one is connecting up (say) 1 and 2 without at the same time influencing 3,but the difficulties are the same as encountered in classical engineering,and many techniques have been developed to deal with them;this is probably the least of our problems.

5.Decoherence

This is usually anticipated to be the most serious problem that has to be overcome before we can realistically contemplate using superconducting devices as practical qubits.Let's review some general results concrning decoherence in a 2-state system.As argued in lecture 3B,we can rather generically model the (quantum) environment giving rise to the decoherence (and to dissipation in the classicallimt) by a bath of SHO's,with a coupling which is linear in the oscillator coordinates.When we are talking about a continuous variable,the coupling may have a complicated form as a function of the system coordinate,but when we project on to the lowest 2D state (qubit basis) it is clear that the most general form allowed is

$$H_{S-E} = -\sigma \cdot \Omega$$

where the vector Ω may be thought of a sort of « fluctuating magnetic field » due to the action of the environment,with components given by the expressions

$$\Omega_i \equiv \sum_\alpha C_\alpha^{(i)} x_\alpha$$

All effects of the environemnet,both dissipative and decoherent,are encpasulated in the « coupling spectral densities » $J^{(i)}(\omega) \equiv (\pi/2)\sum_\alpha (C_\alpha^{(i)2}/m_\alpha\omega_\alpha)\delta(\omega-\omega_\alpha)$,which may be obtained,at least in principle,by comparing the coefficients in the continuous limit with the classical dissipation (cf.lecture 3B) and projecting on the qubit space.

Recap:for any 2-state system,the most general form of the Hamiltonian of the isolated system is of the form

$$H_S = -\sigma \cdot H(t)$$

where the vector H(t) is a (possibly time-dependent) c-number « magnetic field »,and the most general system-environment interaction is of the form

$$H_{S-E} = -\sigma \cdot \Omega$$

where the (vector) operator Ω is a « fluctuating magnetic field » whose effects are completely parametrized by the coupling spectral densities $J^{(i)}(\omega)$ defined above.

In the case of the charge qubit (and,actually,of most kinds of superconducting qubit) we can actually simplify the description a little further.In most of these cases we can choose the « spin »axes so that only σ_z and σ_x correspond to physically important macroscopic physical quantities,while the third Pauli matrix, σ_y ,has no simple physical significance.In this case we can omit the y-component of not only H(t) but also of Ω. We note that,having originally chosen our CB to correspond ,say,to number eigenstates (so that σ_z corresponds to ΔN),we can now rotate the basis in the xz-plane so that the CB now corresponds to (say) the energy eigenstates for the given value of the gate voltage.However,it is extremely important to note that the coupling spectral densities $J^{(i)}(\omega)$ will now in general be functions of the basis, i.e. of the value of the gate voltage. For the ensuing discussion we will assume the energy basis.

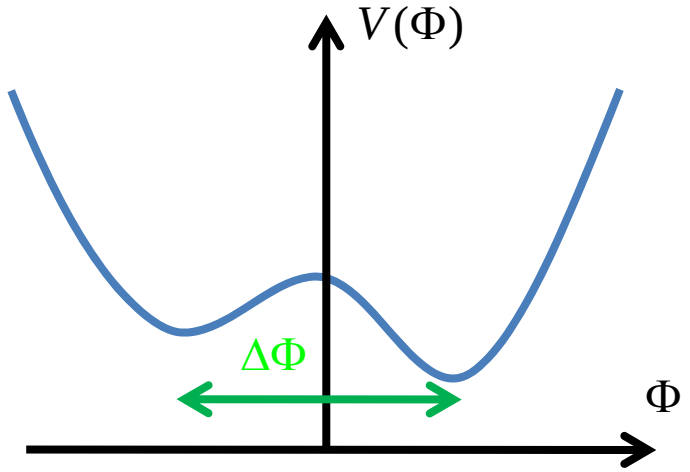
Quite generally,as in traditional NMR theory we can distinguish the energy relaxation time T_1 ,which is the relaxation time from the upper to the lower state,and the phase relaxation time T_2 ,which is the time for the relative phase of the two components in the energy basis to decohere.It is T_2 which is most relevant to the « figure of merit » of the qubit.(Since we shall see below that the order of magnitude of T_1 is never smaller than that of T_2 ,the corresponding figure of merit is always at least as good).

