# Transport and Nonreciprocity in Monitored Quantum Devices: An Exact Study 

João Ferreira© ${ }^{1}$ Tony Jin© © ${ }^{1,2}$ Jochen Mannhart© ${ }^{3}{ }^{3}$ Thierry Giamarchi©, ${ }^{1}$ and Michele Filippone $\oplus^{4}$<br>${ }^{1}$ Department of Quantum Matter Physics, École de Physique University of Geneva, 1211 Geneva, Switzerland<br>${ }^{2}$ Pritzker School of Molecular Engineering, University of Chicago, Chicago, Illinois 60637, USA<br>${ }^{3}$ Max Planck Institute for Solid State Research, Heisenbergstrasse 1, 70569 Stuttgart, Germany<br>${ }^{4}$ IRIG-MEM-L_Sim, Université Grenoble Alpes, CEA, Grenoble INP, Grenoble 38000, France

(Received 28 June 2023; revised 3 January 2024; accepted 25 January 2024; published 27 March 2024)


#### Abstract

We study noninteracting fermionic systems undergoing continuous monitoring and driven by biased reservoirs. Averaging over the measurement outcomes, we derive exact formulas for the particle and heat flows in the system. We show that these currents feature competing elastic and inelastic components, which depend nontrivially on the monitoring strength $\gamma$. We highlight that monitor-induced inelastic processes lead to nonreciprocal currents, allowing one to extract work from measurements without active feedback control. We illustrate our formalism with two distinct monitoring schemes providing measurement-induced power or cooling. Optimal performances are found for values of the monitoring strength $\gamma$, which are hard to address with perturbative approaches.


DOI: 10.1103/PhysRevLett.132.136301

Introduction.-Nonunitary dynamics in quantum systems stems from interactions with the environment [1-4], which usually suppress quantum coherence [5,6]. Nonetheless, nonunitary evolution caused by engineered dissipation [7-11] or measurements [12,13] can stabilize target quantum states, many-body correlations [14-23], and exotic entanglement dynamics [24-31].

Of particular interest are the effects of nonunitarity on quantum transport. Environment-assisted processes can drive currents in coherent systems [32-43] and the impact of losses [41,44-52] is investigated in quantum simulators [22,23,37,40]. Work extraction from dissipative environments [53,54] or active monitoring [55-62] may use quantum effects at the nanoscale to break the operational limits imposed by classical thermodynamics [63].

Quantum devices are usually driven by thermodynamic baths, whose large number of degrees of freedom challenges exact numerical [64] and analytical [65-68] approaches. Local master equation approaches, based on weak-coupling assumptions between system and reservoirs [69], may miss interesting effects [70,71] or imply apparent violations of the second law of thermodynamics [72-76].

In this Letter, we derive exact formulas for the particle and heat currents driven by continuous monitoring of a single-particle observable $\mathcal{O}$ and biased reservoirs in free fermion systems. We exploit an exact self-consistent Born scheme for two-point correlation functions [77,78] and a generalized Meir-Wingreen approach [50,79] to account for reservoirs. Our main result is formula (5), which offers a simple and exact tool to address quantum transport in coherent systems under continuous monitoring and is valid for any coupling strength between system and reservoirs.

We provide two illustrations of our approach, showing monitor-assisted nonreciprocal effects in quantum systems. We consider first the continuous monitoring of a single level (Fig. 1). Under generic assumptions, we find that monitoring triggers a nonreciprocal current between reservoirs without external bias and thus generates power. We then show that monitoring cross-correlations between two sites (Fig. 2) enables quantum measurement cooling [80]. For both cases, we highlight nontrivial dependencies on the measurement strength $\gamma$, showcased by peaks of performances in regimes escaping perturbative approaches. We stress that the measurement-based engines described here do not rely on feedback loops or Maxwell's demons [55-62].

Derivation of monitored currents.-For simplicity, we consider two-terminal setups [81] described by Hamiltonians of the form $\mathcal{H}=\mathcal{H}_{\text {res }}+\mathcal{H}_{\mathrm{T}}+\mathcal{H}_{\text {sys }}$. Left and right $(r=L / R)$ reservoirs are ruled by $\mathcal{H}_{\text {res }}=\sum_{r, k} \varepsilon_{r, k} c_{r, k}^{\dagger} c_{r, k}$, where $c_{k, r}$ annihilates fermions of the reservoir $r$ in mode $k$ of energy $\varepsilon_{r, k}$. Both reservoirs are in thermal equilibrium, with chemical potential $\mu_{r}$, temperature $T_{r}$, and mode occupation obeying Fermi's distribution $f_{r}(\varepsilon)=\left[e^{\left(\varepsilon-\mu_{r}\right) / T_{r}}+1\right]^{-1}$. Free fermions in the system are described by $\mathcal{H}_{\text {sys }}=\sum_{i, j} d_{i}^{\dagger} h_{i j} d_{j}$, where $h_{i j}$ is a single-particle Hamiltonian with labels $i, j$ referring to internal degrees of freedom (orbitals, spin, ...). The coupling between system and reservoirs reads $\mathcal{H}_{\mathrm{T}}=$ $\sum_{r, k, i} t_{r, k i} c_{r, k}^{\dagger} d_{i}+$ H.c., where $t_{r, k i}$ are tunnel amplitudes.

