Theory of Mesoscopic Systems

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CONFÉRENCE UNIVERSITAIRE
DE SUISSE OCCIDENTALE

Lecture 1 01 June 2006

Meso: intermediate between micro and macro

Main Topics:

- Disordered conductors in d=3,2,1 and 0 dimensions
- Anderson localization
- Weak localization
- Sample to sample fluctuations
- Quantum chaos & spectral statistics

- Systems of interacting quantum particles
- Fermi liquid theory without translation invariance
- Decoherence
- Many body localization

ORIGINS

E.P. Wigner, Conference on Neutron Physics by Time of Flight, November 1956

P.W. Anderson, "*Absence of Diffusion in Certain Random Lattices*"; Phys.Rev., 1958, v.109, p.1492

L.D. Landau, "*Fermi-Liquid Theory*" Zh. Exp. Teor. Fiz.,1956, v.30, p.1058

J. Bardeen, L.N. Cooper & J. Schriffer, "*Theory of Superconductivity*"; Phys.Rev., 1957, v.108, p.1175.

Introduction

Einstein's Miraculous Year - 1905

Six papers:

1. The light-quantum and the photoelectric effect.
   Completed March 17.

2. A new determination of molecular dimensions.
   Completed April 30. Published in1906
   Ph.D. thesis.

3. Brownian Motion.
   Received by Annalen der Physik May 11.

4,5. The two papers on special relativity.
   Received June 30 and September 27

6. Second paper on Brownian motion.
   Received December 19.

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Q: Are these papers indeed important enough to stay in the same line with the relativity and photons. Why **?**

Einstein’s Miraculous Year - 1905

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1. The light-quantum and the photoelectric effect.
   Completed March 17.

   Nobel Prize

2. A new determination of molecular dimensions.
   Completed April 30. Published in1906
   Ph.D. thesis.

   By far the largest number of citations

3. Brownian Motion.
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Brownian Motion - history

Robert Brown
(1773-1858)

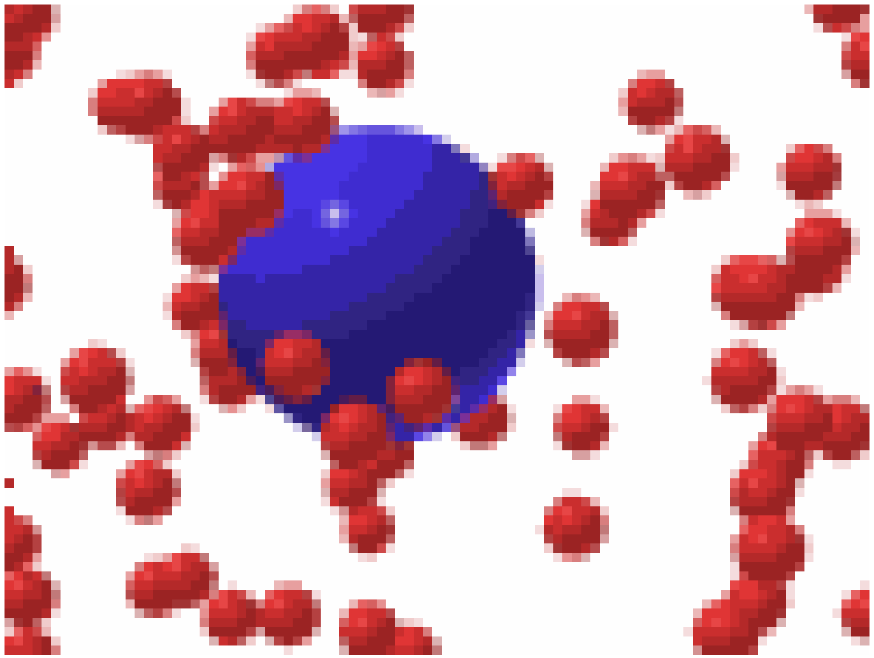
The instrument with which Robert Brown studied Brownian Motion and which he used in his work on identifying the nucleus of the living cell. This instrument is preserved at the Linnean Society in London.

Brownian Motion - history

Robert Brown, ***Phil.Mag***. ***4****,161(1828);* ***6****,161(1829)*

Random motion of particles suspended in water ("dust or soot deposited on all bodies in such quantities, especially in London")

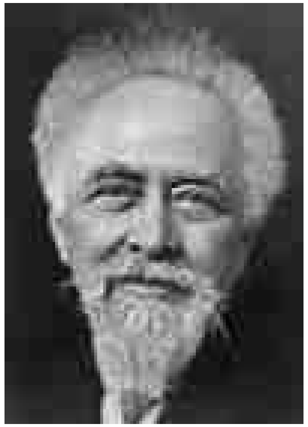
Action of water molecules pushing against the suspended object ?



Giovanni Cantoni (Pavia). ***N.Cimento,*** *27,156(1867).*

The Nobel Prize in Physics 1926

"for his work on the discontinuous structure of matter, and especially for his discovery of sedimentation equilibrium"



Jean Baptiste Perrin
France

b. 1870
d. 1942

... measurements on the Brownian movement showed that Einstein's theory was in perfect agreement with reality. Through these measurements a new determination of Avogadro's number was obtained.

The Nobel Prize in Physics 1926
From the Presentation Speech by Professor C.W. Oseen, member of the Nobel Committee for Physics of The Royal Swedish Academy of Sciences on December 10, 1926

Brownian Motion - history

Robert Brown, ***Phil.Mag.*** *4,161(1828);* ***6****,161(1829)*

Random motion of particles suspended in water ("dust or soot deposited on all bodies in such quantities, especially in London")

Action of water molecules pushing against the suspend object

Problems:

1. Each molecules is too light to change the momentum of the suspended particle.
2. Does Brownian motion violate the second law of thermodynamics ?

Jules Henri Poincaré (1854-1912)

"We see under our eyes now motion transformed into heat by friction, now heat changes inversely into motion. This is contrary to Carnot's principle."

H. Poincare, "The fundamentals of Science", p.305, Scientific Press, NY, 1913

Problems:

1. Each molecules is too light to change the momentum of the suspended particle.
2. Does Brownian motion violate the second law of thermodynamics ?
3. Do molecules exist as real objects and are the laws of mechanics applicable to them?

Kinetic theory

Ludwig Boltzmann
1844 - 1906

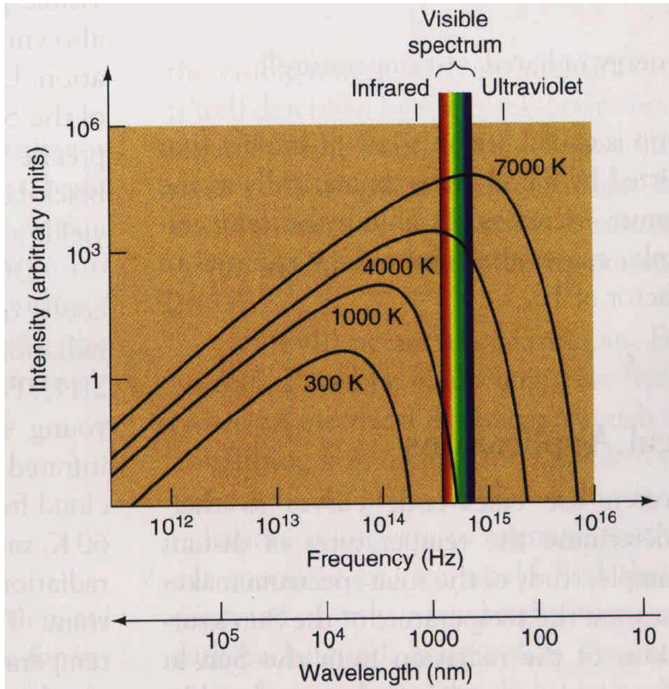
$$S = k \log W + const$$

entropy — probability

k is Boltzmann constant

Max Planck
1858 - 1947

$$\rho(\nu, T) = \frac{8\pi h \nu^3}{c^3\left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]}$$



From Micro to Macro

$$S = k \log W + const$$

From Macro to Micro

"It is of great importance since it permits exact computation of Avogadro number … . The great significance as a matter of principle is, however … that one sees directly under the microscope part of the heat energy in the form of mechanical energy."

Einstein, 1915

Brownian Motion - history

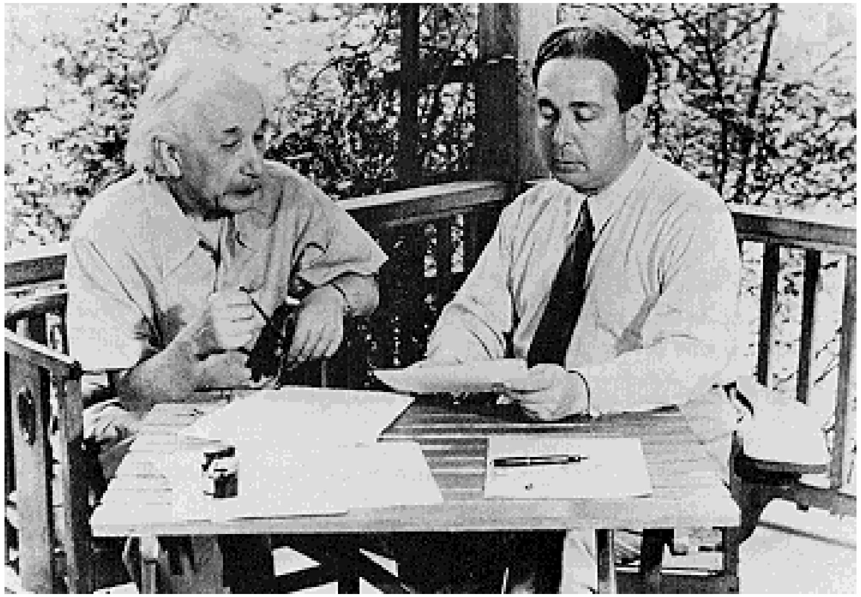
Einstein was not the first to:

1. Attribute the Brownian motion to the action of water molecules pushing against the suspended object

2. Write down the diffusion equation

3. Saved Second law of Thermodynamics

L. Szilard, Z. Phys, 53, 840(1929)



Brownian Motion - history

Einstein was not the first to:

1. Attribute the Brownian motion to the action of water molecules pushing against the suspended object
2. Write down the diffusion equation
3. Saved Carnot's principle [L. Szilard, Z. Phys, 53, 840(1929)]

Einstein was the first to:

1. Apply the diffusion equation to the probability
2. Derive the diffusion equation from the assumption that the process is markovian (before Markov) and take into account nonmarkovian effects
3. Derived the relation between diffusion const and viscosity (conductivity), i.e., connected fluctuations with dissipation

By studying large molecules in solutions sugar in water or suspended particles Einstein made molecules visible

Diffusion Equation

$$\frac{\partial r}{\partial t} - D\nabla^2 r = 0$$

Diffusion constant

Einstein-Sutherland Relation for electric conductivity s

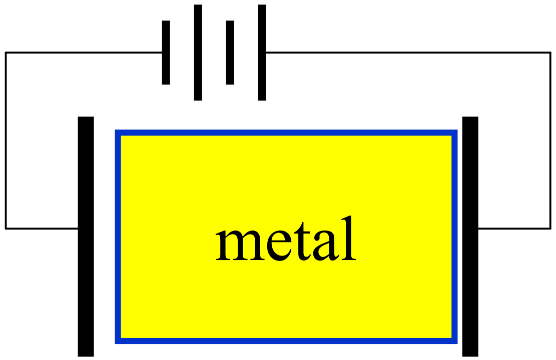
William Sutherland
(1859-1911)

$$s = e^2 D n \qquad n \equiv \frac{dn}{dm}$$

If electrons would be degenerate and form a classical ideal gas

$$n = \frac{1}{T n_{tot}}$$

Einstein-Sutherland Relation for electric conductivity s



$$n = n(\mathrm{m})$$

Density of electrons

Chemical potential

$$\frac{dn}{dx} = \frac{dn}{d\mathrm{m}}\frac{d\mathrm{m}}{dx} = eE\frac{dn}{d\mathrm{m}}$$

Electric field

No current

$$eD\frac{dn}{dx} = \mathrm{s}\,E$$

Conductivity

$$\mathrm{s} = e^2 D\mathrm{n} \qquad \mathrm{n} \equiv \frac{dn}{d\mathrm{m}}$$

Density of states

Diffusion Equation

$$\frac{\partial \rho}{\partial t} - D\nabla^2 \rho = 0$$

Lessons from the Einstein's work:

- **Universality:** the equation is valid as long as the process is marcovian
- Can be applied to the **probability** and thus describes both fluctuations and dissipation
- There is a universal relation between the diffusion constant and the viscosity
- Studies of the diffusion processes brings information about micro scales.

What is a Mesoscopic System?