We now examine the effects of the x-and z-coupling (i.e. of Ω_x and Ω_z) on the characteristic times T_1 and T_2 .It is clear that the x-coupling can induce transitions between the levels, with a rate proportional to $J^{(x)}(\omega_0)$,where $\omega_0 \equiv \Delta E / \hbar$ with $\Delta E \equiv 2H_z$ the energy splitting.It thus contributes to T_1^{-1}(in fact,we shall see that in our « xz » model it is the only contribution). It is slightly less obvious,but follows with a little thought,that this coupling also contributes to T_2^{-1}an amount one half of its contribution to T_1^{-1} .Thus,as already stated,T_2 can never be longer than T_1 in order of magnitude.

Now consider the effects of the z-coupling.From the symmetry,this cannot induce relaxation of the z-component of spin,so cannot contribute to T_1^{-1} .However,it can and does contribute to precession of the xy-plane component (cf.lecture 1B),so can contribute to the dephasing rate T_2^{-1} (this contribution is often called « pure dephasing » in the literature).The theory of this term is a little more delicate;in the model considered it is proportional to the quantity $\lim_{\omega\to 0} \coth(\hbar\omega / 2k_B T)\cdot J^{(z)}(\omega)$,and thus vanishes in the limit T->0 for any form of $J^{(z)}(\omega)$ which tends to zero faster than a constant (e.g.for the « ohmic » form $J^{(z)}(\omega) \propto \omega$).In the case of 1/f noise one can get a nonzero effect even in the limit T->0,(but the decay of the phase is not in general pure exponential);I will not go into the details here.The main point to note is that in general there are effects which influence the dephasing but cannot be seen in the energy relaxation.

In the case of interest,that of a charge qubit,the principal contributions to the coupling spectral densities are likely to come from the fluctuations in the gate voltage,so that in the number basis they appear in the z-coupling.This then means that at the crossing point,if we use the energy representation,they will appear only in the x-coupling,which means that their influence on the two T’s is of the same order of magnitude .(At the crossing point,the energy eigenstates are superpositions of the two number states with equal weight,so are insensitive to gate voltage fluctuations).Unfortunately there may also be a nonnegligible contribution to the z-coupling from fluctuations of the Josephson energy,on which more below.

2.Flux qubit



Recall that the general expression for the potential energy of a SQUID ring in an external flux Φ_{ext} is ,as a function of the total trapped flux Φ

$$V(\Phi) = (\Phi - \Phi_{ext})^2 / 2L - (I_C \Phi_0 / 2\pi)\cos(2\pi\Phi / \Phi_{0)}$$

and that if the quantity $\beta_L \equiv 2\pi L I_C / \Phi_0$ is just greater than 1 this reduces,near its minimum, to the form $(x \equiv \Phi / \Phi_0 - 1/2)$

$$V(x) = -\alpha x^2 + (\beta/2)x^4 - \gamma x$$

where if V is measured in units of $\Phi_0{}^2 / 2L$,

$$\alpha = \beta_L - 1, \beta = 6\beta_L, \gamma = 2(\Phi_{ext} / \Phi_0 - 1/2)$$

Let us now apply QM to this situation,with the KE given by $Q^2 / 2C$ where Q is the « momentum » canonically conjugate to the trapped flux Φ .It is convenient to define a « semiclassicality parameter » λ by

$$\lambda \equiv (8CI_C\Phi_0{}^3 / \pi^3\hbar^2)^{1/2} \sim 4\cdot 10^{11}(CI_C)^{1/2}$$

