From Light-Cone to Supersonic Propagation of Correlations by Competing Short- and Long-Range Couplings

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We investigate the dynamical spreading of correlations in many-body quantum systems with competing short- and global-range couplings. We monitor the nonequilibrium dynamics of the correlations following a quench, showing that for strong short-range couplings the propagation of correlations is dominated at short and intermediate distances by a causal, light-cone dynamics, resembling the purely short-range quantum systems. However, the interplay of short- and global-range couplings leads to a crossover between space-time regions in which the light cone persists to regions where a supersonic, distance-independent spreading of the correlations occurs. We identify the important ingredients needed for capturing the supersonic spreading and demonstrate our findings in systems of interacting bosonic atoms, in which the global-range coupling is realized by a coupling to a cavity light field or atomic long-range interactions, respectively. We show that our results hold in both one and two dimensions and in the presence of dissipation. Furthermore, we characterize the short-time power-law scaling of the distance-independent growth of the density-density correlations.

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In recent decades, a fundamental question attracting enormous interest is what are the speed limits on how fast physical effects can propagate in nonrelativistic quantum systems [1–4]. In addition to its fundamental character, this problem is also relevant for quantum technologies, as it bounds the timescale of quantum operations and algorithms [5]. The knowledge of how correlations propagate enables one to select the most suitable platform to engineer the dynamical features leading to the desired speed.

In quantum systems with short-range interactions, Lieb and Robinson have shown that correlations can only travel within a light-cone defined by a maximal velocity [1]. The Lieb-Robinson bound is a key concept across different topics in quantum many-body physics, quantum thermodynamics, and quantum information [2], e.g., the exponential clustering for correlation functions [6], the scrambling of information [7], and even the area law of entanglement [8,9]. Although the original derivation of this bound is not strictly applicable to bosonic systems, an effective light cone can be derived at low densities [10–12] and in the Bose-Hubbard model [13–19], while the spreading of correlations breaking the light cone bound is typically only present in tailored models [20–22]. The typical light-cone spreading of correlations for short-range couplings has been observed experimentally with ultracold atoms in optical lattices [15,16].

The extension of these concepts to long-range interacting systems, where the interactions can directly connect distant sites, is nontrivial and partially counterintuitive [3,4]. An important class of long-range interactions are algebraically decaying with the distance, i.e., $x^{-\alpha}$, naturally occurring in trapped ions, Rydberg, and dipolar systems [4]. The exponent can also be tailored by engineering interactions with lasers and resonators [23,24]. While, in general, the concept of Lieb-Robinson bounds is not applicable to arbitrary long-range interacting systems [25–29], an effective bound exists for sufficiently large values of α [18,19,27–39] or special setups [40]. The spreading of correlation has been experimentally measured in trapped ion chains with tailored interactions [41,42], reporting the expected deviations from the light-cone propagation [28,36,37].

For the special case of strong long-range interactions, where α is smaller than the spatial dimension, theoretical bounds allow for correlation propagation that can be almost instantaneous [33]. On the other hand, for certain initial states, local perturbations can remain frozen, exhibiting effects analogous to localization [43,44] originating from large energy gaps in the spectra of strong long-range interacting systems. Remarkably, little is known on how correlations spread in systems with competing short- and long-range interactions. While previous work shed light on the intricate thermalization behavior in the presence of long- and short-range interactions [3,45–48], theoretical analyses of correlation spreading in this situation relevant for state-of-the art experiments are rare.

In this Letter, we investigate the dynamical spreading of correlations after a quench in a system with competing global- and short-range couplings. We consider a paradigmatic model of experimental relevance, namely, the

Bose-Hubbard model of cavity quantum electrodynamics [49,50], where the short-range kinetic and on-site processes compete with global density-density interactions mediated by photon scattering. We perform numerically exact simulations based on matrix product states and supplement them with approximate numerical and analytical techniques. We characterize the interplay of light-cone dynamics and of the noncausal propagation of the globally interacting dynamics, reporting a crossover between the light-cone behavior and the supersonic spreading of correlations, where correlations can spread across the system almost simultaneously, independent of the distance. We also show that the supersonic spreading can be triggered by light-cone dynamics. While the exact simulations are crucial in obtaining the details of the nonequilibrium dynamics, the approximate approaches allow us to identify the key ingredients necessary for the supersonic propagation and to pinpoint fluctuations of the global coupling that act as the carriers of the long-range correlations.