We consider that an observable of the system $\mathcal{O}$ is continuously monitored with strength $\gamma$. If the measurement outcomes are discarded, the averaged dynamics of the system density matrix $\rho$ obeys Lindblad's equation $\partial_{t} \rho=-i[\mathcal{H}, \rho]+\mathcal{D}[\rho]$, where $\left(\hbar=e=k_{\mathrm{B}}=1\right)$ [82-84]

$$
\begin{equation*}
\mathcal{D}[\rho]=\gamma\left(2 \mathcal{O} \rho \mathcal{O}-\left\{\mathcal{O}^{2}, \rho\right\}\right) \tag{1}
\end{equation*}
$$

We are interested in the average particle $(\zeta=0)$ and heat $(\zeta=1)$ currents flowing into a reservoir $r$,

$$
\begin{equation*}
J_{r}^{\zeta}=i \sum_{k, i}\left(\varepsilon_{r, k}-\mu_{r}\right)^{\zeta}\left[t_{r, k i}^{*}\left\langle d_{i}^{\dagger} c_{r, k}\right\rangle-t_{r, k i}\left\langle c_{r, k}^{\dagger} d_{i}\right\rangle\right], \tag{2}
\end{equation*}
$$

with averages $\langle 0\rangle=\operatorname{tr}[\circ \rho]$ calculated on the steady state. When single-particle observables $\mathcal{O}=\sum_{i j} d_{i}^{\dagger} O_{i j} d_{j}$ are monitored, calculating Eq. (2) becomes a difficult task, since Eq. (1) is nonquadratic. Even though, for quadratic Hamiltonians, correlation functions obey closed systems of equations [85-87], efficient numerical calculations can be performed only for finite systems [78,88,89]. We show now that analytical solutions can be obtained with infinite reservoirs thanks to the validity of the self-consistent Born scheme for two-point correlation functions, extensively discussed in Refs. [77,78] and the Supplemental Material [90].

We consider the retarded, advanced, and Keldysh Green's functions: $\mathcal{G}_{i j}^{R}\left(t, t^{\prime}\right)=-i \theta\left(t-t^{\prime}\right)\left\langle\left\{d_{i}(t), d_{j}^{\dagger}\left(t^{\prime}\right)\right\}\right\rangle$, $\mathcal{G}_{i j}^{A}\left(t, t^{\prime}\right)=\left[\mathcal{G}_{j i}^{R}\left(t^{\prime}, t\right)\right]^{*}$ and $\mathcal{G}_{i j}^{K}\left(t, t^{\prime}\right)=-i\left\langle\left[d_{i}(t), d_{j}^{\dagger}\left(t^{\prime}\right)\right]\right\rangle$, which we collect in the matrix $\mathcal{G}=\left(\begin{array}{cc}\mathcal{G}^{R} & \mathcal{G}^{K} \\ 0 & \mathcal{G}^{A}\end{array}\right)$ [93-96]. The matrix $\mathcal{G}$ obeys Dyson's equation $\mathcal{G}^{-1}=\mathcal{G}_{0}^{-1}-\mathbf{\Sigma}$, where $\mathcal{G}_{0}$ is the Green's function of the isolated system $\left(t_{r, k i}=\gamma=0\right)$ and $\boldsymbol{\Sigma}$ is the self-energy, encoding the effects of the reservoir $r\left(\boldsymbol{\Sigma}_{r}\right)$ and monitoring $\left(\boldsymbol{\Sigma}_{\gamma}\right)$. The contribution $\boldsymbol{\Sigma}_{r}$ is obtained by integration of the modes $c_{r, k}$. In frequency space, $\boldsymbol{\Sigma}_{r, i j}(\omega)=\sum_{k} t_{r, k i}^{*} t_{r, k j} \mathcal{C}_{0, r, k}(\omega)$, where $\mathcal{C}_{0, r, k}$ is the Green's function of the isolated reservoir [e.g., $\left.\quad \mathcal{C}_{0, r, k}^{K}\left(t-t^{\prime}\right)=-i\left\langle\left[c_{r, k}(t), c_{r, k}^{\dagger}\left(t^{\prime}\right)\right]\right\rangle_{0}\right]$. Importantly, particle exchange with the system is treated exactly and described by the hybridization matrix $\Gamma_{r}(\omega)=\left[\Sigma_{r}^{A}(\omega)-\right.$ $\left.\Sigma_{r}^{R}(\omega)\right] / 2$ [97]. The Keldysh component $\Sigma_{r}^{K}(\omega)=$ $-2 i \Gamma_{r}(\omega) \tanh \left[\left(\omega-\mu_{r}\right) / 2 T_{r}\right]$ carries information about the reservoirs' equilibrium state.

Monitoring contributes to the self-energy following the self-consistent Born scheme [77,78], which involves the full Green's matrix $\mathcal{G}$, including baths and monitoring,

$$
\begin{equation*}
\boldsymbol{\Sigma}_{\gamma, i j}(\omega)=2 \gamma \sum_{p q} O_{i p} \mathcal{G}_{p q}(t, t) O_{q j} \tag{3}
\end{equation*}
$$

To derive the retarded and advanced components of $\boldsymbol{\Sigma}_{\gamma}$, we exploit the prescription $\mathcal{G}_{i j}^{R / A}(t, t)=\mp i \delta_{i j} / 2$ [93] and obtain $\left[\mathcal{G}^{R / A}\right]_{i j}^{-1}(\omega)=\omega-h_{i j}-\sum_{r} \Sigma_{r, i j}^{R / A}(\omega) \pm i \gamma \sum_{p} O_{i p} O_{p j}$. In this expression, monitoring appears as a frequencyindependent lifetime $\gamma \sum_{p} O_{i p} O_{p j}$, in analogy with sin-gle-particle gains or losses [37,40,41,48-52].

The difference between monitoring and losses appears in the Keldysh component of Eq. (3). Inserting $\mathcal{G}_{i j}^{K}(t, t)=$ $2 i\left\langle d_{j}^{\dagger} d_{i}\right\rangle-i \delta_{i j}$ [98] and inverting the Dyson equation, one
finds a self-consistent equation for the correlation matrix $\mathcal{D}_{i j}=\left\langle d_{j}^{\dagger} d_{i}\right\rangle$,
$\mathcal{D}=\int \frac{d \omega}{\pi} \mathcal{G}^{R}(\omega)\left[\sum_{r} f_{r}(\omega) \Gamma_{r}(\omega)+\gamma O \mathcal{D} O\right] \mathcal{G}^{A}(\omega)$.
The solution of Eq. (4) completes the derivation of the Green's function $\mathcal{G}$, which is sufficient to derive currents [50,79]. After straightforward algebra, detailed in the Supplemental Material [90], we find closed, exact, and nonperturbative expressions for the particle and heat currents,