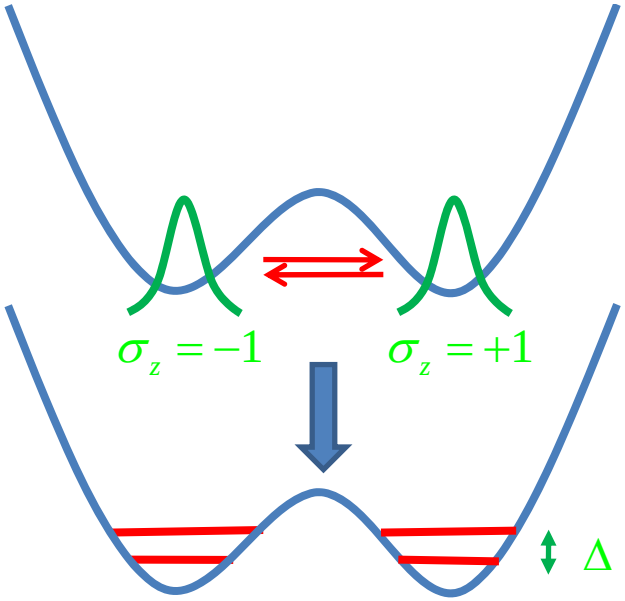
in SI units.In the real-life SQUIDs used for qubits*, λ is invariably >>1 (typically ~5-10).Then the behavior of the system,for γ small enough that the two wells are well-defined,is criticallly sensitive to the value of the quantity ζ defined by

$$\zeta \equiv (\beta_L - 1)^{3/2}\lambda$$

For $\zeta \leq 1$ the groundstate is delocalized between the two wells,and the general structure of the levels is harmonic-oscillator-like,so that at least prima facie this situation is not much use for quantum computing (though cf.below on the phase qubit).

*though not in charge-mode qubits,where it may be ~1 or even <1.

On the other hand,for $\zeta >> 1$ the groundstate is a doublet, corresponding in the first approximation to states localized In one or other of the two wells,with a zero-point width which is of order $\zeta^{-1/2}$ relative to their separation (see figure).For the moment we will take these states to form the CB (computational basis),so that at this level the « qubit Hamiltonian is simply $-\gamma\sigma_z$ .However, tunnelling between the wells provides a term proportional to $-\Delta\sigma_x$,where the tunnelling matrix element Δ can be straightforwardly calculated in the WKB approximation and is given by



$$\Delta = A(\beta_L - 1)^{1/2}(2\pi I_C / C\Phi_0)^{1/2} \cdot \exp(-2^{-1/2}\zeta)$$

with A a numerical factor of order unity.Note that since β_L is by hypothesis close to 1,the prefactor is of order $(\beta_L - 1)^{1/2}\omega_{LC}$,where ω_{LC} is the LC-resonance frequency of the ring interrupted by the junction capacitance C;typically this of the order of tens of GHz.

Thus,the Hamiltonian truncated to the lowest 2D Hilbert space of the system (the « qubit basis » is of the form

$$H = -\gamma(\Phi_{ext})\sigma_z - \Delta\sigma_x$$

It is thus exactly analogous to that of the charge qubit,with the CB now corresponding to flux rather than charge eigenstates and the control parameter V_g replaced by Φ_{ext} .Just as in that case,the levels drawn as a function of Φ_{ext} show the typical level-repulsion diagram of fig.7 , and just as there ,while one needs to make rough estimates to get the parameters in the right ballpark,it may be more sensible to determine the exact values not a priori but by spectroscopic experiments which in effect measure this diagram.

We now consider the satisfaction of the various criteria for qubit operation for a flux qubit;many of the considerations are formally closely analogous to those already met for a charge qubit,so we can go quite fast.

(1)Initialization:to initialize in the flux basis we apply a large exteranl flux and allow the system to reach thermal equilibrium.Provided the bias energy is >> the thermal energy, we are guaranteed to end up in the groundstate.(Note that just as in the case of the charge qubit,to avoid complications due to quasiparticles we need $T \ll \Delta / k_B \ln N \sim 80mK$,so since typical level splittings are ~a few tenths of a K this condition is likely to be automatically fulfilled).