- Statistical description
- Can be effected by a microscopic system and the effect can be macroscopically detected

Meso can serve as a microscope to study micro

Brownian particle was the first mesoscopic device in use

Brownian particle was the first mesoscopic device in use

First paper on Quantum Theory of Solid State (Specific heat)
Annalen der Physik, 22, 180, 800 (1907)

First paper on Mesoscopic Physics
Annalen der Physik, 17, 549 (1905)

Finite size quantum physical systems

Atoms

Nuclei

Molecules

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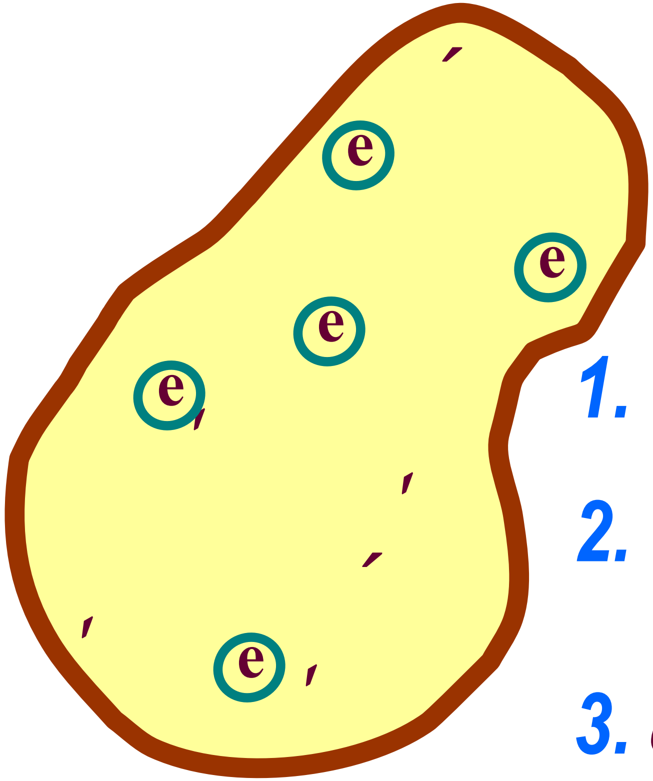
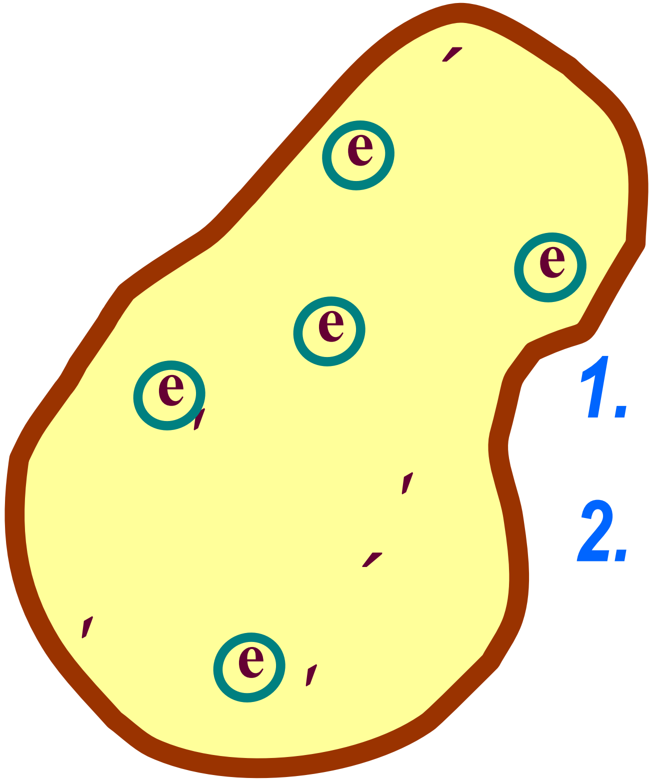
} Quantum Dots

Quantum Dot



1. *Disorder (´ - impurities)*
2. *Complex geometry*
3. *e-e interactions*



1. *Disorder (´ - impurities)*
2. *Complex geometry*

How to deal with disorder?

- ~~Solve the Shrodinger equation exactly~~
- Start with plane waves, introduce the mean free path, and derive Boltzmann equation

How to take quantum interference into account

?

Electrons in nanostructures

Clean systems without boundaries:

- Electrons are characterized by their momenta or quasimomenta
  [electronic wave functions are plane waves

- Physics is essentially local

- Interaction between electrons is often apparently not important

In mesoscopic systems:

- Due to the scattering of the electrons off disorder (impurities) and/or boundaries the momentum is not a good quantum number

- Response to external perturbation is usually nonlocal

- Interaction between electrons is often crucial

Lesson 1:

Beyond Markov chains:

Anderson Localization
and
Magnetoresistance

Т. 18 Журнал экспериментальной и теоретической физики. Вып.

1948

ОБ ИЗМЕНЕНИИ ЭЛЕКТРИЧЕСКОГО СОПРОТИВЛЕНИЯ ТЕЛЛУРА В МАГНИТНОМ ПОЛЕ ПРИ НИЗКИХ ТЕМПЕРАТУРАХ

Р. А. Ченцов

R.A. Chentsov ***“On the variation of electrical conductivity of tellurium in magnetic field at low temperatures”****, Zh. Exp. Theor. Fiz. v.18, 375-385, (1948).*

Таблица 2

Уменьшение сопротивления теллура в магнитном поле

Образец	Температура (°K)	Максимальное уменьшение сопротивления
Te-1	2,13	$0{,}7 \cdot 10^{-1}$
Te-2	2,15	$1{,}0 \cdot 10^{-3}$
Te-4	1,96	$1{,}1 \cdot 10^{-1}$
Te-5	1,96	$0{,}5 \cdot 10^{-4}$

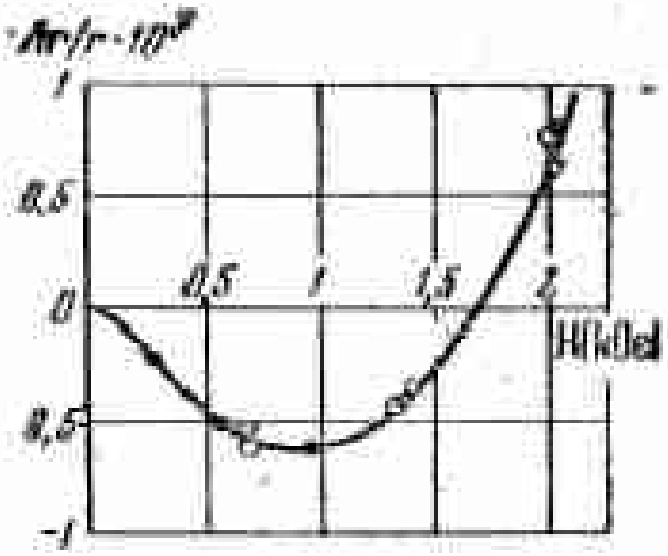


Рис. 2

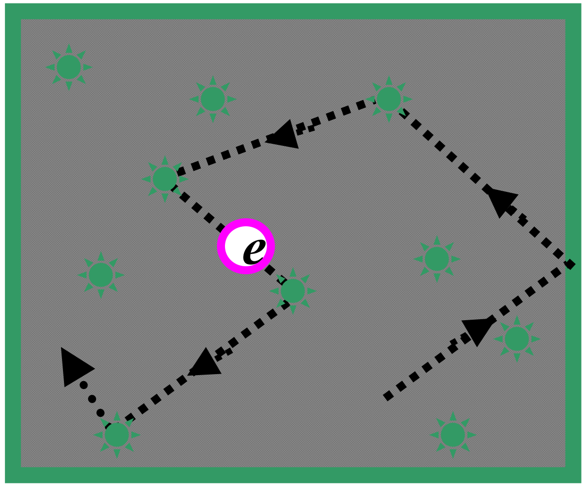
Quantum particle in random quenched potential

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.



ORIGINS

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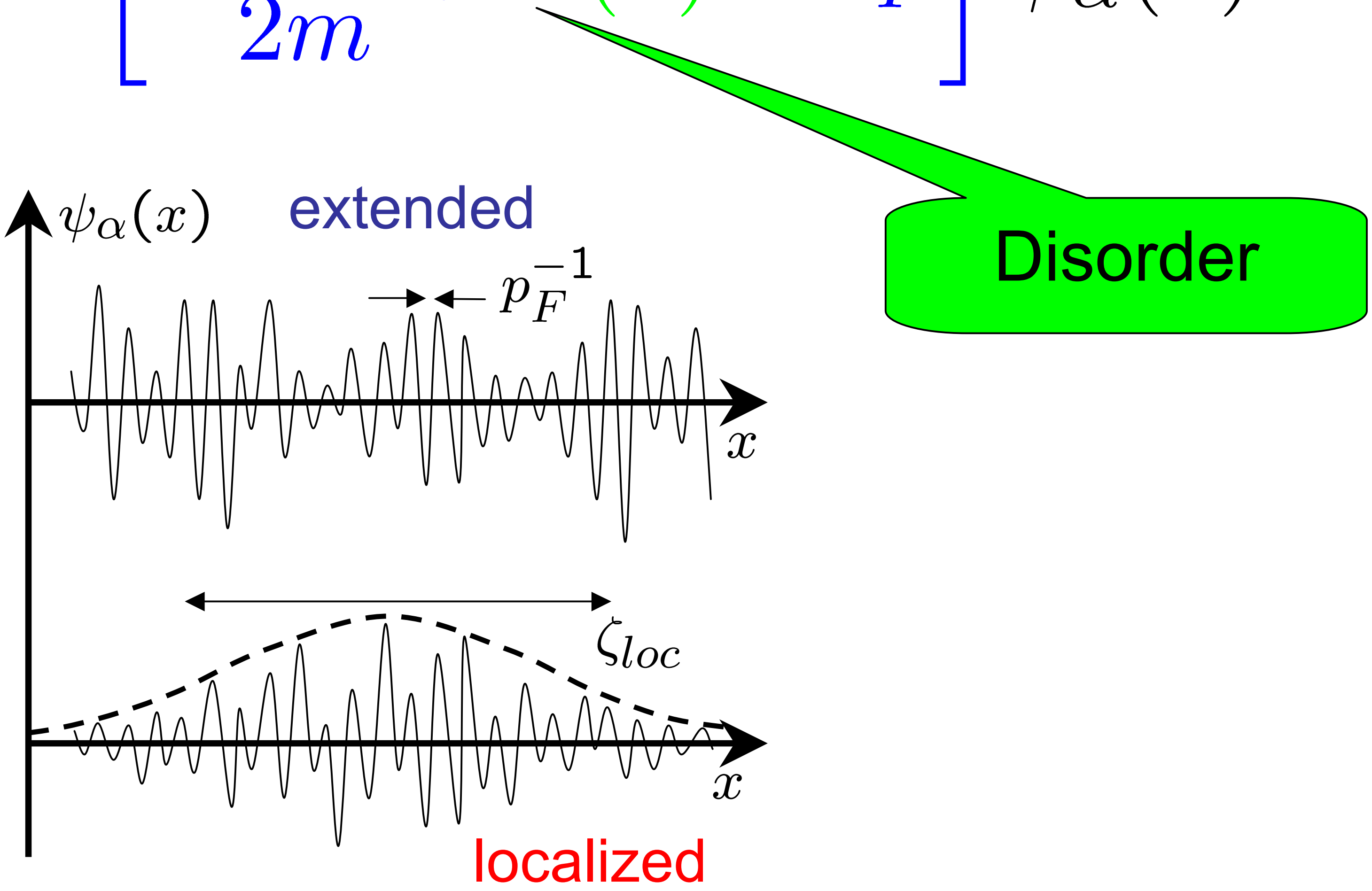
J. Bardeen, L.N. Cooper & J. Schriffer, "*Theory of Superconductivity*"; Phys.Rev., 1957, v.108, p.1175.

Localization of single-particle wave-functions:

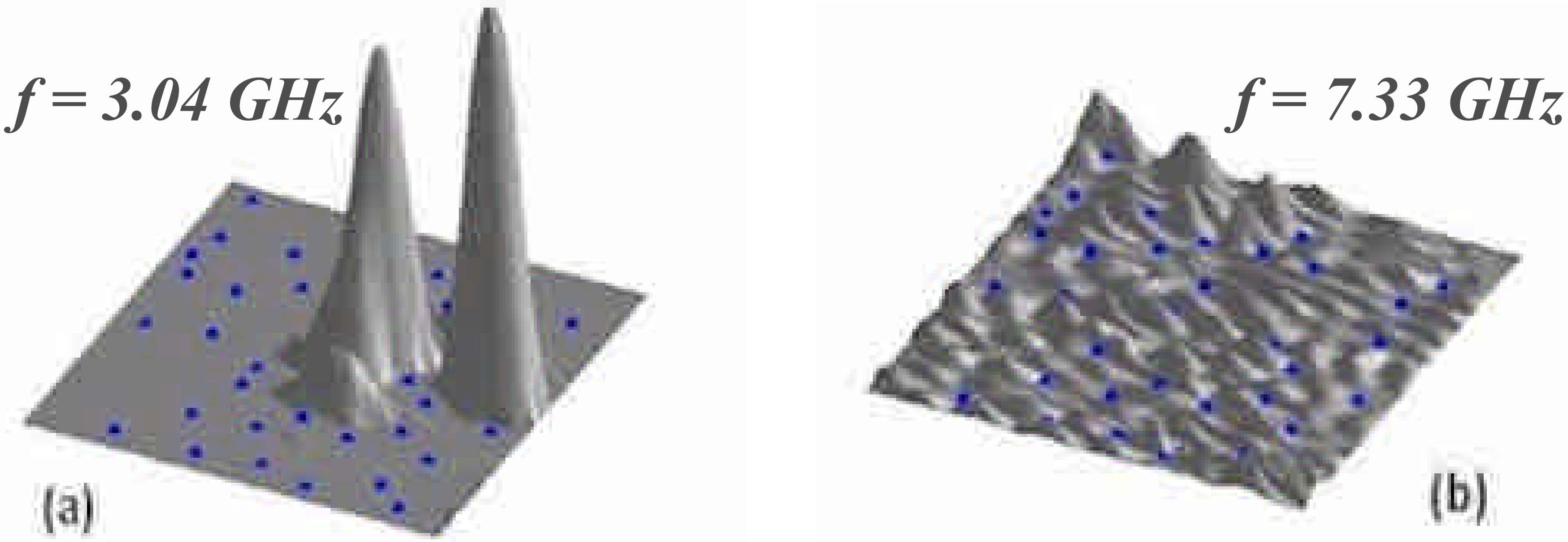
$$\left[-\frac{\boldsymbol{\nabla}^2}{2m} + U(\boldsymbol{r}) - \epsilon_F\right]\psi_\alpha(\boldsymbol{r}) = \xi_\alpha\psi_\alpha(\boldsymbol{r})$$