$$
\begin{align*}
J_{r}^{\zeta}= & \frac{2}{\pi} \int d \omega \underbrace{\left(\omega-\mu_{r}\right)^{\zeta}\left(f_{\bar{r}}-f_{r}\right) \operatorname{tr}\left[\Gamma_{r} \mathcal{G}^{R} \Gamma_{\bar{r}} \mathcal{G}^{A}\right]}_{\text {elastic }} \\
& +\gamma \frac{2}{\pi} \int d \omega \underbrace{\left(\omega-\mu_{r}\right)^{\zeta} \operatorname{tr}\left[\Gamma_{r} \mathcal{G}^{R} O\left(\mathcal{D}-f_{r} \mathbb{1}\right) O \mathcal{G}^{A}\right]}_{\text {inelastic }}, \tag{5}
\end{align*}
$$

with $\bar{r}=R$ if $r=L$ and vice versa. This expression is the main result of this Letter. It allows us to draw general conclusions on monitor-assisted transport and, combined with Eq. (4), can be applied to all settings described by Lindbladians of the form (1).

Equation (5) appears as a sum of two terms. The first term reproduces the Landauer-Büttiker formula for currents in noninteracting systems [97,99]. It describes the energypreserving transfer of particles between reservoirs at energy $\omega$ with transmission probability $\mathcal{T}(\omega)=4 \operatorname{tr}\left[\Gamma_{r} \mathcal{G}^{R} \Gamma_{\bar{r}} \mathcal{G}^{A}\right]$. As $\mathcal{T}(\omega)$ depends on $\mathcal{G}^{R / A}$, where measurements only reduce lifetimes, monitoring affects elastic transport exactly as single-particle gains or losses [37,40,41,48-52].

The second term in Eq. (5) is controlled by monitoring. The implicit dependence of the correlation matrix $\mathcal{D}$ on additional energy integrals, see Eq. (4), indicates that measurements inelastically change the energy of particles in the system. A rough inspection of Eq. (5) shows that the inelastic contribution peaks as a function of the observation rate $\gamma$, interpolating between a linear growth for small $\gamma$ and a $\gamma^{-1}$ decay in the strong measurement limit, as $\mathcal{G}^{R / A} \propto \gamma^{-1}$ for $\gamma \rightarrow \infty$, see Figs. 1(b)-2(c). The position and intensity of this maximum vary with setups, typically occurring for $\gamma$ comparable to the system spectral width and its coupling strength to the baths. These maxima fall beyond the reach of perturbative approaches. Importantly, the inelastic current is not proportional to $f_{L}-f_{R}$ and can thus be finite even without a bias. This describes the generation of nonreciprocal currents from measurement and can be harnessed for work generation.

We now illustrate these considerations on different monitor-assisted devices.

Monitored density engine.-We first consider a monitored setting, sketched in Fig. 1, where a single level of energy $\varepsilon_{d}$, described by the Hamiltonian $\mathcal{H}_{\text {sys }}=\varepsilon_{d} d^{\dagger} d$,


FIG. 1. (a) Monitored level of energy $\varepsilon_{d}$, coupled to left and right reservoirs with asymmetric hybridization functions $\Gamma_{L}(\omega) \neq \Gamma_{R}(\omega)$. The level occupation is measured with strength $\gamma$, providing the inelastic mechanism promoting particles from energy $\omega_{1}$ to $\omega_{2}$ and inducing a current against the bias (arrows). The blue-shaded areas correspond to the Fermi distributions of the reservoirs. For all plots, we use the two-filter model discussed in the main text, with $\varepsilon_{R}=1.48 t=-\varepsilon_{L}, \Delta=0.55 t$, which maximizes the unbiased particle current at $\mu_{L, R}=T_{L, R}=0$. (b) Peaked structure of the unbiased particle current as function of the measurement strength $\gamma$ for varying $\varepsilon_{d}$. Inset: the unbiased current decays for increasing temperatures $(\gamma=1$ ). (c) Differential conductance $G$ as a function of the chemical potential $\mu$ at $T=0$ for increasing $\gamma$. The measurement suppresses the resonance associated with the single level and favors those from the filters, as highlighted by arrows. (d) Electric power as function of a symmetric bias $\mu_{R}-\mu_{L}$ around $\mu=0$, for different values of $\gamma$. Dashed lines correspond to linear response calculations.
evolves under the continuous measurement of its occupation, associated with the operator $\mathcal{O}=n=d^{\dagger} d$. Solving Eq. (4) gives the occupation of the level

$$
\begin{equation*}
\langle n\rangle=\frac{\int d \omega \mathcal{A}(\omega)\left[f_{L}(\omega) P_{L}(\omega)+f_{R}(\omega) P_{R}(\omega)\right]}{\int d \omega \mathcal{A}(\omega)\left[P_{L}(\omega)+P_{R}(\omega)\right]} \tag{6}
\end{equation*}
$$

where $\mathcal{A}(\omega)=-\operatorname{Im}\left[\mathcal{G}_{d d}^{R}(\omega)\right] / \pi=(1 / \pi)\left[\left(\Gamma_{L}+\Gamma_{R}+\gamma\right) / \mid \omega-\right.$ $\left.\varepsilon_{d}-\Sigma_{L}-\Sigma_{R}+\left.i \gamma\right|^{2}\right]$ is the level's spectral function. We have introduced the quantity $P_{r}(\omega)=\Gamma_{r} /\left[\Gamma_{\mathrm{L}}+\Gamma_{\mathrm{R}}+\gamma\right]$, which highlights the nonequilibrium effects of monitoring. For instance, in the unbiased case $\left(f_{L, R}=f\right)$, the absence of dephasing $(\gamma=0)$ is needed to recover $P_{R}+P_{L}=1$ and the standard equilibrium expression $\langle n\rangle_{\text {eq }}=$ $\int d \omega \mathcal{A}(\omega) f(\omega)$ [100]. Injecting Eq. (6) in Eq. (5), we obtain the particle current $J^{0}=J_{R}^{0}=-J_{L}^{0}$ flowing through the system

$$
\begin{align*}
J^{0}= & 2 \int d \omega \mathcal{A} \frac{\Gamma_{L} \Gamma_{R}}{\Gamma_{L}+\Gamma_{R}+\gamma}\left(f_{L}-f_{R}\right) \\
& +\frac{2 \gamma}{\int d \omega \mathcal{A}\left(P_{L}+P_{R}\right)} \int d \omega d \omega^{\prime} \mathcal{A} \mathcal{A}^{\prime} P_{L} P_{R}^{\prime}\left(f_{L}-f_{R}^{\prime}\right) \tag{7}
\end{align*}
$$

where we omit all frequency dependency for compactness and use the shorthand notation $f^{\prime}=f\left(\omega^{\prime}\right)$.