(2)Readout:this is typically done by coupling the flux through the system –(« qubit ») to a dc SQUID (« detector »),biassing the latter with an external current of magnitude such that if in the flux basis the σ_z -value of the qubit is (say) -1 then $I_C^{eff} > I_{ext}$,so that the detector remains in the zero-voltage state,while if the qubit state is +1 then the critical current is lowered so that $I_C^{eff} < I_{ext}$ and the detector jumps into the running (voltage) state.As in the charge case,this arrangement has the advantage that the detector can be turned on or off by manipulating the externally input current.In real life,one does not usually couple the qubit to the detector sufficiently strongly that every « measurement » is 100% efficient,this would risk inducing back-action from the detector which might itself dephase the qubit.Rather,one couples very weakly,so that the probablity of the dc SQUID « firing » is only slightly different for the two values of the qubit σ_z ,and repeats the measurement many times so as to accumulate meaningful statistics.(This is often also true for the SET measurement of a charge qubit).This is by no means ideal from the point of view of quantum computing,cf.lecture 4B.)

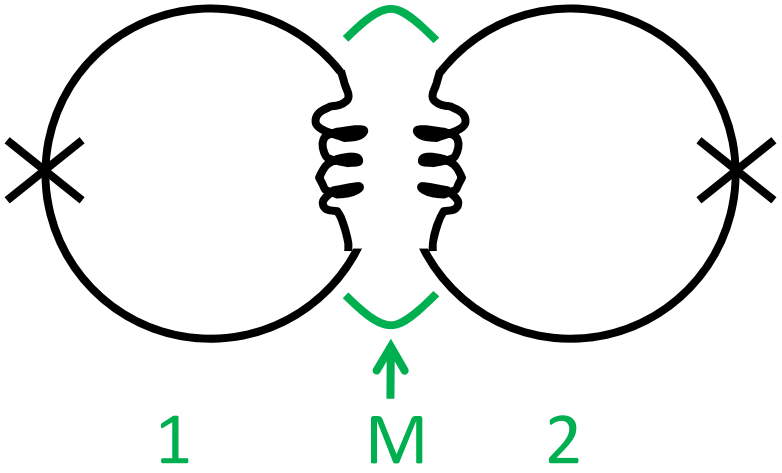
(3)Scaling:see discussion of charge qubit.Let’s note at this point that contrary to the case of the charge qubit,the two « flux » basis states do differ in the behavior of a macroscopic number of electrons (exactly how many is a matter of rather tricky definition) and may therefore be regarded as genuinely «macroscopically distinct ».

4.Qubit gates

a)Single-qubit gates

In analogy to the charge case,one can implement an arbitrary single-qubit gate by appropriate manipulations of the control variable,in this case the external flux.The general principles are identical to those for the charge qubit.

b) 2-qubit gates



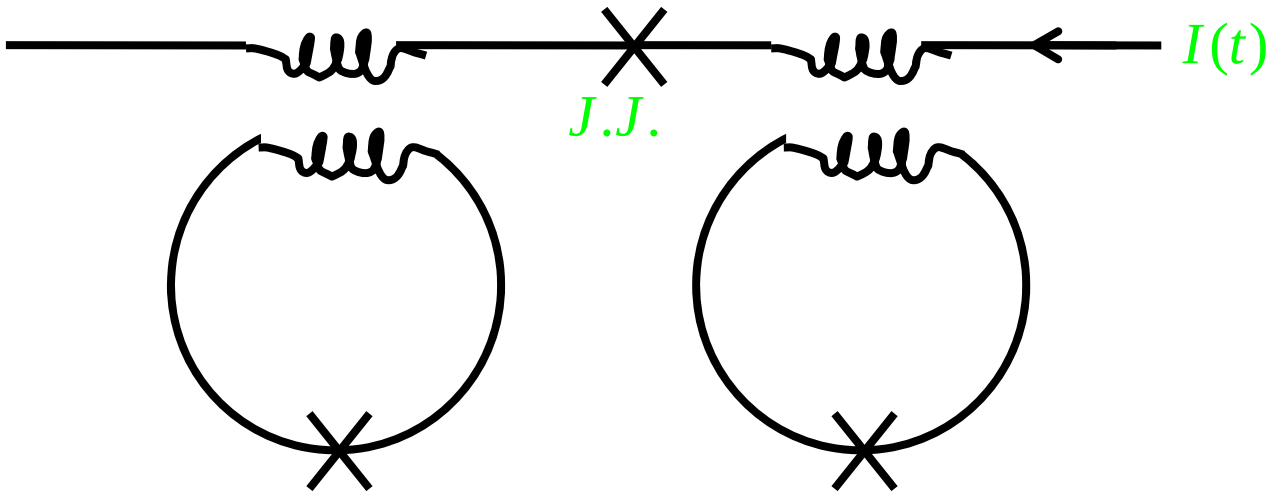
The simplest way of coupling two flux qubits together is inductive.In the setup shown,there is a mutual inductance energy which for arbitrary values Φ_1, Φ_2 of the fluxes through the two rings is given by