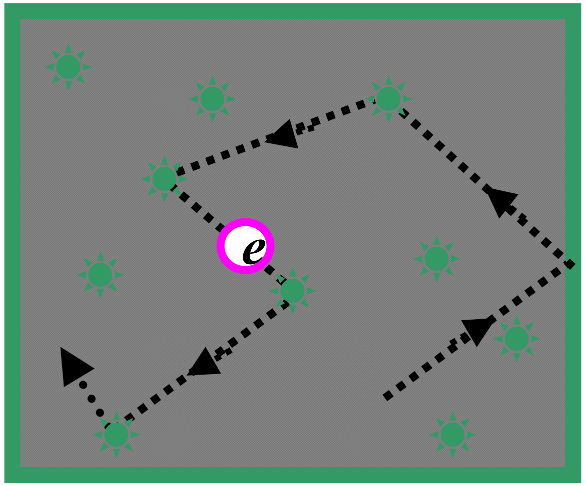
VOLUME 85, NUMBER 11 PHYSICAL REVIEW LETTERS 11 SEPTEMBER 2000

Correlations due to Localization in Quantum Eigenfunctions of Disordered Microwave Cavities

Prabhakar Pradhan and S. Sridhar
Department of Physics, Northeastern University, Boston, Massachusetts 02115
(Received 28 February 2000)



Anderson Insulator ***Anderson Metal***



Scattering centers, e.g., impurities

Models of disorder:

Randomly located impurities

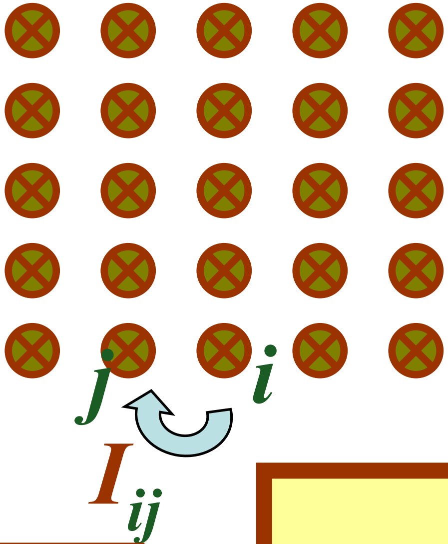
White noise potential

Lattice models

Anderson model

Lifshits model

Anderson Model



- *Lattice - tight binding model*
- *Onsite energies* e_i *- random*
- *Hopping matrix elements* I_{ij}

$-W < e_i < W$

uniformly distributed

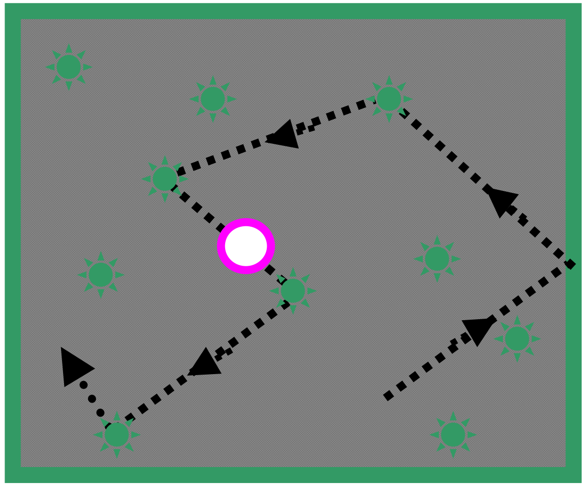
$$I_{ij} = \begin{cases} I & i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases}$$

Anderson Transition

$I < I_c$	$I > I_c$
Insulator	***Metal***
All eigenstates are localized	*There appear states extended all over the whole system*
Localization length X	

Classical particle in a random potential

Diffusion



1 particle – random walk

Density of the particles ρ

Density fluctuations $\rho(\boldsymbol{r},t)$ at a given point in space $\boldsymbol{r}$ and time t.

$$\frac{\partial \rho}{\partial t} - D\nabla^2 \rho = 0$$

Diffusion Equation

D - Diffusion constant

$$D = \frac{l^2}{d\tau}$$

l mean free path

τ mean free time

d # of dimensions

Einstein - Sutherland Relation for electric conductivity S

$$s = e^2 D n \qquad n \equiv \frac{dn}{dm}$$

Conductance

$$G = s\, L^{d-2}$$

for a cubic sample of the size L

$$G = \underbrace{\left(n L^d\right) \frac{Dh}{L^2}}_{g(L)}$$

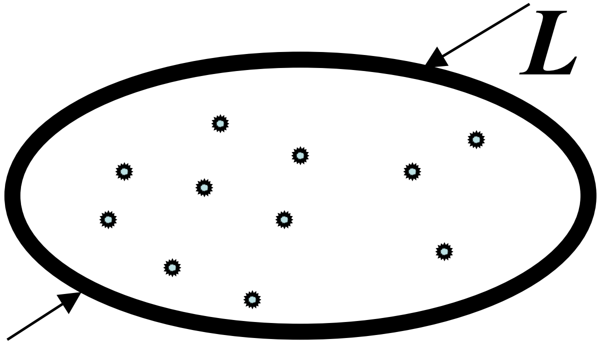
$$g(L) = \frac{hD/L^2}{1/n L^d} = \frac{\text{Thouless energy}}{\text{mean level spacing}}$$

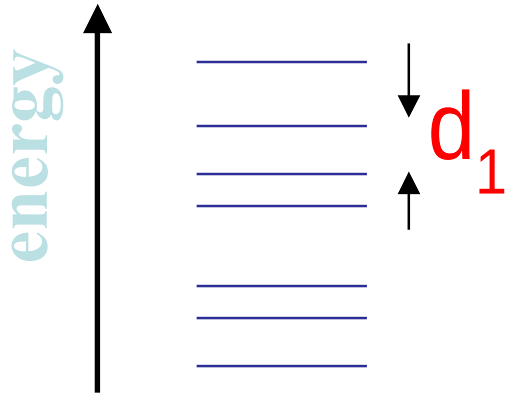
Dimensionless Thouless conductance

Energy scales (*Thouless, 1972*)

1. Mean level spacing

$d_1 = 1/n' \, L^d$





L *is the system size;*

d *is the number of dimensions*

2. Thouless energy

$E_T = hD/L^2$

D *is the diffusion const*

E_T *has a meaning of the **inverse diffusion time** of the traveling through the system or the **escape rate** (for open systems)*

$g = E_T / d_1$

dimensionless
Thouless
conductance

$g = Gh/e^2$

$$g(L) = \frac{hD/L^2}{1/n\,L^d} = \frac{\text{Thouless energy}}{\text{mean level spacing}}$$

Thouless conductance is Dimensionless

Corrections to the diffusion come from the large distances (infrared corrections)

Scaling theory of Localization

(Abrahams, Anderson, Licciardello and Ramakrishnan, 1979)

Universal description!

Scaling theory of Localization

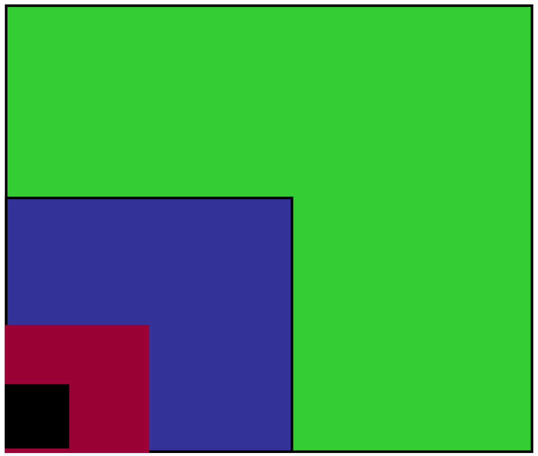
Abrahams, Anderson, Licciardello and Ramakrishnan

1979

$g = E_T / d_1$

Dimensionless Thouless conductance

$g = Gh/e^2$



$L = 2L = 4L = 8L$ ....

without quantum corrections

E_T μ L^{-2} d_1 μ L^{-d}

$E_T \rightarrow E_T \rightarrow E_T \rightarrow E_T$

$d_1 \quad d_1 \quad d_1 \quad d_1$

$g \rightarrow g \rightarrow g \rightarrow g$

$$\frac{d(\log g)}{d(\log L)} = b(g)$$

$$\frac{d(\log g)}{d(\log L)} = \beta(g)$$

β – function

Is **universal**, i.e., material independent

But

It depends on the global symmetries, e.g., it is different with and without T-**invariance**

Limits:

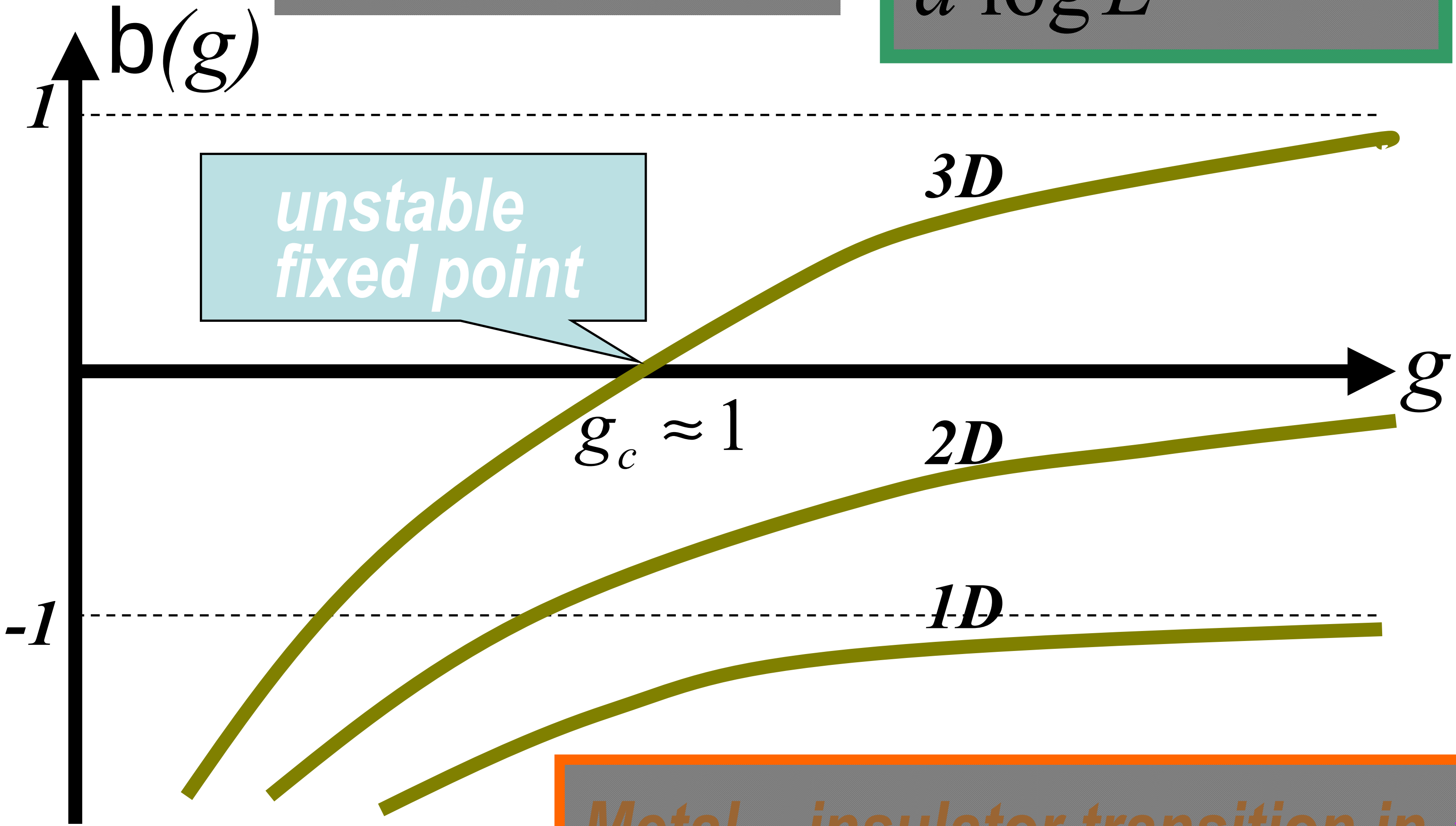
$$g >> 1 \quad g \propto L^{d-2} \quad \beta(g) = (d-2) + O\left(\frac{1}{g}\right)$$