The first term reproduces the well-known expression of the current flowing through a Breit-Wigner resonance [101,102], with an additional suppression controlled by $\gamma$.

The inspection of the inelastic term in Eq. (7) shows that, without bias, monitoring can trigger the flow of a finite,
nonreciprocal current through the system. The latter is finite provided that at least one of the hybridization functions $\Gamma_{L / R}$ depends on energy and that mirror and particle-hole symmetry are simultaneously broken [103-105]. Such conditions are satisfied when $\Gamma_{L} \neq \Gamma_{R}$ and at least one function among $\mathcal{A}$ or $\Gamma_{L / R}$ is not symmetric around the chemical potential $\mu$. The mechanism generating this current is sketched in Fig. 1(a): electrons at energy $\omega_{1}$ are emitted from one reservoir onto the level and the measurement provides the energy for the electron to exit into an empty state of the other reservoir at energy $\omega_{2}$. Asymmetrical injection and emission rates allow the generation of this current. The emergence of a nonreciprocal current can be also understood based on the fact that averaging over the measurement outcomes is equivalent, in this specific case [106,107], to coupling the system to an infinite-temperature bosonic bath (see Supplemental Material [90]), which induces a thermoelectric flow in the system if mirror and particle-hole symmetry are broken [108-110].

Figure 1(b) shows that the inelastic current displays the aforementioned peak as a function of the measurement strength $\gamma$ at zero bias $\delta \mu=\mu_{L}-\mu_{R}=0$. For all numerical applications, we consider a minimal model with the level coupled to two metallic reservoirs via two energy filters of energy $\varepsilon_{L / R}$. Here, $\Sigma_{r}^{R}(\omega)=t^{2} /\left(\omega-\varepsilon_{r}+i \Delta\right)$, where $t$ is the level-filter tunnel coupling and $\Delta$ is the hybridization constant of the filter with the reservoirs, see Supplemental Material [90]. The resulting hybridization function $\Gamma_{r}(\omega)=-\operatorname{Im} \Sigma_{r}^{R}(\omega)$ is peaked around $\varepsilon_{r}$, as sketched in Fig. 1(a). We have found the maximum nonreciprocal current for $\gamma \simeq t$-that is out of weak coupling $(\gamma \gg t)$ when $\varepsilon_{d}=0$ and when mirror and particle-hole symmetry


FIG. 2. (a) Two-level system under continuous monitoring of its cross-correlations, coupled to a left (hot) and right (cold) reservoir. For applications, we consider the same filters as in Fig. 1, aligned with the levels $\varepsilon_{L / R}$. (b) Parameter region where a reservoir at temperature $T_{R}=t$ can be cooled by measurement $\Delta=0.5 t$ and varying $\gamma$. The range of parameters where quantum measurement cooling is possible reduces with increasing $\gamma$. The black dot corresponds to $\varepsilon_{L}=10 t$ and $\varepsilon_{R}=3 t$, used in (c) and (d). (c) Heat flowing into the right reservoirs for increasing temperature bias $T_{L}>T_{R}$. QmC occurs in the colored region below some critical temperature bias. (d) Parametric plot of the coefficient of performance (COP) of QmC. Curves are obtained by varying the measurement strength $\gamma$.
are broken by antisymmetric reservoirs with $\varepsilon_{L}=-\varepsilon_{R}$. The peak roughly follows $\varepsilon_{d}$ and is suppressed by finite temperatures, see inset of Fig. 1(b). Similar nonreciprocal effects and peaks were also discussed, from a real-time perspective, in Refs. [38,39]. Analogous peaks also arise for fixed $\gamma$ as function of the coupling to the reservoirs $(t, \Delta)$, which are not captured in the weak-coupling limit, see Supplemental Material [90].

Figure 1(c) shows the differential conductance $G=$ $\partial J^{0} /\left.\partial \delta \mu\right|_{\delta \mu=0}$ for the same system. $G$ also features elastic and inelastic contributions [111,112], scaling differently with $\gamma$. For small rates, the elastic term dominates, showing as many peaks as resonances in the systemthree in the application of Fig. 1. As only the central level is monitored, increasing $\gamma$ suppresses its associated resonance, transferring spectral weight to the filters [arrows in Fig. 1(c)]. Consequently, for intermediate monitoring strengths $\gamma \simeq t$, the conductance increases out of resonance $(\mu \neq 0)$, before being suppressed in the $\gamma \gg t$ limit.

The fact that monitoring generates currents at zero bias implies that they can flow against externally imposed biases to generate work. We consider here the generated power $\mathcal{P}=\delta \mu \cdot J^{0}$ and show the importance of nonperturbative and out-of-equilibrium effects on this quantity. In linear response, $\left.J^{0} \simeq J^{0}\right|_{\delta \mu=0}-\delta \mu G$ and the power depends parabolically on $\delta \mu$, with a maximum $\mathcal{P}_{\max }=\left.J^{0}\right|_{\delta \mu=0} ^{2} / 2 G$ and a change of sign at the stopping voltage $\delta \mu_{\text {stop }}=\left.J^{0}\right|_{\delta \mu=0} / G$. Figure 1(d) shows that maximum power is found for monitoring strengths $\gamma>t$, which is beyond the weakcoupling regime. Moreover, nonequilibrium effects associated with strongly biased reservoirs cannot be neglected. They are exactly derived via Eq. (7), and the dashed lines in Fig. 1(d) show that linear response greatly overestimates $\mathcal{P}_{\text {max }}$ and $\delta \mu_{\text {stop }}$ when $\gamma \simeq t$.