$$E_{\text{int}} = \Phi_1 \Phi_2 / M$$

where M^{-1} is actually a shorthand for the relevant element of the inverse of the (self- and mutual) inductance matrix.When projected on to the 2D qubit space,this takes the standard form

$$E_{\text{int}} = A\sigma_{z1}\sigma_{z2} \qquad A \equiv \Delta\Phi_1 \Delta\Phi_2 / 4M$$

where $\Delta\Phi_j$ is the distance in flux space betwen the two states of the j-th qubit.As in the charge case,it is not easy to change this coupling as a function of time,but as there one can now implement (e.g.) a CNOT operation spectroscopically.A somewhat more flexible alternative couples the two qubits indirectly,via an element such as a superconducting inductor itself incorporating a Josephson junction;since the effective inductance of the latter,and hence the effective coupling mediated by it,is a function of the externally input current,this enables the constant A to be changed as a function of time.



5.Decoherence

The formal treatment of decoherence goes through exactly as in the charge case,with the (c-number or fluctuating) external flux replacing the (c-number or fluctuating) gate voltage. In analogy to that case,the most obvious mechanism of decoherence is the fluctuating external flux,e.g.due to the 60 Hz background;to some extent this can be minimized by using a more elaborate geometry than a single ring,so as to minmize the effcetive area which the circuit presents to « naturally occurring » magnetic fields while leaving it sensitive to that applied by the experimenter.However,even when the contributions of this fluctuating flux are minimized, there still appears to be a residual decoherence,which is usually attributed to fluctuations in the critical current of the Josephson junction itself:note that since the tunneling splitting depends exponentially on this quantity (see above) the effect is considerably magnified.We will come back to this question in the context of the phase qubit,where it has been particularly extensively investigated.

3.Phase qubit

At first sight a single Josephson junction biassed by an externally input current would not seem to be a particularly promising candidate for a qubit;if we consider the Hamiltonian of such a system in the absence of a bias current,namely

$$H = p_{\Delta\phi}{}^2 / 2C - I_C \Phi_0 / 2\pi \cos\Delta\phi$$

where $\Delta\phi$ is the phase difference acrossthe Junction and $p_{\Delta\phi}$ the « momentum » conjugate to it,we find that the ratio of the height of the barrier between neighboring (actually equivalent) wells to the zero-point energy is of the order of the « quasiclassicality parameter »

$$\lambda \equiv (8CI_C\Phi_0{}^3 / \pi^3\hbar^2)^{1/2} \sim 4\cdot 10^{11}(CI_C)^{1/2}$$

and since in practice this quantity is almost always >>1,the lowest levels of the system are very harmonic-oscillator-like and hence of little use for quantum computing.