$$g << 1 \quad g \propto e^{-L/\xi} \quad \beta(g) \approx \log g < 0$$

> 0	$d > 2$
??	$d = 2$
< 0	$d < 2$

b - function

$$\frac{d\log g}{d\log L} = \mathrm{b}(g)$$

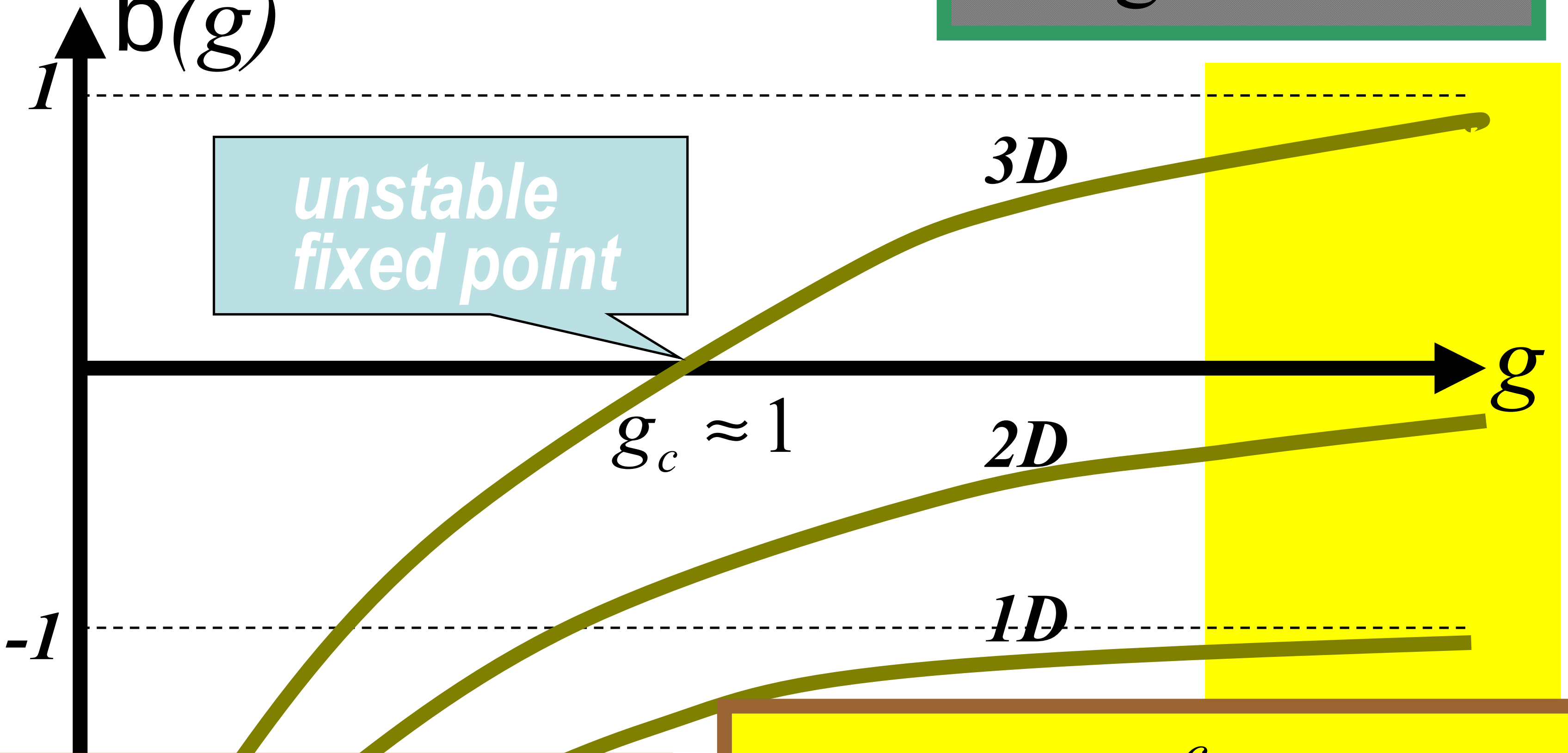


Metal – insulator transition in 3D

All states are localized for d=1,2

β - function

$$\frac{d\log g}{d\log L} = \beta(g)$$



$$\beta(g) = d - 2 + \frac{c_d}{g}$$

$c_d = ? \quad \pm ?$

$$g(L) = \begin{cases} \sigma_{cl} L^{d-2} - \dfrac{c_d}{d-2} & d \neq 2 \\ c_2 \log\left(\dfrac{L}{l}\right) & d = 2 \end{cases}$$

Questions:

Why

- the scaling theory is correct?
- the corrections of the diffusion constant and conductance are negative?

Why diffusion description fails at large scales ?

Diffusion description fails at large scales

Why?

Einstein: there is no diffusion at too short scales – there is memory, i.e., the process is not marcovian.

$$r(t) = \sqrt{Dt}$$

$$\frac{dr}{dt} = \sqrt{\frac{D}{2t}}$$

Does velocity diverge at $t \longrightarrow 0$?
No because at times shorter than mean free time process is not marcovian and there is no diffusion

Diffusion description fails at large scales

Why?

Einstein: there is no diffusion at too short scales – there is memory, i.e., the process is not marcovian.

Why there is memory at large distances in quantum case ?

Quantum corrections at large Thouless conductance – weak localization

Universal description

Quantum corrections

$$\beta(g) = d - 2 + \frac{c_d}{g}$$

$$c_d = ? \quad \pm ?$$

$$g(L) = \sigma_{cl} L^{d-2} - \begin{cases} \dfrac{c_d}{d-2} & d \neq 2 \\ c_2 \log\left(\dfrac{L}{l}\right) & d = 2 \end{cases}$$

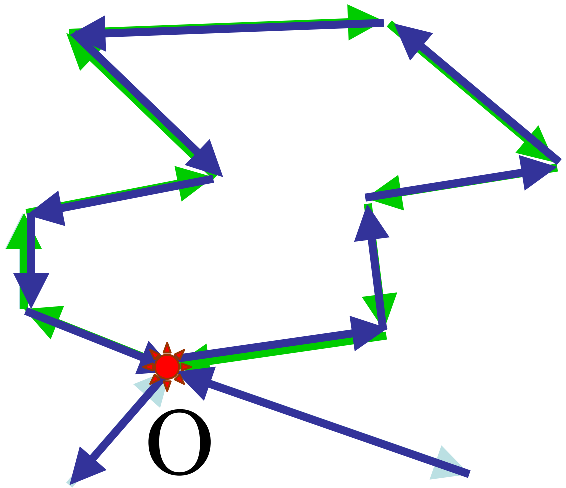
Suggested homework:

1. Derive the equation for $g(L)$ from this limit of the β-function
2. Suppose you know $\beta(g)$ for some number of dimensions d. Let g at some size of the system L_0 be close to the critical value: $g(L_0) = g_c + \delta g;\ \delta g \ll 1$ Estimate the localization length ξ (for $\delta g < 0$) and the conductivity σ in the limit $L \to \infty$ (for $\delta g > 0$)

WEAK LOCALIZATION

$$j = \oint \vec{p}\,d\vec{r}$$

Phase accumulated when traveling along the loop



The particle can go around the loop in two directions

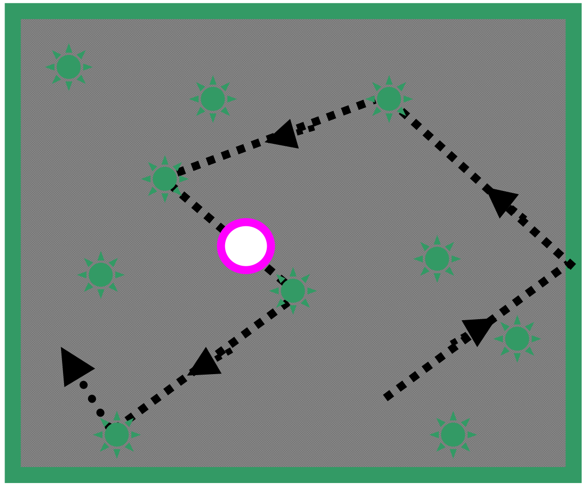
Memory!

$$j_1 = j_2$$

Constructive interference → probability to return to the origin gets enhanced → diffusion constant gets reduced. Tendency towards localization

b *- function is negative for* ***d*=2**

Diffusion



Random walk

Density fluctuations $\rho(\boldsymbol{r},t)$ at a given point in space $\boldsymbol{r}$ and time t.

$$\frac{\partial \rho}{\partial t} - D\nabla^2 \rho = 0$$

Diffusion Equation

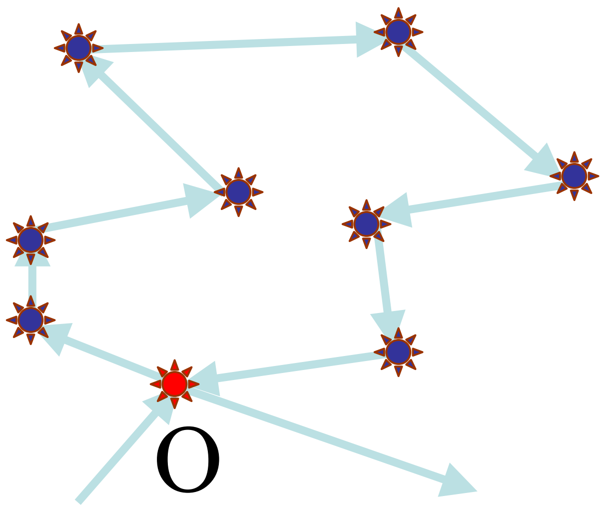
D - Diffusion constant

Mean squared distance from the original point at time t

$$\langle r(t)^2 \rangle = Dt$$

Probability to come back (to the element of the volume dV centered at the original point)

$$P\big(r(t)=0\big)\,dV = \frac{dV}{(Dt)^{d/2}}$$



What is the probability *P(t)* the particle comes back in a time t ?

Probability to come back (to the element of the volume dV around the original point)

$$P\left(r\left(t\right)=0\right)dV=\frac{dV}{\left(Dt\right)^{d/2}}$$

Q: $dV=?$

A: $dV=\lambda^{d-1}v_F\,dt$

$$P\left(t\right)=-\lambda^{d-1}\int_{\tau}^{t}\frac{v_F\,dt'}{\left(Dt'\right)^{d/2}}$$

$$\frac{\delta\gamma}{\gamma}\approx P\left(t_{\max}\right)$$

$$P(t) = -\lambda^{d-1}\int_{\tau}^{t}\frac{v_F\,dt'}{(Dt')^{d/2}} \qquad \frac{\delta g}{g} \approx P(t_{\max})$$

Q: $t_{\max} = ?$

A: $t_{\max} \approx \min\left\{\frac{L^2}{D}, \frac{1}{\omega}, \tau_{\varphi}\right\}$

Decoherence in the leads

AC conductivity

Dephasing time (inelastic processes)

$$P(t) = -\bar{\lambda}^{d-1} \int_{\tau}^{t} \frac{v_F \, dt'}{(Dt')^{d/2}}$$

$$\frac{\delta g}{g} \approx P(t_{\max})$$

$$t_{\max} \propto \frac{L^2}{D} = \frac{h}{E_T}$$

$$d = 2$$

$$\frac{\delta g}{g} \approx -\frac{\bar{\lambda} v_F}{D} \log \frac{L^2}{D\tau}$$

$$P(t) = \bar{\lambda}^{d-1}\int_{\tau}^{t}\frac{v_F\,dt'}{(Dt')^{d/2}}$$

$$\frac{\delta g}{g} \approx P(t_{\max})$$

$$\frac{\delta g}{g} \approx -\frac{\bar{\lambda} v_F}{D}\log\frac{L^2}{D\tau} = -\frac{2\bar{\lambda} v_F}{D}\log\frac{L}{l}$$

$$\bar{\lambda} v_F = \frac{1}{\pi\nu}$$

$$g = \nu D\hbar$$

$$\delta g = -\frac{2}{\pi}\log\frac{L}{l}$$

$$\beta(g) = -\frac{2}{\pi g}$$

Universal !!!

Q: What does it mean $d=2$ **?**

A: Transverse dimension is much less than

$$\sqrt{Dt_{\max}}$$

$$d\,g = -\frac{2}{p}\log\frac{L}{l}$$

$$b(g) = -\frac{2}{p\,g}$$

for a film with a thickness a much smaller than L, L_j

Т. 18 Журнал экспериментальной и теоретической физики. Вып.
1948

ОБ ИЗМЕНЕНИИ ЭЛЕКТРИЧЕСКОГО СОПРОТИВЛЕНИЯ ТЕЛЛУРА В МАГНИТНОМ ПОЛЕ ПРИ НИЗКИХ ТЕМПЕРАТУРАХ

Р. А. Ченцов

R.A. Chentsov ***"On the variation of electrical conductivity of tellurium in magnetic field at low temperatures"***, *Zh. Exp. Theor. Fiz. v.18, 375-385, (1948).*

Таблица 2

Уменьшение сопротивления теллура в магнитном поле

Образец	Температура (°K)	Максимальное уменьшение сопротивления
Te-1	2,13	$0{,}7 \cdot 10^{-3}$
Te-2	2,15	$1{,}0 \cdot 10^{-3}$
Te-4	1,96	$1{,}1 \cdot 10^{-3}$
Te-5	1,96	$0{,}5 \cdot 10^{-4}$