Quantum measurement cooling.-We consider two independent sites $H_{\text {sys }}=\varepsilon_{L} d_{L}^{\dagger} d_{L}+\varepsilon_{R} d_{R}^{\dagger} d_{R}$ coupled via the monitoring process $O_{i j}=\delta_{i L} \delta_{j R}+\delta_{i R} \delta_{j L}$, see Fig. 2(a). This process can be, in principle, realized by adding an interferometer measuring cross-correlations between the two sites [113,114]. Also, here, we take $t$ as the level-filter tunnel coupling and rely on Eq. (4) to find the level occupations $\left\langle n_{r}\right\rangle=\left\langle d_{r}^{\dagger} d_{r}\right\rangle$,

$$
\begin{equation*}
\left\langle n_{r}\right\rangle=\frac{\int d \omega\left[f_{\bar{r}} P_{\bar{r}} \mathcal{A}_{\bar{r}}+\left(1-\int d \omega^{\prime} P_{\bar{r}}^{\prime} \mathcal{A}_{\bar{r}}^{\prime}\right) f_{r} P_{r} \mathcal{A}_{r}\right]}{\sum_{r^{\prime}} \int d \omega P_{r^{\prime}} \mathcal{A}_{r^{\prime}}-\prod_{r^{\prime}} \int d \omega P_{r^{\prime}} \mathcal{A}_{r^{\prime}}}, \tag{8}
\end{equation*}
$$

with modified notation $P_{r}=\Gamma_{r} /\left(\Gamma_{r}+\gamma\right)$ and spectral functions $\mathcal{A}_{r}(\omega)=-\operatorname{Im} \mathcal{G}_{r r}^{R} / \pi=(1 / \pi)\left[\left(\Gamma_{r}+\gamma\right) / \mid \omega-\varepsilon_{r}-\right.$ $\left.\Sigma_{r}+\left.i \gamma\right|^{2}\right]$.

Because of the absence of coherent hopping between sites, $\mathcal{G}_{L R}^{R / A}=0$ and only the inelastic component of the currents in Eq. (5) is finite [for which the knowledge of Eq. (8) is needed]. We are interested in exact expressions for quantum measurement cooling ( QmC ) [80]. We thus consider the heat current flowing in the right reservoir,

$$
\begin{align*}
J_{R}^{1}= & \frac{2 \gamma}{\mathcal{N}} \int d \omega d \omega^{\prime}\left(\omega-\mu_{R}\right) \mathcal{A}_{R} P_{R}\left[\mathcal{A}_{L}^{\prime} P_{L}^{\prime}\left(f_{L}^{\prime}-f_{R}\right)\right. \\
& \left.+\left(1-\int d \omega^{\prime \prime} \mathcal{A}_{L}^{\prime \prime} P_{L}^{\prime \prime}\right) \mathcal{A}_{R}^{\prime} P_{R}^{\prime}\left(f_{R}^{\prime}-f_{R}\right)\right] \tag{9}
\end{align*}
$$

where $\mathcal{N}$ is the denominator appearing in Eq. (8). To get physical insight on the physical requirements for QmC and the multiple processes described by Eq. (9), we first inspect the $\gamma \rightarrow 0$ limit. To leading order in $\gamma, P_{r}=1$ and only the first term in Eq. (9) remains. It can be cast in the compact form

$$
\begin{equation*}
J_{R}^{1}=2 \gamma \int d \omega\left(\omega-\mu_{R}\right) \mathcal{A}_{R}(\omega)\left[\left\langle n_{L}\right\rangle-f_{R}(\omega)\right] \tag{10}
\end{equation*}
$$

If we further approximate the spectral function by $\mathcal{A}_{R}(\omega)=$ $\delta\left(\omega-\varepsilon_{R}\right)$, we get $J_{R}^{1}=2 \gamma\left(\varepsilon_{R}-\mu_{R}\right)\left(\left\langle n_{L}\right\rangle-\left\langle n_{R}\right\rangle\right)$. Here, the heat flow into the right reservoir is controlled by the position of the right level relative to the chemical potential and the difference in occupation compared to the left level. The condition for cooling the right reservoir is $J_{R}^{1}<0$. Without bias, such condition requires $\mu_{R} \lessgtr \varepsilon_{R}$ and $\varepsilon_{L} \lessgtr \varepsilon_{R}$, as sketched in Fig. 2(a). Analogous conditions were found to achieve cooling by heating [115,116], where the role of measurement is played by a third hot reservoir. The second term in Eq. (9) acts at order $\gamma^{2}$ and describes the reinjection of heat in the right reservoir by particles hopping back and forth to the left level via the monitoring process.

In Fig. 2, we explore QmC and its performances for strong temperature biases and large values of $\gamma$. For numerical applications, we consider $\mu_{L / R}=0$ and take the same hybridization functions $\Gamma_{r}(\omega)$ of the previous section, with peaks aligned with $\varepsilon_{r}$. Figure 2(b) shows the regions where QmC occurs, in the absence of bias and for increasing monitoring strength $\gamma$. QmC indeed occurs when $\varepsilon_{L} \lessgtr \varepsilon_{R} \lessgtr 0$. Nonetheless, its parameter region shrinks with increasing monitoring strength $\gamma$ : the stronger the measurement process is, the more heat is injected into both reservoirs. Figure 2(c) shows the behavior of $J_{R}^{1}$ for increasing temperature biases as a function of $\gamma$. As the nonreciprocal current discussed in the previous section [Fig. 1(b)], the heat current peaks for $\gamma \approx t$. However, increasing the temperature bias changes the sign of $J_{R}^{1}$, signaling that the left reservoir is hot enough to heat the right one.