Setup—We consider bosonic atoms in a lattice, which interact with a quantum field, representing a photonic mode mediating global interactions between the atoms. The evolution of the density matrix of the coupled system $\hat{\rho}$ is given by the Lindblad master equation [49–53]

$$\frac{\partial}{\partial t}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H}, \hat{\rho}\right] + \frac{\Gamma}{2}(2\hat{a}\,\hat{\rho}\,\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\,\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}), \quad (1)$$

where we include the losses of the cavity with rate Γ. The Hamiltonian reads

$$\hat{H} = \hat{H}_{\mathrm{cav}} + \hat{H}_{\mathrm{BH}}, \qquad \hat{H}_{\mathrm{cav}} = \hbar\delta\hat{a}^\dagger\hat{a} - \hbar\Omega(\hat{a} + \hat{a}^\dagger)\hat{\Delta},$$
$$\hat{H}_{\mathrm{BH}} = \frac{U}{2}\sum_j \hat{n}_j(\hat{n}_j - 1) - J\sum_{\langle j,j'\rangle}(\hat{b}_j^\dagger\hat{b}_{j'} + \mathrm{H.c.}). \quad (2)$$

The local processes, captured by the Bose-Hubbard Hamiltonian $\hat{H}_{\mathrm{BH}}$, consist of the repulsive on-site interactions of strength U and atomic tunneling between neighboring sites $\langle j, j'\rangle$ with the amplitude J. The model provides a very good description of the dynamics realized in experiments [54–56], where the atomic transition is far detuned and spontaneous decay can be neglected. The effective light-matter coupling strength Ω is controllable by a pump laser and δ is the cavity-pump detuning. The period of the lattice is twice the period of the cavity mode coupling the cavity to the density imbalance $\hat{\Delta} = \sum_{j\in A}\hat{n}_j - \sum_{j\in B}\hat{n}_j$ of a bipartite lattice with sublattices A and B [55,57,58]. Theoretical efforts characterized mostly the nature of steady states in such models [59–70] and certain aspects of their dynamics [71–73].

Quench scenario—We simulate the propagation of correlations in a quench scenario in which initially the atoms are in an uncorrelated product state with one atom every two sites, $|1010\ldots\rangle$, and the cavity is empty. The initial state breaks the $\mathbb{Z}_2$ symmetry of $\hat{H}$, $(\hat{a}, \hat{\Delta}) \to (-\hat{a}, -\hat{\Delta})$. We analyze the dynamics of connected density-density correlations

$$\mathcal{C}_{nn}(d, t) = \frac{1}{\mathcal{N}}\sum_{j,j',|j-j'|=d}|\langle\hat{n}_j\hat{n}_{j'}\rangle - \langle\hat{n}_j\rangle\langle\hat{n}_{j'}\rangle|(t), \quad (3)$$

where we average over all correlations at a certain distance $d \equiv |j - j'|$ with $\mathcal{N}$ being the number of sites at the given distance d. In $\mathcal{C}_{nn}(d, t)$ we subtract the disconnected part of the density-density correlations, $\langle\hat{n}_j\rangle\langle\hat{n}_{j'}\rangle$, which describes the reorganization of the atomic density in the cavity-induced potential.

As in the initial state the different sites are uncorrelated, this scenario allows us to investigate cleanly the space-time propagation of correlations through the system. We remark that one particularity of the considered correlations is that, in the limit of vanishing tunneling, $J = 0$, $\hat{H}$ commutes with density operators and the correlations $\mathcal{C}_{nn}$ would be constant in time. We investigate the propagation of correlations for $J \neq 0$, in the presence of competing short- and global-range terms for the one-dimensional (1D) version of $\hat{H}$, Eq. (2), and of the Liouvillian, Eq. (1), using a recently developed method based on time-dependent matrix product states (tMPS) employing swap gates for the cavity coupling and quantum trajectories for the dissipative dynamics [74,75].