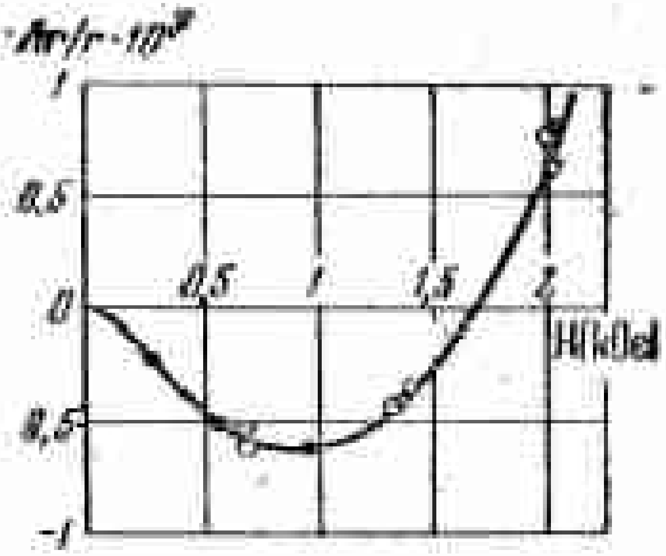
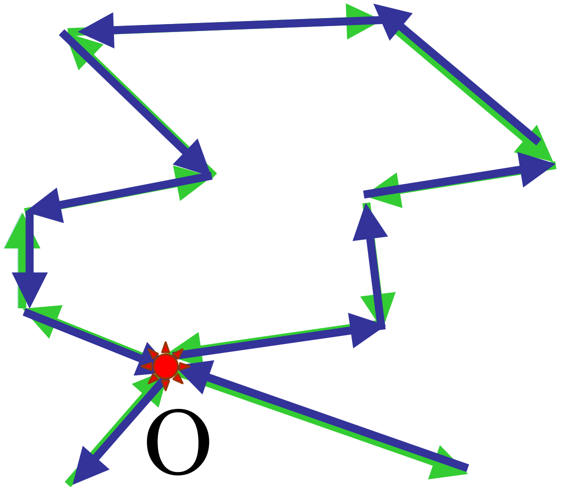
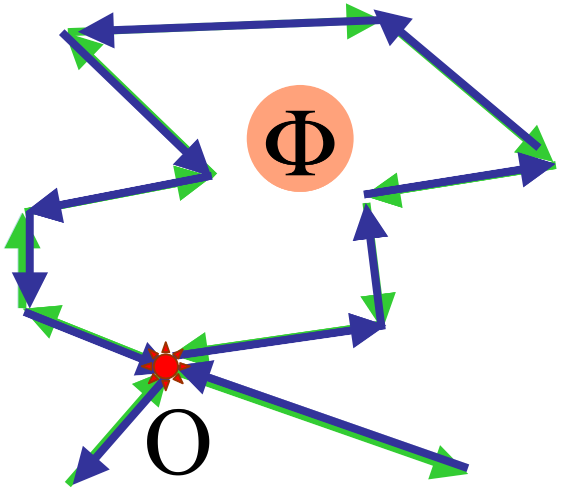


Рис. 2

Magnetoresistance





No magnetic field

$j_1 = j_2$

With magnetic field H

$j_1 - j_2 = 2 * 2p\ \Phi / \Phi_0$

Length Scales

Magnetic length $L_H = (hc/eH)^{1/2}$

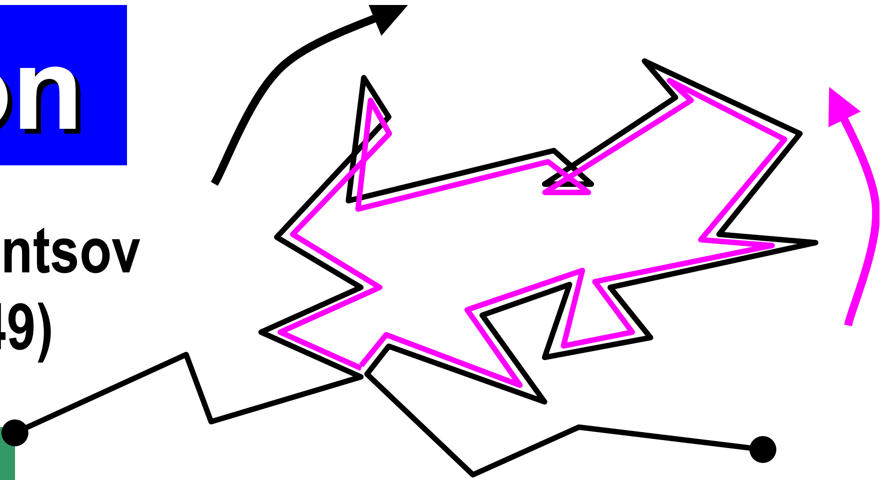
Dephasing length $L_\varphi = (D\tau_\varphi)^{1/2}$

$$\delta g(H) = f_d\left(\frac{L_H}{L_\varphi}\right)$$

Universal functions

Magnetoresistance measurements allow to study inelastic collisions of electrons with phonons and other electrons

Weak Localization



Negative Magnetoresistance

Chentsov (1949)

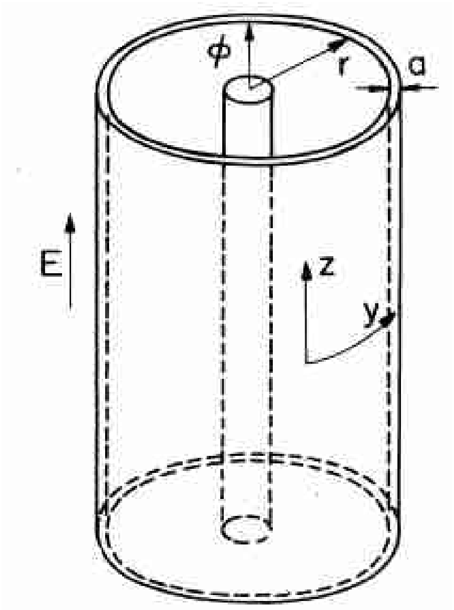
Aharonov-Bohm effect

Theory

B.A., Aronov & Spivak (1981)

Experiment

Sharvin & Sharvin (1981)



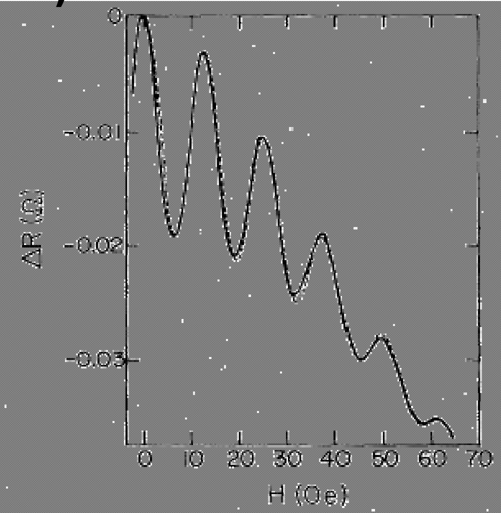


FIG. 8. Longitudinal magnetoresistance $\Delta R(H)$ at $T=1.1$ K for a cylindrical lithium film evaporated onto a 1-cm-long quartz filament. $R_{4.2}=2$ kΩ, $R_{300}/R_{4.2}=2.8$. Solid line: averaged from four experimental curves. Dashed line: calculated for $L_\varphi=2.2$ μm, $\tau_\varphi/\tau_{so}=0$, filament diameter $d=1.31$ μm, film thickness 127 nm. Filament diameter measured with scanning electron microscope yields $d=1.30\pm0.03$ μm (Altshuler et al., 1982; Sharvin, 1984).

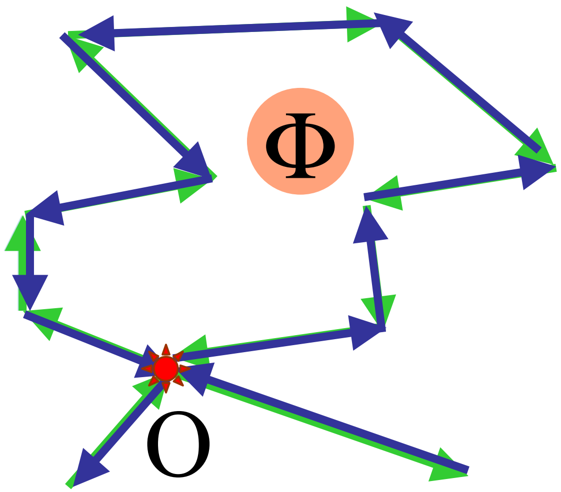
Aharonov-Bohm effect

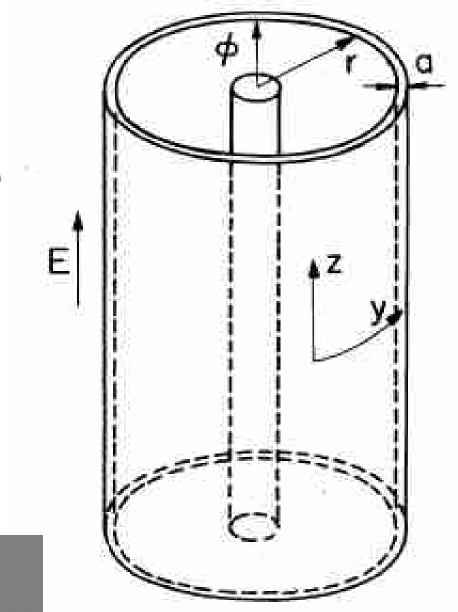
Theory

B.A., Aronov & Spivak (1981)

Experiment

Sharvin & Sharvin (1981)





With magnetic field H

j 1 - j 2 = 2*2p Φ/Φ 0

Resistance is a periodic function of the magnetic flux with the period F o/2

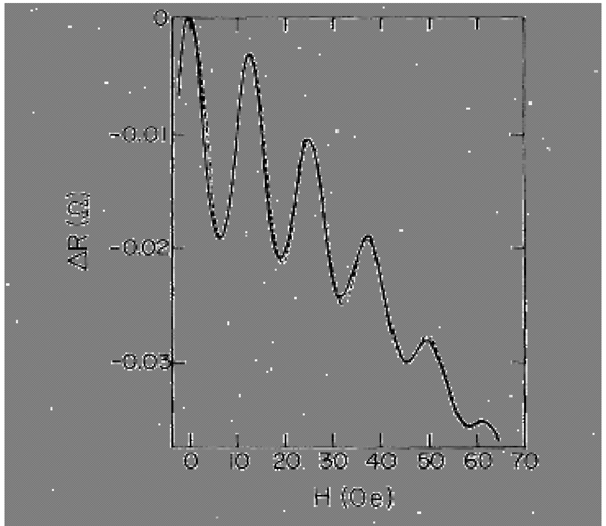


FIG. 8. Longitudinal magnetoresistance $\Delta R(H)$ at $T = 1.1$ K for a cylindrical lithium film evaporated onto a 1-cm-long quartz filament. $R_{4.2} = 2$ kΩ, $R_{300}/R_{4.2} = 2.8$. Solid line: averaged from four experimental curves. Dashed line: calculated for $L_\varphi = 2.2$ μm, $\tau_\varphi/\tau_{so} = 0$, filament diameter $d = 1.31$ μm, film thickness 127 nm. Filament diameter measured with scanning electron microscope yields $d = 1.30 \pm 0.03$ μm (Altshuler et al., 1982; Sharvin, 1984).

Lesson 2:

Brownian Particle as a mesoscopic system

PHYSICAL REVIEW B VOLUME 30, NUMBER 7 1 OCTOBER 1984

Magnetoresistance of small, quasi-one-dimensional, normal-metal rings and lines

C. P. Umbach, S. Washburn, R. B. Laibowitz, and R. A. Webb
IBM Thomas J. Watson Research Center, P. O. Box 218, Yorktown Heights, New York 10598

(Received 6 July 1984)

The magnetoresistance of sub-0.4-μm-diam Au and $Au_{60}Pd_{40}$ rings was measured in a perpendicular magnetic field at temperatures as low as 5 mK in search of simple, periodic resistance oscillations that would be evidence of flux quantization in norm[...] very complex structure developed in the magneto[...] data did not reveal convincing evidence for flux[...] that observed in the rings was also found in th[...] lines. This structure appears to be associated with[...]