Finally, we discuss the efficiency of this process, characterized by the coefficient of performance, $\mathrm{COP}=$ $\left|J_{R}^{1} /\left(J_{R}^{1}+J_{L}^{1}\right)\right|$ measuring how much heat can be extracted from monitoring [117]. We depict the COP in Fig. 2(d) as a parametric plot on $\gamma$. For fixed temperatures in the reservoirs, the maximum COP is found near the critical $\gamma$, where the heat flow changes sign in Fig. 2(c). This monitoring strength, also on the order of $t$, lies beyond the weak-coupling limit.

Conclusions.-We have derived exact analytic expressions for particle and heat currents in a large class of driven monitored systems. These formulas were applied to investigate power harvesting and cooling assisted by measurements. Notably, we identified peaks in the current with respect to measurement strengths $\gamma$ outside the weakcoupling limit [Figs. 1(b)-2(c)]. These nontrivial features could provide valuable insights for future experiments investigating measurement effects in open quantum systems. Our findings can be generalized to different setups, enabling investigation of unexplored regimes beyond standard perturbative approaches. We have shown that these regimes are important, as they manifest the best
performances in terms of power generation and quantum measurement cooling.

On a more fundamental level, we have provided exact expressions for quantum transport in the presence of nonelastic effects caused by monitoring. It would be of great interest to establish whether formulas like Eq. (5) also apply for interacting quantum impurity models driven out of equilibrium and/or for systems coupled to bosonic baths at finite or even zero temperature [42,118-121].