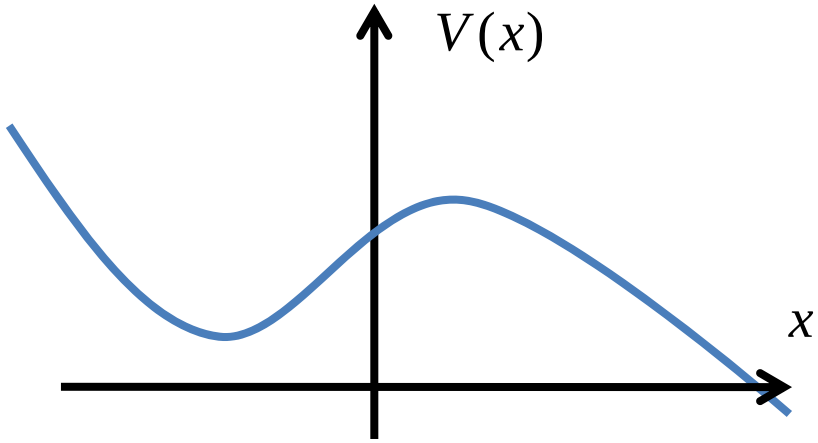
However,the situation changes markedly when we inject an external current and add the corresponding term $-I_{ext}\Delta\phi$ to the Hamiltonian (see lecture 3B),so that

$$H = p_{\Delta\phi}{}^2 / 2C + V(\Delta\phi), V(\Delta\phi) \equiv -I_C\Phi_0 / 2\pi \cos\Delta\phi - I_{ext}\Delta\phi$$

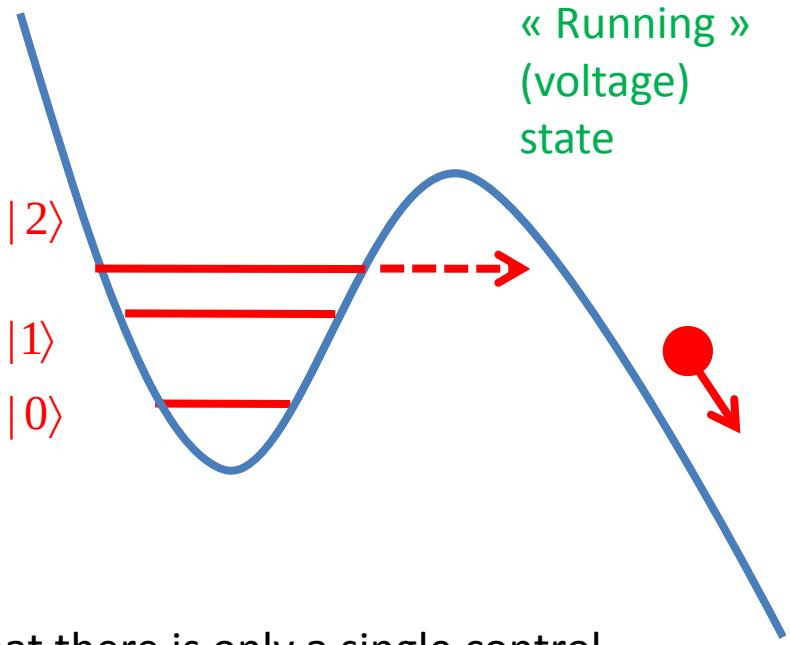
Consider in particular the case where I_{ext} is just slightly smaller than I_C ,so that the value of $\Delta\phi$ in the zero-voltage state is close to $\pi/2$;then the potential has the form, up to a constant,

$$V(x) = \alpha x^2 - \beta x^3, x \equiv \Delta\phi - \sin^{-1}(I_{ext} / I_{C)}$$

as represented in the figure.Note that in the « interesting » region $(\Delta\phi \le \alpha/\beta)$ the « excursion » of $\Delta\phi$ is $<< 2\pi$,so the fact that it is only defined modulo 2π need not worry us.



The crucial point,now,is that because of the strong anhrmonicity of the potential in the region close to x=0 the energy levels are not equally spaced. Moreover,it is possible to tune the parameters so that while the groundstae (|0>)and first excited state (|1>) have negligible probability of decay by tunnelling through the barrier into the running (voltage) state,the lifetime of the second excited state (|2>) against this process is quite small.Consequently,it is possible to use the states |0\ and |1\ as the qubit basis,and the state |2> for readout (see below).