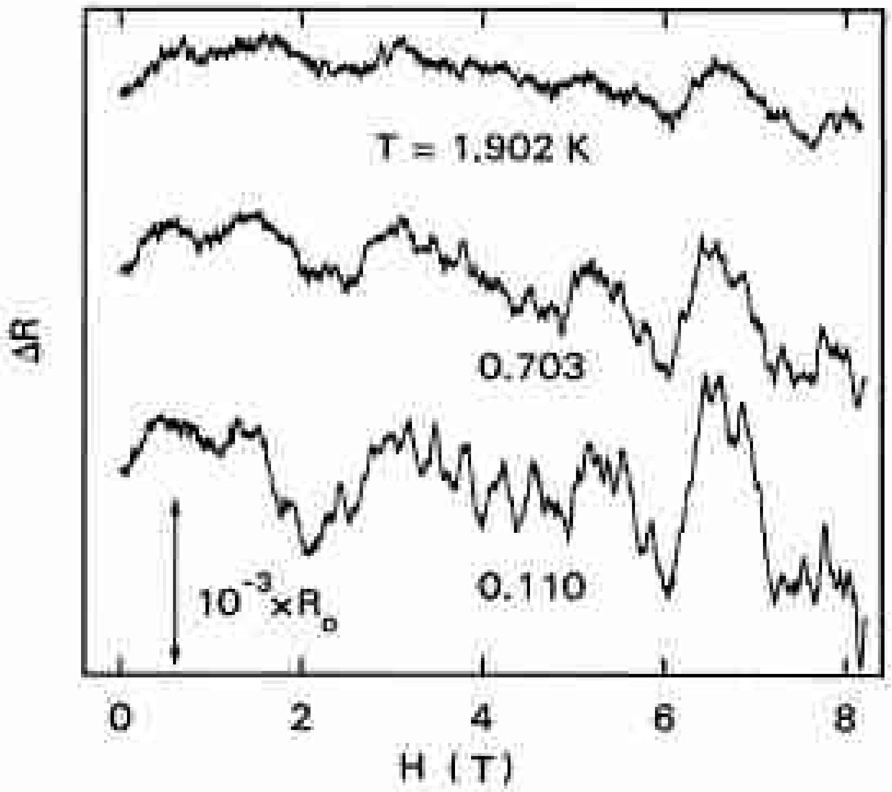


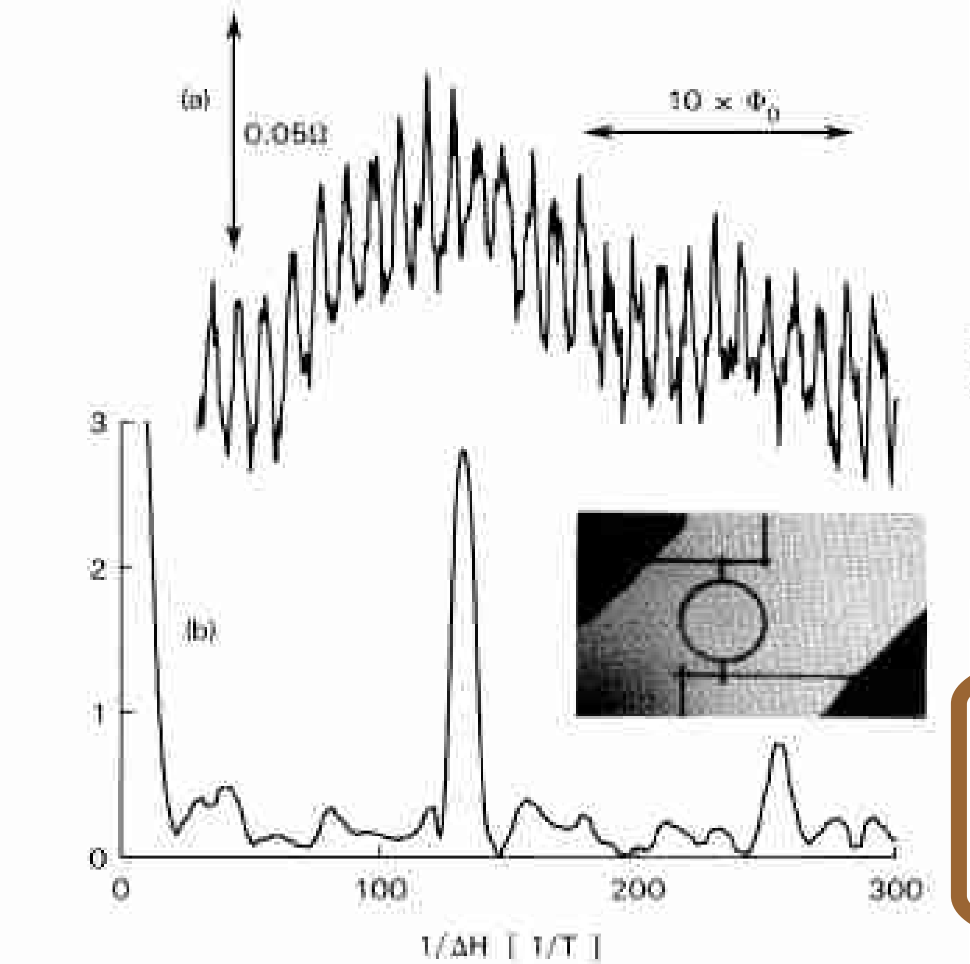
FIG. 4. Temperature dependence of the magnetoresistance from 0–8 T of a 60-nm-diam by 790-nm-long $Au_{60}Pd_{40}$ line. The zero-field resistance of the line, R_0, was 101.7 Ω.

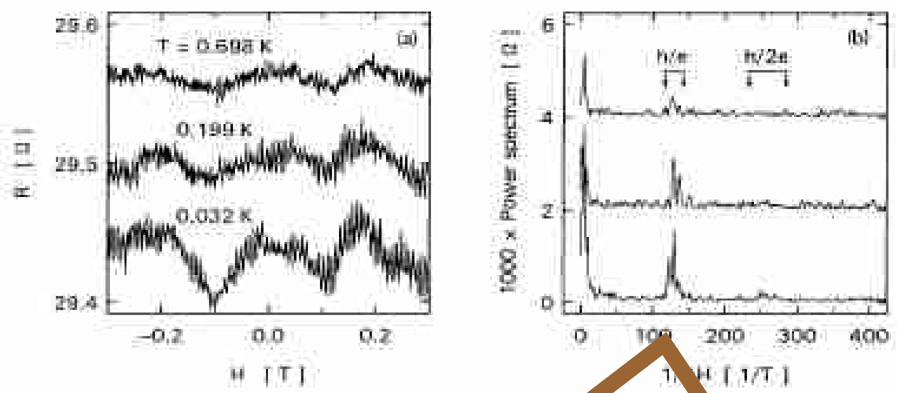
Mesoscopic fluctuations

VOLUME 54, NUMBER 25 PHYSICAL REVIEW LETTERS 24 JUNE 1985

Observation of *h/e* Aharonov-Bohm Oscillations in Normal-Metal Rings

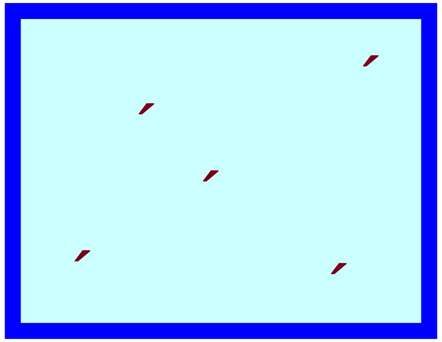
R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 27 March 1985)



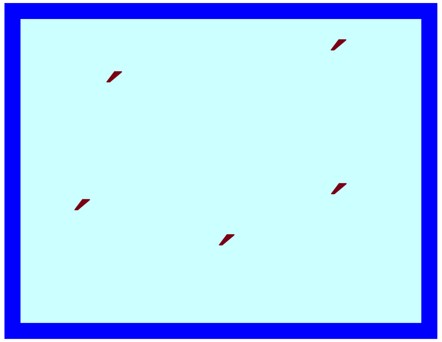


Resistance is a periodic function of the magnetic flux with the period F_o

Mesoscopic Fluctuations.



$$g_1 \neq g_2$$



Properties of systems with identical set of macroscopic parameters but different realizations of disorder are different!

$g(H)$ is sample-dependent

$\langle \ldots \rangle$ - ensemble averaging $\qquad \langle g \rangle >> 1$

Before Einstein:

Correct question would be: describe $\vec{r}(t)$

OK, maybe you can restrict yourself by $\langle \vec{r}(t) \rangle$

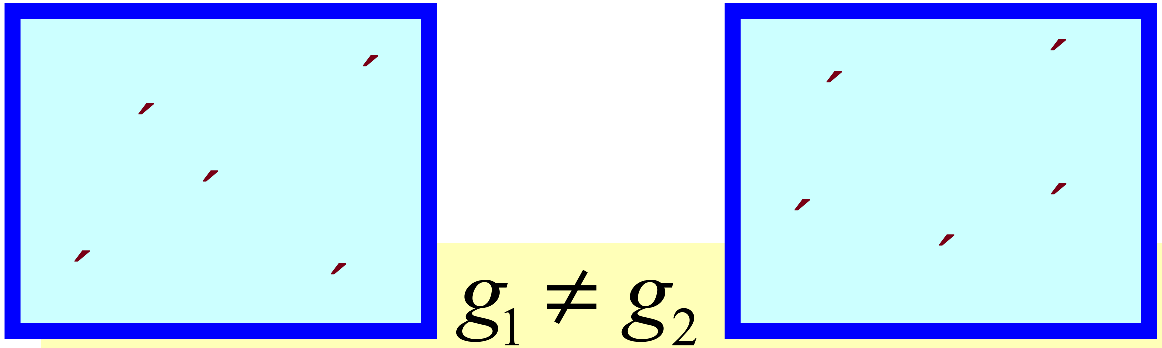
Einstein: What is $\left\langle \left[\vec{r}(0) - \vec{r}(t) \right]^2 \right\rangle$?

$$\left\langle \left[\vec{r}(0) - \vec{r}(t) \right]^n \right\rangle = ?$$

Mesoscopic physics: Not only $\langle g(H) \rangle$

But also $\left\langle \left[g(H) - g(H+h) \right]^2 \right\rangle$

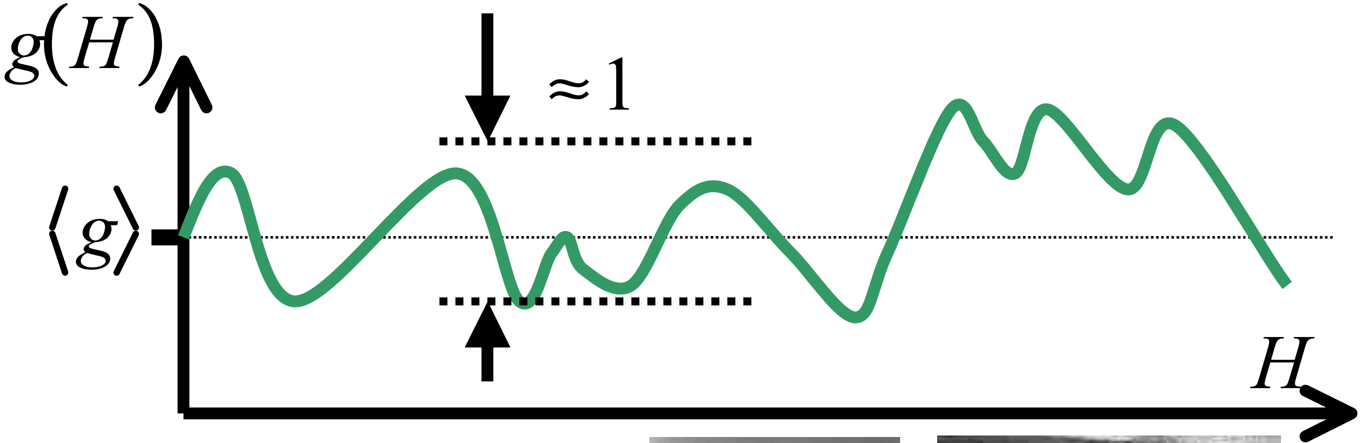
	Brownian motion	**Conductance fluctuations**
ensemble	Set of brownian particles	Set of small conductors
observables	Position of each particle $\vec{r}$	Conductance of each sample g
evolves as function of	Time t	Magnetic field H or any other external tunable parameter
Interested in	Statistics of $\vec{r}(t)$	Statistics of $g(H)$
Example	$\left\langle \left[\vec{r}(t_1) - \vec{r}(t_2)\right]^2 \right\rangle$	$\left\langle \left[g(H_1) - g(H_2)\right]^2 \right\rangle$



$g_1 \neq g_2$

$g_1 - g_2 \cong 1$ $\qquad$ $G_1 - G_2 \cong e^2/\hbar$

Magnetoresistance



Statistics of the functions of $g(H)$ are universal

B.A.(1985); Lee & Stone (1985)

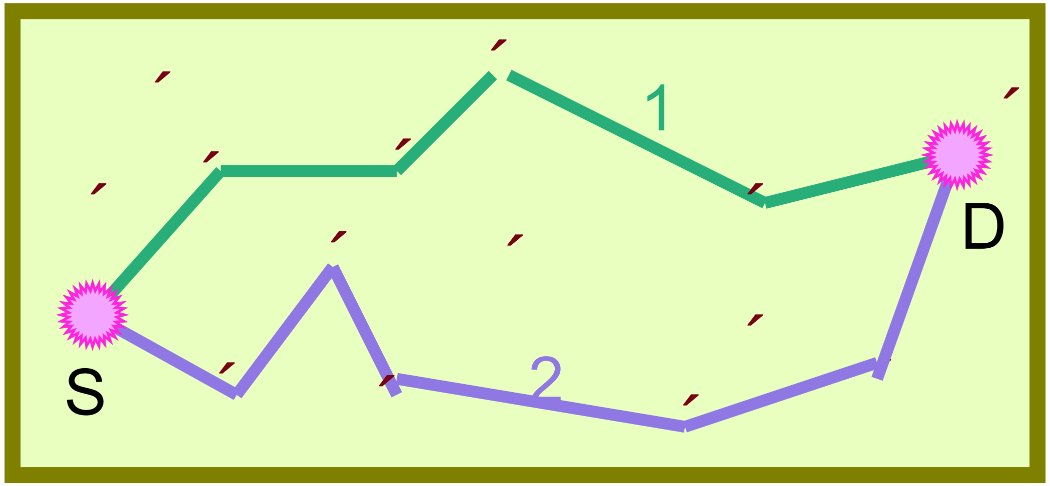
Statistics of random function(s) $g(H)$ are universal **!**

In particular, $\left\langle (\mathrm{d}g)^2 \right\rangle \approx 1$

$$g \mu L^{d-2} \quad ® \quad \frac{\left\langle (\mathrm{d}\, g)^2 \right\rangle}{g^2} \mu L^{4-2d} >> L^{-d}$$