We are grateful to Christophe Berthod, Daniel Braak, Géraldine Haack, Manuel Houzet, Rafael Sánchez, Kyrylo Snizhko, Clemens Winkelmann, and Robert Whitney for helpful comments and discussions. This work has been supported by the Swiss National Science Foundation under Division II under Grant No. 200020-188687. J. S. F. and M. F. acknowledge support from the FNS/SNF Ambizione Grant No. PZ00P2_174038. M. F. acknowledges support from EPiQ ANR-22-PETQ-0007 part of Plan France 2030.
[1] R. P. Feynman and F. Vernon Jr., Ann. Phys. (N.Y.) 281, 547 (2000).
[2] A. Caldeira and A. Leggett, Physica (Amsterdam) 121A, 587 (1983).
[3] C. Gardiner and P. Zoller, Quantum Noise: A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics (Springer Science, New York, 2004).
[4] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. 82, 1155 (2010).
[5] D. F. Walls and G. J. Milburn, Phys. Rev. A 31, 2403 (1985).
[6] W. H. Zurek, Rev. Mod. Phys. 75, 715 (2003).
[7] J. F. Poyatos, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 77, 4728 (1996).
[8] M. Müller, S. Diehl, G. Pupillo, and P. Zoller, Adv. At. Mol. Opt. Phys. 61, 1 (2012).
[9] A. Grimm, N. E. Frattini, S. Puri, S. O. Mundhada, S. Touzard, M. Mirrahimi, S. M. Girvin, S. Shankar, and M. H. Devoret, Nature (London) 584, 205 (2020).
[10] R. Lescanne, M. Villiers, T. Peronnin, A. Sarlette, M. Delbecq, B. Huard, T. Kontos, M. Mirrahimi, and Z. Leghtas, Nat. Phys. 16, 509 (2020).
[11] D. Poletti, J.-S. Bernier, A. Georges, and C. Kollath, Phys. Rev. Lett. 109, 045302 (2012).
[12] D. Gottesman, Ph.D. thesis, CalTech, 1997, arXiv:quantph/9705052.
[13] A. Calderbank, E. Rains, P. Shor, and N. Sloane, IEEE Trans. Inf. Theory 44 (1998).
[14] G. Barontini, R. Labouvie, F. Stubenrauch, A. Vogler, V. Guarrera, and H. Ott, Phys. Rev. Lett. 110, 035302 (2013).
[15] H. P. Lüschen, P. Bordia, S. S. Hodgman, M. Schreiber, S. Sarkar, A. J. Daley, M. H. Fischer, E. Altman, I. Bloch, and U. Schneider, Phys. Rev. X 7, 011034 (2017).
[16] T. Tomita, S. Nakajima, I. Danshita, Y. Takasu, and Y. Takahashi, Sci. Adv. 3, e1701513 (2017).
[17] M. Fitzpatrick, N. M. Sundaresan, A. C. Y. Li, J. Koch, and A. A. Houck, Phys. Rev. X 7, 011016 (2017).
[18] R. Ma, B. Saxberg, C. Owens, N. Leung, Y. Lu, J. Simon, and D. I. Schuster, Nature (London) 566, 51 (2019).
[19] K. Yamamoto, M. Nakagawa, N. Tsuji, M. Ueda, and N. Kawakami, Phys. Rev. Lett. 127, 055301 (2021).
[20] N. Dogra, M. Landini, K. Kroeger, L. Hruby, T. Donner, and T. Esslinger, Science 366, 1496 (2019).
[21] F. Ferri, R. Rosa-Medina, F. Finger, N. Dogra, M. Soriente, O. Zilberberg, T. Donner, and T. Esslinger, Phys. Rev. X 11, 041046 (2021).
[22] N. Syassen, D. M. Bauer, M. Lettner, T. Volz, D. Dietze, J. J. Garcia-Ripoll, J. I. Cirac, G. Rempe, and S. Durr, Science 320, 1329 (2008).
[23] K. Sponselee, L. Freystatzky, B. Abeln, M. Diem, B. Hundt, A. Kochanke, T. Ponath, B. Santra, L. Mathey, K. Sengstock, and C. Becker, Quantum Sci. Technol. 4, 014002 (2018).
[24] B. Skinner, J. Ruhman, and A. Nahum, Phys. Rev. X 9, 031009 (2019).
[25] Y. Li, X. Chen, and M. P. A. Fisher, Phys. Rev. B 98, 205136 (2018).
[26] S. Choi, Y. Bao, X.-L. Qi, and E. Altman, Phys. Rev. Lett. 125, 030505 (2020).
[27] M. J. Gullans and D. A. Huse, Phys. Rev. X 10, 041020 (2020).
[28] O. Alberton, M. Buchhold, and S. Diehl, Phys. Rev. Lett. 126, 170602 (2021).
[29] T. Müller, S. Diehl, and M. Buchhold, Phys. Rev. Lett. 128, 010605 (2022).
[30] J. C. Hoke, M. Ippoliti, D. Abanin, R. Acharya, M. Ansmann, F. Arute, K. Arya, A. Asfaw, J. Atalaya, J. C. Bardin et al., Nature (London) 622, 481 (2023).
[31] I. Poboiko, P. Pöpperl, I. V. Gornyi, and A. D. Mirlin, Phys. Rev. X 13, 041046 (2023).
[32] M. B. Plenio and S. F. Huelga, New J. Phys. 10, 113019 (2008).
[33] P. Rebentrost, M. Mohseni, I. Kassal, S. Lloyd, and A. Aspuru-Guzik, New J. Phys. 11, 033003 (2009).
[34] F. Caruso, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. 105, 190501 (2010).
[35] S. Viciani, M. Lima, M. Bellini, and F. Caruso, Phys. Rev. Lett. 115, 083601 (2015).
[36] C. Maier, T. Brydges, P. Jurcevic, N. Trautmann, C. Hempel, B. P. Lanyon, P. Hauke, R. Blatt, and C. F. Roos, Phys. Rev. Lett. 122, 050501 (2019).
[37] L. Corman, P. Fabritius, S. Häusler, J. Mohan, L. H. Dogra, D. Husmann, M. Lebrat, and T. Esslinger, Phys. Rev. A 100, 053605 (2019).
[38] P. Bredol, Phys. Rev. B 103, 035404 (2021).
[39] P. Bredol, H. Boschker, D. Braak, and J. Mannhart, Phys. Rev. B 104, 115413 (2021).
[40] M.-Z. Huang, J. Mohan, A.-M. Visuri, P. Fabritius, M. Talebi, S. Wili, S. Uchino, T. Giamarchi, and T. Esslinger, Phys. Rev. Lett. 130, 200404 (2023).
[41] A.-M. Visuri, T. Giamarchi, and C. Kollath, Phys. Rev. Lett. 129, 056802 (2022).
[42] O. Entin-Wohlman, Y. Imry, and A. Aharony, Phys. Rev. B 70, 075301 (2004).
[43] K. Yamamoto, Y. Ashida, and N. Kawakami, Phys. Rev. Res. 2, 043343 (2020).
[44] D. Rossini, A. Ghermaoui, M. B. Aguilera, R. Vatré, R. Bouganne, J. Beugnon, F. Gerbier, and L. Mazza, Phys. Rev. A 103, L060201 (2021).
[45] M. Nakagawa, N. Tsuji, N. Kawakami, and M. Ueda, Phys. Rev. Lett. 124, 147203 (2020).
[46] L. Rosso, D. Rossini, A. Biella, and L. Mazza, Phys. Rev. A 104, 053305 (2021).
[47] G. Mazza and M. Schirò, Phys. Rev. A 107, L051301 (2023).
[48] A.-M. Visuri, T. Giamarchi, and C. Kollath, Phys. Rev. Res. 5, 013195 (2023).
[49] A.-M. Visuri, J. Mohan, S. Uchino, M.-Z. Huang, T. Esslinger, and T. Giamarchi, Phys. Rev. Res. 5, 033095 (2023).
[50] T. Jin, M. Filippone, and T. Giamarchi, Phys. Rev. B 102, 205131 (2020).
[51] H. Fröml, A. Chiocchetta, C. Kollath, and S. Diehl, Phys. Rev. Lett. 122, 040402 (2019).
[52] T. Müller, M. Gievers, H. Fröml, S. Diehl, and A. Chiocchetta, Phys. Rev. B 104, 155431 (2021).
[53] F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, Rev. Mod. Phys. 78, 217 (2006).