Light-cone to supersonic evolution—We investigate the crossover between the light-cone evolution and distance-independent propagation of the correlations in Fig. 1, for the 1D chain of atoms coupled to the cavity, assuming no photon losses ($\Gamma = 0$). For low coupling to the cavity field [Fig. 1(a) for $\hbar\Omega/J = 0.03$] and the considered distances, a light-cone propagation is found following the blue line, which marks approximately the speed limit for correlation spreading we found in the absence of the global-range coupling. For the same value of the coupling, we present in Fig. 1(b) $\mathcal{C}_{nn}(d, t)$ for certain distances as a function of time. The correlations exhibit an algebraic increase with time until reaching a maximum following the light cone. By comparing with the $\Omega = 0$ case, we observe for correlations at longer distances, $d \gtrsim 10$, finite values outside of the light cone, signaling the supersonic spreading. The light-cone spreading at small times and distances can be understood by expanding the time-evolution operator, i.e., $e^{-i\hat{H}t/\hbar} \approx e^{-i\hat{H}_{\mathrm{BH}}t} + O(t^2\Omega)$. Thus, the time evolution is dominated by the Bose-Hubbard evolution beside corrections on the order of $O(t^2\Omega)$ or higher, depending on the initial state and the form of the coupling (see Supplemental Material [75]).

The features of the supersonic propagation become apparent for larger values of the coupling strength, $\hbar\Omega/J = 0.42$ in Figs. 1(c) and 1(d). Here, we observe a coexistence of the light cone induced by short-range processes and a distance-independent increase of correlations, for $d > 4$, due to the presence of the cavity-induced global-range

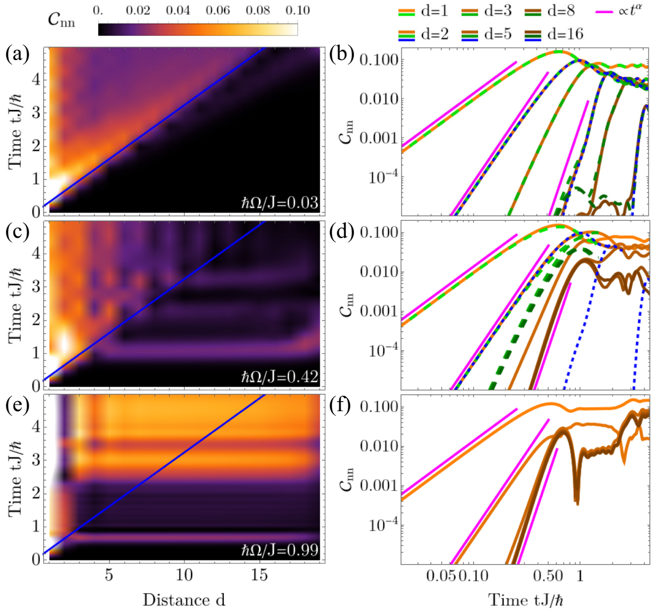

FIG. 1. (a),(c),(e) The propagation of $\mathcal{C}_{nn}(d,t)$, computed with $\hat{H}$, Eq. (2), for the atom-cavity coupling strength $\hbar\Omega/J \in \{0.03, 0.42, 0.99\}$. The blue line is a guide to the eye, approximating the front of the light-cone propagation for $\Omega = 0$. (b),(d), (f) $\mathcal{C}_{nn}(d,t)$ for several distances and $\hbar\Omega/J \in \{0.03, 0.42, 0.99\}$. The curves depicted with shades of orange to brown correspond to $\hat{H}$, Eq. (2), the dashed curves with shades of green to $\hat{H}_{\text{atom-only}}$, Eq. (5), and the dashed blue curves to the $\Omega = 0$ case. The magenta lines show the algebraically scaling $\propto t^\alpha$, with $\alpha \in \{2, 4, 8\}$. The parameters used are $N = 10$ particles, $L = 20$ sites, $U/J = 2$, $\hbar\delta/J = 2$, $\hbar\Gamma/J = 0$.