Fluctuations are large and nonlocal

Waves in Random Media



W_1, W_2 probabilities

A_1, A_2 probability amplitudes

$$W_{1,2} = \left| A_{1,2} \right|^2$$

$$A_{1,2} = \left| A_{1,2} \right| e^{i j_{1,2}}$$

Total probability

$$W = \left| A_1 + A_2 \right|^2 = W_1 + W_2 + 2\,\mathrm{Re}\left(A_1 A_2^{\,*} \right)$$

interference term:

$$2\,\mathrm{Re}\left(A_1 A_2^{\,*} \right) = 2\sqrt{W_1 W_2}\cos\left(j_1 - j_2 \right)$$

$$W = |A_1 + A_2|^2 = W_1 + W_2 + 2\,\mathrm{Re}\left(A_1 A_2^*\right)$$

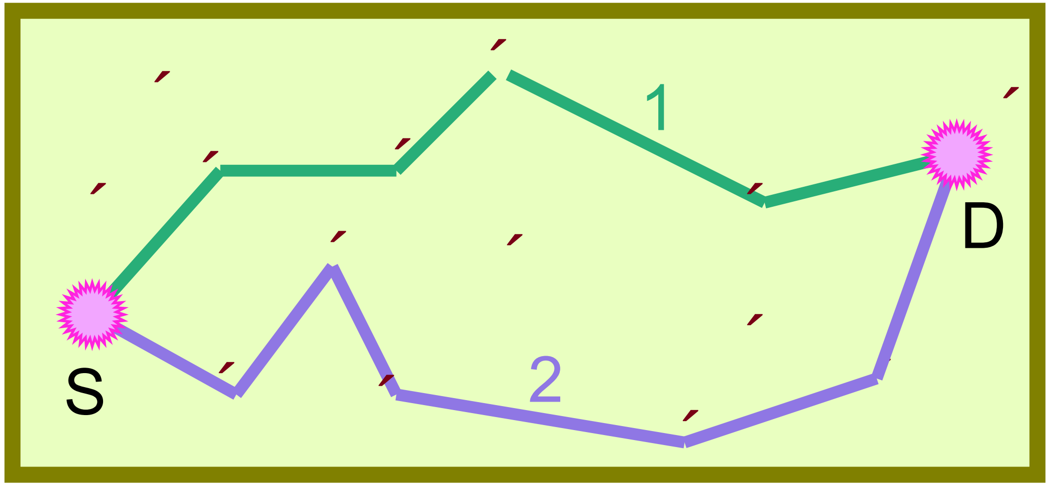
$$2\,\mathrm{Re}\left(A_1 A_2^*\right) = 2\sqrt{W_1 W_2}\cos(j_1 - j_2)$$

1. $A_{1,2} = \sqrt{W_{1,2}}\exp(i j_{1,2})$
2. Phases $j_{1,2}$ are random
3. $|j_1 - j_2| >> 2p$

The interference term disappears after averaging

$$\langle \cos(j_1 - j_2) \rangle = 0$$

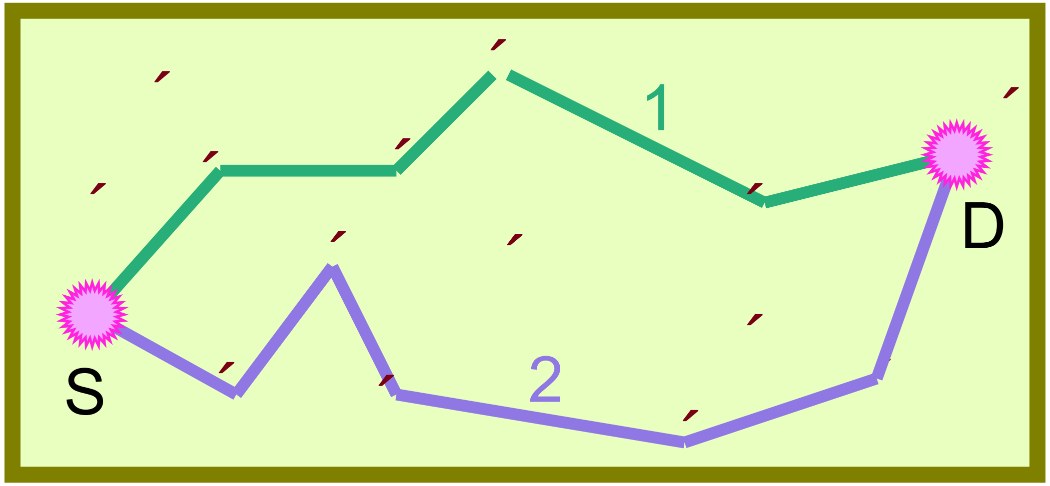
$$\langle W \rangle = \langle W_1 \rangle + \langle W_2 \rangle$$



$$W = |A_1 + A_2|^2 = W_1 + W_2 + 2\,\mathrm{Re}\left(A_1 A_2^{\ *}\right)$$

Classical result for average probability:

$$\langle W \rangle = W_1 + W_2$$



Consider now square of the probability

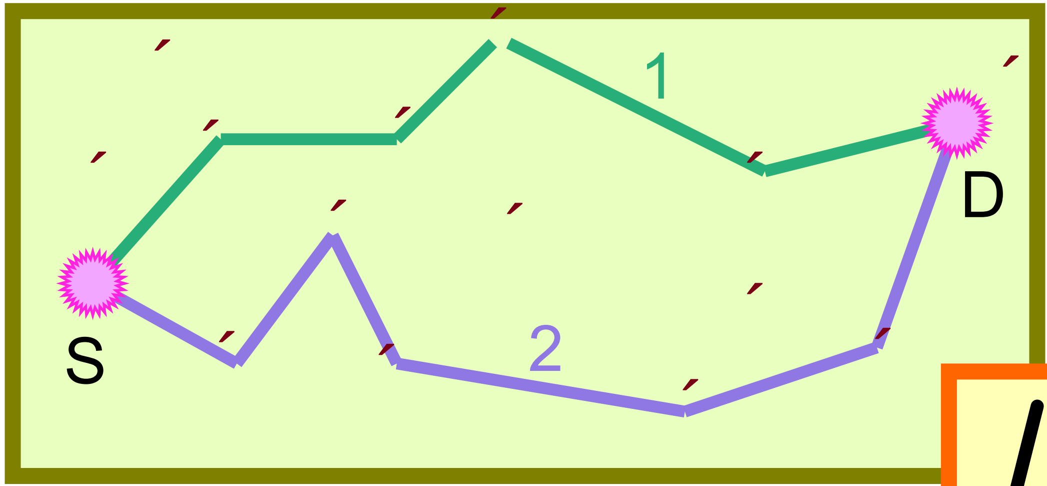
$$\langle W^2 \rangle = (W_1 + W_2)^2 + 2W_1W_2$$

Reason:

$$\langle \cos(j_1 - j_2) \rangle = 0$$

$$\langle \cos^2(j_1 - j_2) \rangle = 1/2$$

$$\langle W^2 \rangle \neq \langle W \rangle^2$$



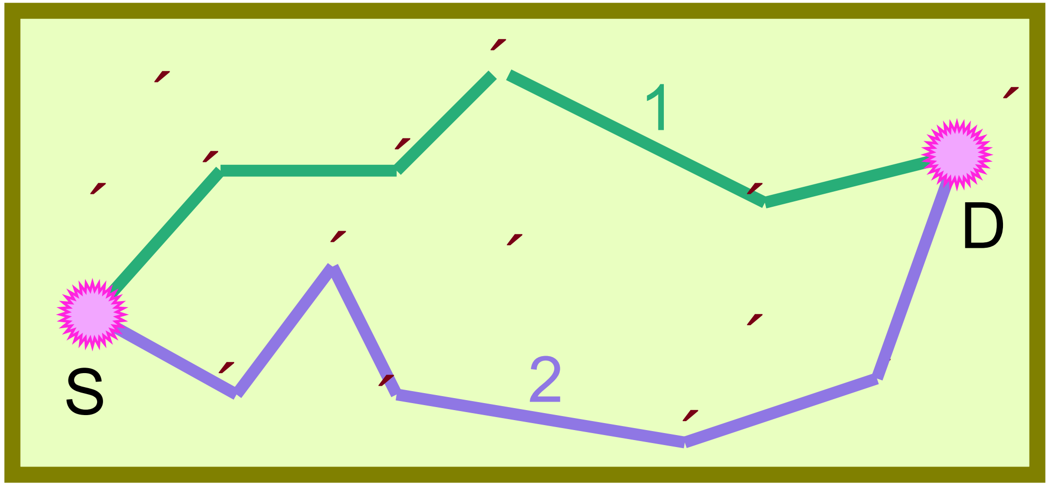
$$\langle W^2 \rangle \neq \langle W \rangle^2$$

CONCLUSIONS:

1. There are fluctuations!

2. Effect is nonlocal.

Now let us try to understand the effect of magnetic field. Consider the correlation function

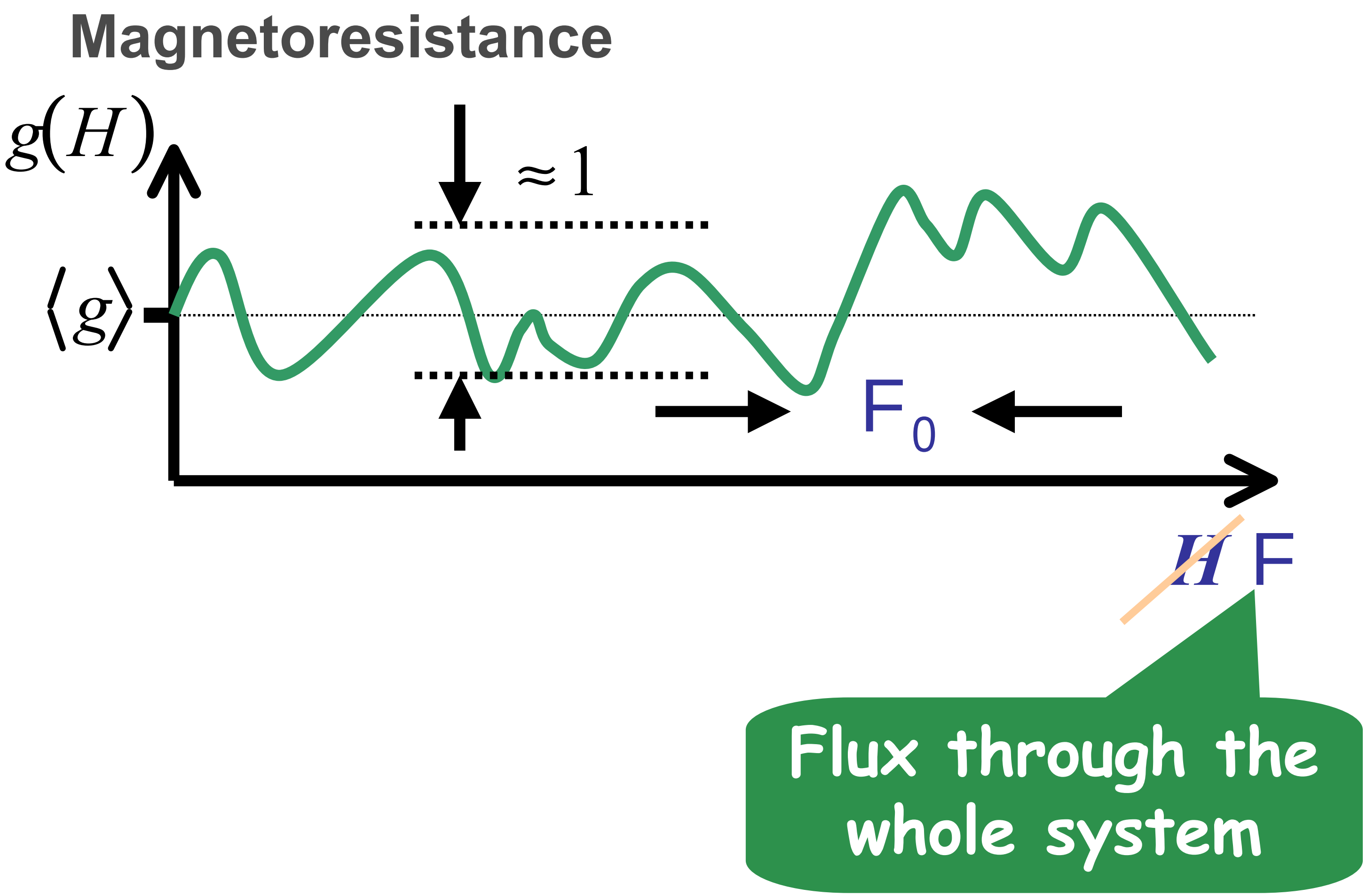


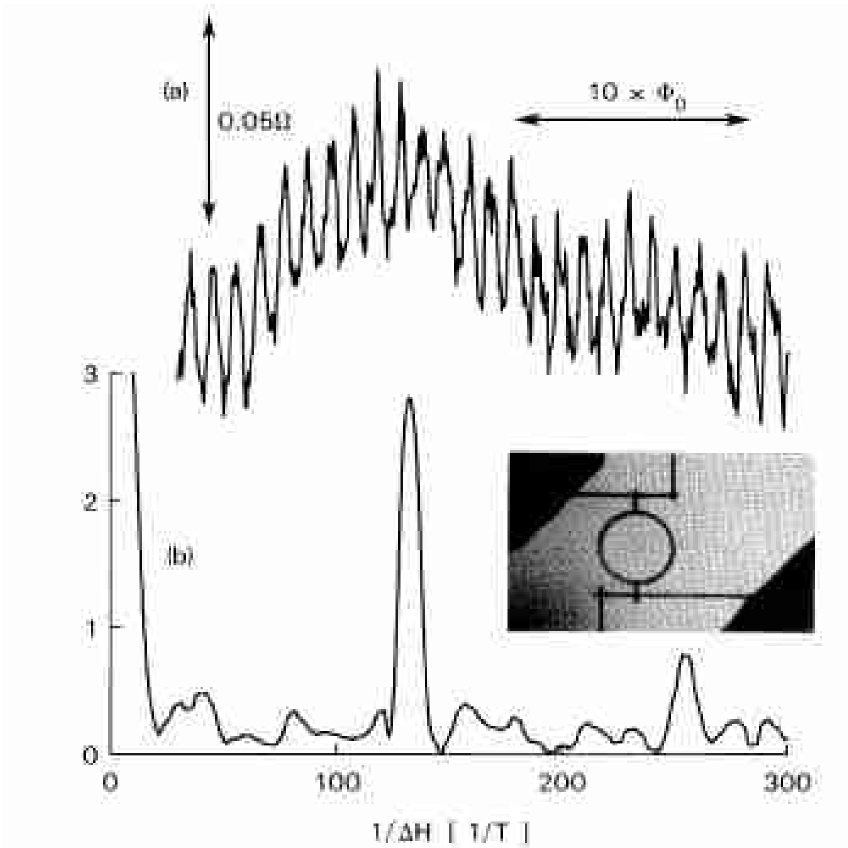
$$\langle W(H)W(H+h)\rangle = \langle W(H)\rangle\langle W(H+h)\rangle + 2W_1W_2\langle\cos(\delta\varphi(H))\cos(\delta\varphi(H+h))\rangle$$