[54] G. Benenti, G. Casati, K. Saito, and R. S. Whitney, Phys. Rep. 694, 1 (2017).
[55] C. Elouard, D. Herrera-Martí, B. Huard, and A. Auffèves, Phys. Rev. Lett. 118, 260603 (2017).
[56] C. Elouard and A. N. Jordan, Phys. Rev. Lett. 120, 260601 (2018).
[57] M. Naghiloo, J. J. Alonso, A. Romito, E. Lutz, and K. W. Murch, Phys. Rev. Lett. 121, 030604 (2018).
[58] L. Bresque, P. A. Camati, S. Rogers, K. Murch, A. N. Jordan, and A. Auffèves, Phys. Rev. Lett. 126, 120605 (2021).
[59] J. Stevens, D. Szombati, M. Maffei, C. Elouard, R. Assouly, N. Cottet, R. Dassonneville, Q. Ficheux, S. Zeppetzauer, A. Bienfait, A. N. Jordan, A. Auffèves, and B. Huard, Phys. Rev. Lett. 129, 110601 (2022).
[60] X. Linpeng, L. Bresque, M. Maffei, A. N. Jordan, A. Auffèves, and K. W. Murch, Phys. Rev. Lett. 128, 220506 (2022).
[61] K. Liu, M. Nakagawa, and M. Ueda, arXiv:2303.08326.
[62] B. Annby-Andersson, F. Bakhshinezhad, D. Bhattacharyya, G. De Sousa, C. Jarzynski, P. Samuelsson, and P. P. Potts, Phys. Rev. Lett. 129, 050401 (2022).
[63] S. Ryu, R. López, L. Serra, and D. Sánchez, Nat. Commun. 13 (2022).
[64] A. J. Daley, Adv. Phys. 63, 77 (2014).
[65] T. Prosen, New J. Phys. 10, 043026 (2008).
[66] M. Žnidarič, J. Stat. Mech. (2010) L05002.
[67] L. M. Sieberer, M. Buchhold, and S. Diehl, Rep. Prog. Phys. 79, 096001 (2016).
[68] M. V. Medvedyeva, F. H. L. Essler, and T. Prosen, Phys. Rev. Lett. 117, 137202 (2016).
[69] U. Harbola, M. Esposito, and S. Mukamel, Phys. Rev. B 74, 235309 (2006).
[70] C. Müller and T. M. Stace, Phys. Rev. A 95, 013847 (2017).
[71] P. Strasberg, G. Schaller, T. L. Schmidt, and M. Esposito, Phys. Rev. B 97, 205405 (2018).
[72] R. S. Whitney, arXiv:2304.03106.
[73] A. Levy and R. Kosloff, Europhys. Lett. 107, 20004 (2014).
[74] P. P. Hofer, M. Perarnau-Llobet, L. D. M. Miranda, G. Haack, R. Silva, J. B. Brask, and N. Brunner, New J. Phys. 19, 123037 (2017).
[75] T. Novotný, Europhys. Lett. 59, 648 (2002).
[76] L. P. Bettmann, M. J. Kewming, and J. Goold, Phys. Rev. E 107, 044102 (2023).
[77] P. E. Dolgirev, J. Marino, D. Sels, and E. Demler, Phys. Rev. B 102, 100301(R) (2020).
[78] T. Jin, J. S. Ferreira, M. Filippone, and T. Giamarchi, Phys. Rev. Res. 4, 013109 (2022).
[79] Y. Meir and N. S. Wingreen, Phys. Rev. Lett. 68, 2512 (1992).
[80] L. Buffoni, A. Solfanelli, P. Verrucchi, A. Cuccoli, and M. Campisi, Phys. Rev. Lett. 122, 070603 (2019).
[81] Generalizations with more terminals and internal degrees of freedom (e.g., spin) are straightforward.
[82] J. Dalibard, Y. Castin, and K. Molmer, Phys. Rev. Lett. 68, 580 (1992).
[83] K. Jacobs and D. A. Steck, Contemp. Phys. 47, 279 (2006).
[84] X. Cao, A. Tilloy, and A. D. Luca, SciPost Phys. 7, 024 (2019).
[85] H. Haken and G. Strobl, Z. Phys. A Hadrons Nucl. 262, 135 (1973).
[86] D. Bernard, T. Jin, and O. Shpielberg, Europhys. Lett. 121, 60006 (2018).
[87] D. Bernard and T. Jin, Phys. Rev. Lett. 123, 080601 (2019).
[88] X. Turkeshi and M. Schiró, Phys. Rev. B 104, 144301 (2021).
[89] X. Turkeshi, L. Piroli, and M. Schiró, arXiv:2306.09893.
[90] See Supplemental Material at http://link.aps.org/ supplemental/10.1103/PhysRevLett.132.136301 for technical details and additional information, which includes Refs. [91,92].
[91] B. Oksendal, Stochastic Differential Equations (Springer, Berlin, Heidelberg, 2003).
[92] A. I. Larkin and Y. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 73, 299 (1977).
[93] A. Kamenev, Field Theory of Non-Equilibrium Systems (Cambridge University Press, Cambridge, England, 2011).
[94] J. Schwinger, J. Math. Phys. (N.Y.) 2, 407 (1961).
[95] L. V. Keldysh et al., Sov. Phys. JETP 20, 1018 (1965).
[96] L. P. Kadanoff and G. Baym, Quantum Statistical Mechanics (CRC Press, Boca Raton, Florida, 1962).
[97] S. Datta, Electronic Transport in Mesoscopic Systems (Cambridge University Press, Cambridge, England, 1995).
[98] As we consider the stationary regime, $\left\langle d_{j}^{\dagger}(t) d_{i}(t)\right\rangle$ does not depend on time.
[99] R. Landauer, IBM J. Res. Dev. 1, 223 (1957).
[100] Remarkably, it is also possible to have $\langle n\rangle=\langle n\rangle_{\text {eq }}$ with $\gamma \neq 0$, provided that both the hybridization functions $\Gamma_{r}$ do not depend on frequency, which is also a condition to suppress the nonreciprocal current.
[101] G. Breit and E. Wigner, Phys. Rev. 49, 519 (1936).
[102] M. Buttiker, IBM J. Res. Dev. 32, 63 (1988).
[103] R. Sánchez and M. Büttiker, Phys. Rev. B 83, 085428 (2011).
[104] R. Sánchez, H. Thierschmann, and L. W. Molenkamp, New J. Phys. 19, 113040 (2017).
[105] G. Rosselló, R. López, and R. Sánchez, Phys. Rev. B 95, 235404 (2017).
[106] Z. Cai and T. Barthel, Phys. Rev. Lett. 111, 150403 (2013).
[107] M. Bauer, D. Bernard, and T. Jin, SciPost Phys. 3, 033 (2017).
[108] F. Mazza, R. Bosisio, G. Benenti, V. Giovannetti, R. Fazio, and F. Taddei, New J. Phys. 16, 085001 (2014).
[109] B. Sothmann, R. Sánchez, and A. N. Jordan, Nanotechnology 26, 032001 (2014).
[110] G. Fleury, C. Gorini, and R. Sánchez, Appl. Phys. Lett. 119, 043101 (2021).
[111] H. M. Pastawski, Phys. Rev. B 44, 6329 (1991).
[112] H. M. Pastawski, Phys. Rev. B 46, 4053 (1992).
[113] M. J. Gullans and D. A. Huse, Phys. Rev. X 9, 021007 (2019).
[114] L. Hruza and D. Bernard, Phys. Rev. X 13, 011045 (2023).
[115] B. Cleuren, B. Rutten, and C. Van den Broeck, Phys. Rev. Lett. 108, 120603 (2012).
[116] A. Mari and J. Eisert, Phys. Rev. Lett. 108, 120602 (2012).
[117] H. B. Callen, Thermodynamics and an Introduction to Thermostatistics (American John Wiley \& Sons, Inc., Hoboken, New Jersey, 1998).
[118] L. I. Glazman and R. I. Shekhter, Sov. J. Exp. Theor. Phys. 67, 163 (1988).
[119] N. S. Wingreen, K. W. Jacobsen, and J. W. Wilkins, Phys. Rev. Lett. 61, 1396 (1988).
[120] A.-P. Jauho, N. S. Wingreen, and Y. Meir, Phys. Rev. B 50, 5528 (1994).
[121] D. Braak and J. Mannhart, Found. Phys. 50, 1509 (2020).