One rather inconvenient feature of the simple CBJ is that there is only a single control parameter,namely the external bias current I_{ext} .One can improve the situation by replacing the simple junction with a dc SQUID,so as to take advantage of the dependence of the critical current on the flux through the latter.

On the conditions for use as a qubit:

1)Initialization is achieved as in other cases by thermalization.

2)Readout is achieved in a novel way:namely,teh system is detected to be in state |1\ by applying an rf pulse (of the external current) tuned to the |1>->|2> transition (because of the unequal level spacing,it is possible to do this while avoiding the excitation of $|0>\rightleftarrows|1>$ transitions).Once it is in state |2>,the systen rapidly tunnels out through the barrier into the running state,generating a voltage across the junction which is picked up in the external circuit.

3)Scaling:see on charge qubits

4.Qubit gates

a)1-qubit gates

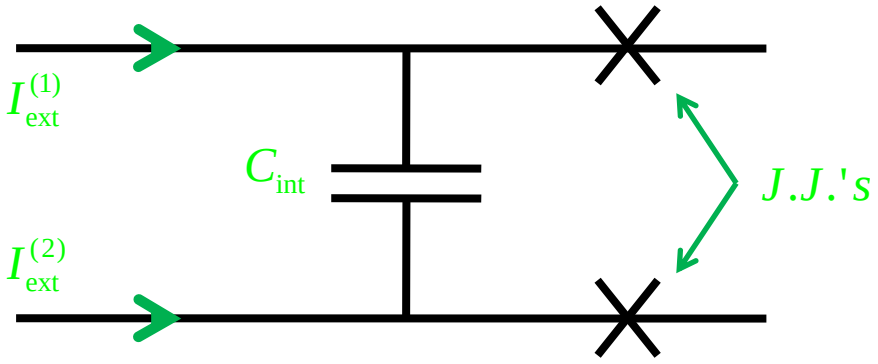
It is perhaps not imediately obvious how,using the single control parameter $I_{ext}(t)$,one can implement arbitray 1-qubit gates on a pahse qubit.However,the principel is teh same as in the charge qubit:if one sets

$$I_{ext}(t) = I_0 + \delta I_{ext} \cos \omega t$$

then the dc splitting will be set by I_0 ,while the ac current term will have matrix elements between (inter alia) the states |0> and |1>,and thus contribute an ac field in the x-direction. The only difference with the charge case (operated at the crossing point) is that the ac current term will also modulate the splitting,i.e.contribute an ac field in the z-direction; however,provided $\delta I_{ext} << I_0$ the effect of this term should be small.Cosnsequently,one can carry out all the usual single-qubit manipulations.

b)2-qubit gates

The only immediately obvious way of coupling two phase qubits is via a common capacitance, and this is what has usually been done.In the standard notation with the computational basis |0> and |1>,the result is to couple all components of the « spins »,but in the approximation of small anharmonicity the leading term turns out to be of the form $const \cdot \sigma_{1y}\sigma_{2y}$ .Note that,unlike the coupling terms in previous cases,this does not commute with the single-qubit Hamiltonain,so while it is still possible to implement (e.g.) a CNOT gate the analysis is more complicated.



5.Decoherence

The only obvious sourecof decoherence in a CBJ (phase qubit),apart from the leads, is fluctuations in the critical current of the junction itself,and this question has been intensively investigated over the last 2-3 years.There is a theoretical expectation that a realistic junction may contain a number of « tunnelling two-level systems (TTLS) » analogous to those commonly supposed to occur in amorphous materials and to dominate their properties below 1K.It is thought that when such a system tunnels between its two equilibrium positions,this should affectc the critical current of the junction and thus give rise to phasenoise.Recently, more or lesss direct spectroscopic evidence,from the details of the junction I-V characteristics,has been obtained for this scenario.Much work is currently going into attempts to improve the fabrication of tunnel oxide junctions so as to reduce the number of these TTLS or get rid of them entirely.