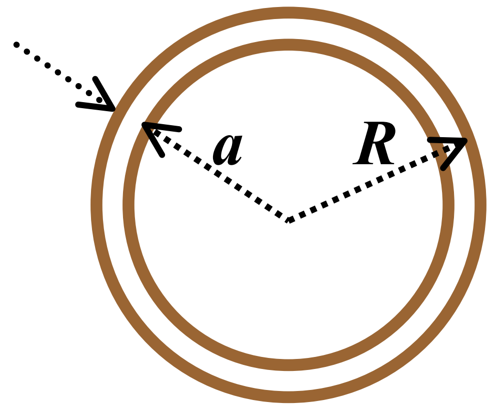
$$\delta\varphi \equiv \varphi_1 - \varphi_2$$

$$\langle\cos(\delta\varphi(H))\cos(\delta\varphi(H+h))\rangle \Rightarrow \begin{cases} \frac{1}{2} \text{ for } h \to 0 \quad (\Phi(h) << \Phi_0) \\ 0 \text{ for } \Phi(h) >> \Phi_0 \end{cases}$$

$$\Phi(h) = h \cdot (\text{area of the loop})$$


Magnetoresistance
g(H)
≈1
⟨g⟩
Φ0
H Φ
Flux through the whole system

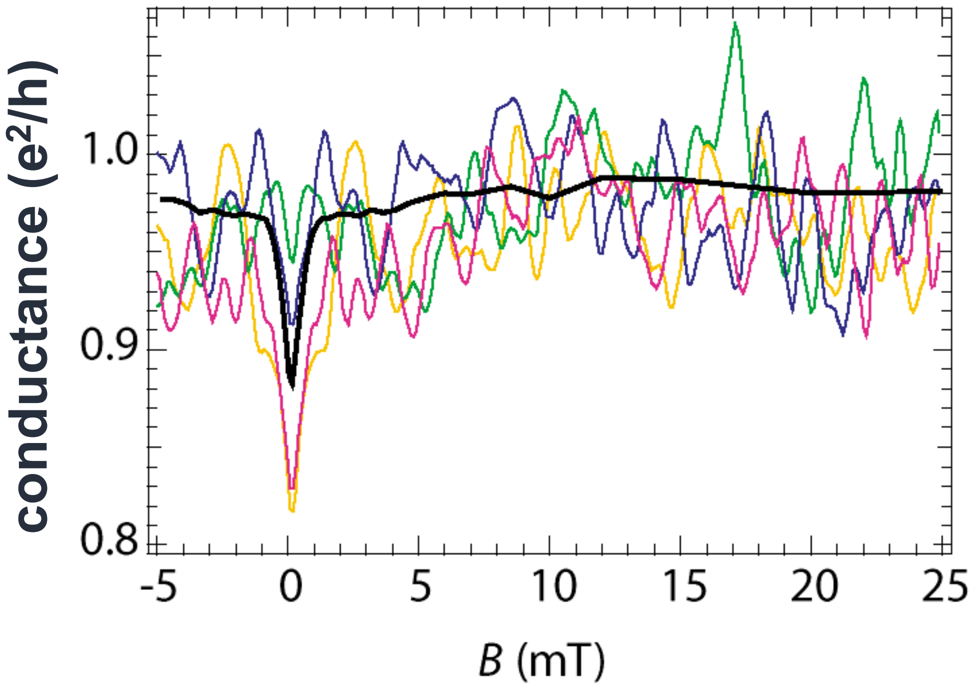


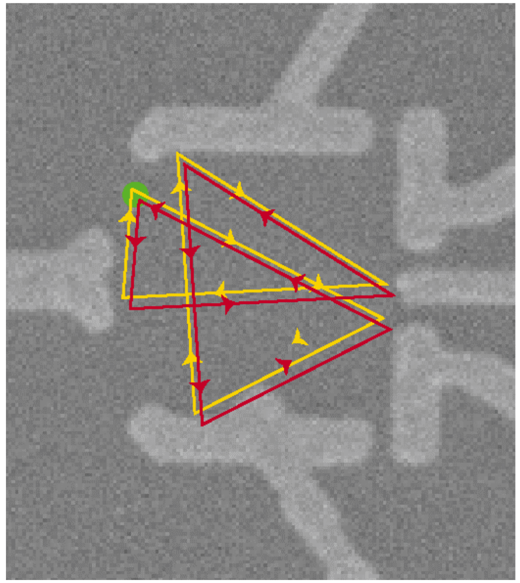


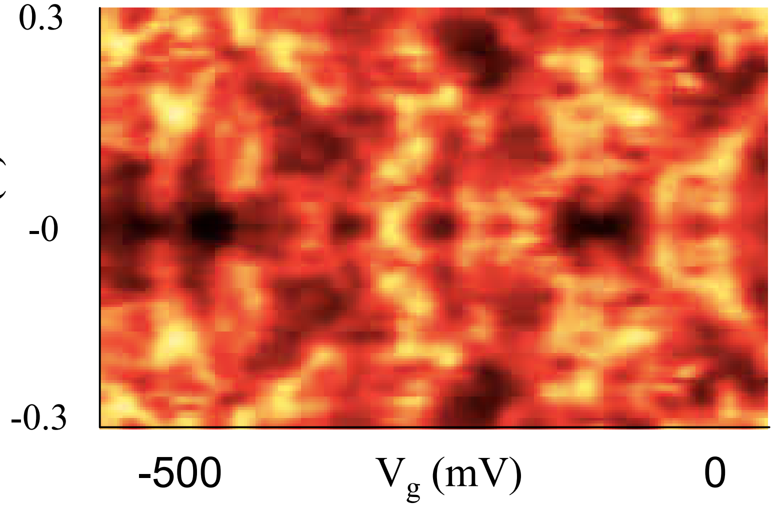
$$\frac{\Phi_0}{pR^2}$$ **period of oscillations**

$$\frac{\Phi_0}{2pRa}$$ **scale of aperiodic fluctuations**

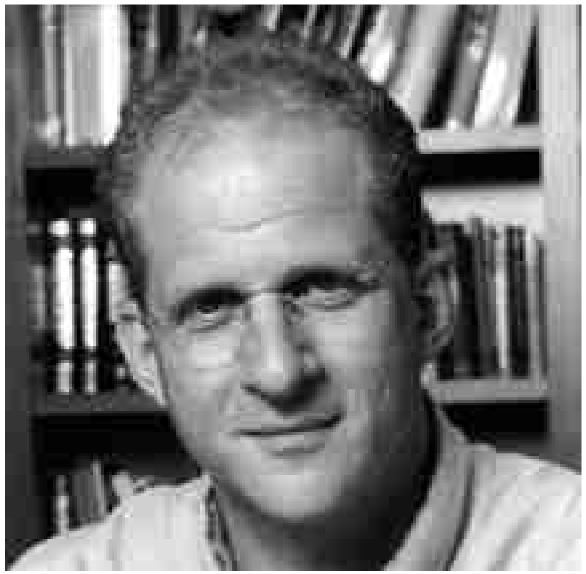
Quantum Chaos

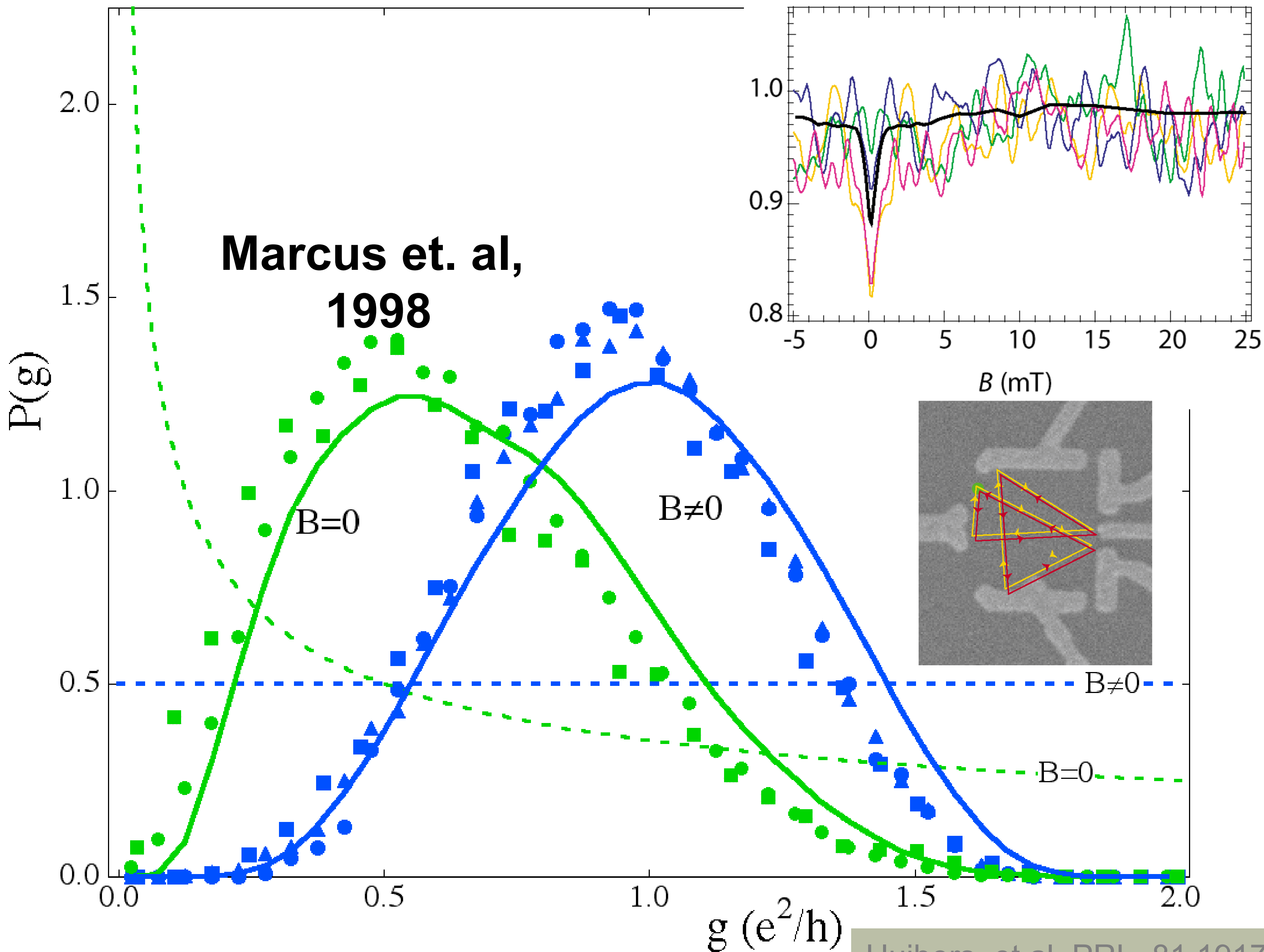




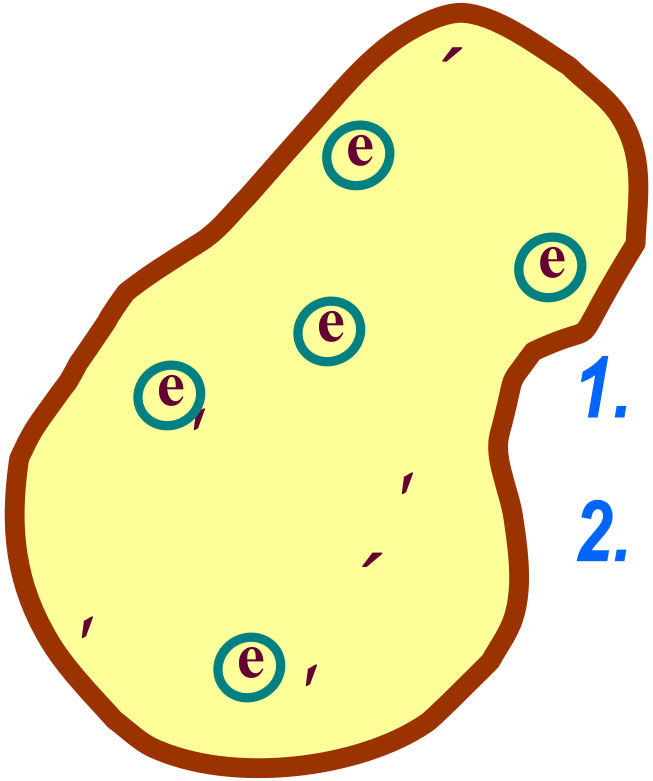


Marcus et al





Huibers, et al. PRL, 81 1917(1998).



1. *Disorder (´ - impurities)*
2. *Complex geometry*

How to deal with disorder?

- ~~Solve the Shrodinger equation exactly~~
- Make statistical analysis

What if there in no disorder?