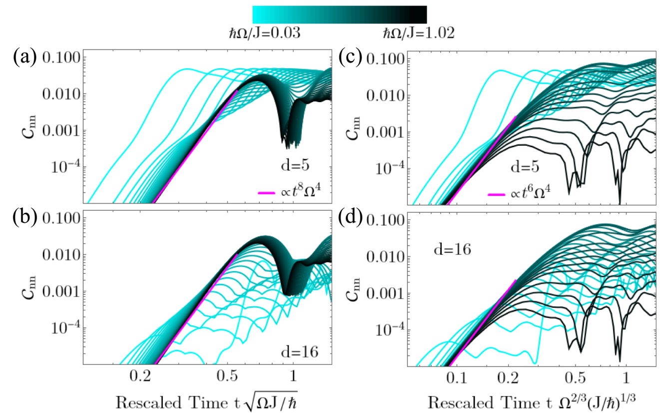

FIG. 2. (a),(b) The dependence of the correlations $\mathcal{C}_{nn}$ as a function of rescaled time $t\sqrt{\Omega J/\hbar}$ for (a) $d = 5$ and (b) $d = 16$, obtained for the atom-cavity Hamiltonian $\hat{H}$, Eq. (2). (c),(d) The dependence of the correlations $\mathcal{C}_{nn}$ as a function of rescaled time $t\Omega^{2/3}(J/\hbar)^{1/3}$ for (c) $d = 5$ and (d) $d = 16$, obtained for the atom-only Hamiltonian $\hat{H}_{\text{atom-only}}$, Eq. (5). The magenta lines represent the algebraic scaling (a), (b) $\propto t^8\Omega^4$ and (c), (d) $\propto t^6\Omega^4$. The values of the coupling are $0.03 \leq \hbar\Omega/J \leq 1.02$, with the same parameters as in Fig. 1.

interactions. In Fig. 1(d), we observe that $\mathcal{C}_{nn}(d,t)$ follows the light cone at very short distances and times ($tJ/\hbar \lesssim 1$, $d \leq 3$), while for $d \geq 5$ all correlations increase simultaneously. This is due to density correlations commuting with the entire Hamiltonian except the tunneling terms, implying that an initial change in the correlations can only be induced by tunneling. Thus, the cavity-mediated long-range interactions can start spreading the correlations only triggered by the short-range hopping term. By increasing the coupling strength to the cavity even further, $\hbar\Omega/J = 0.99$ in Figs. 1(e) and 1(f), we observe that the instantaneous spreading of the correlations occurs at a shorter times and only for $d \leq 2$ do the correlations grow beforehand. Furthermore, for such strong long-range interactions, we do not observe the light-cone dynamics, the role of the short-range processes being restricted to generating locally the density-density correlations, which propagate via the cavity coupling. After the initial rise, an intricate oscillatory behavior is observed in space and time, stemming from the interplay of the Bose-Hubbard terms [16] and the odd-even global ordering induced by $\hat{\Delta}$. This behavior could be related to metastability typical of global-range interacting systems [3,43,88] and its study is left for future work.

Scaling of the supersonic spreading—The short-time behavior of the distance-independent spreading can be understood with the minimal model of a simplified tunneling term and the global coupling to the cavity [75]. The correlations build up following the scaling

$$\mathcal{C}_{nn}(d,t) \propto \Omega^4 J^4 t^8 [\langle(\hat{a} + \hat{a}^\dagger)^2\rangle - \langle\hat{a} + \hat{a}^\dagger\rangle^2] \tag{4}$$

for $\hbar\Omega \gg J$ and an initial uncorrelated Fock state. Here the amplitude is proportional to the cavity field fluctuations, implying that the fluctuations are crucial to induce the supersonic spreading. In Figs. 2(a) and 2(b), we show that this scaling is dominant in the large coupling regime of the atom-cavity model $\hat{H}$ by collapsing the numerical results for a wide range of different values of Ω onto a single curve using $(t\sqrt{\Omega J})^8$. This scaling changes when other energy scales, such as the full kinetic term, the detuning δ, or preexisting long-range density correlations in the initial state, become relevant. The curves at lower Ω values deviate from the scaling gradually, showing a gradual change between the light-cone and supersonic propagation of the correlations. Thus, the interplay between the kinetic process and the global coupling to the cavity field is sufficient to cause the crossover; however, the global coupling fluctuations are required in this minimal model to obtain the supersonic spreading.

Propagation mediated by atom-only global interactions—In the full model, Eq. (2), the cavity field acts as an explicit carrier of the long-range correlations. The question arises if a global purely atomic interaction leads to similar behavior. Thus, we contrast the results of the coupled atom-cavity system with the atom-only model,

$$\hat{H}_{\text{atom-only}} = \hat{H}_{\text{glo}} + \hat{H}_{\text{BH}}, \qquad \hat{H}_{\text{glo}} = -\frac{\hbar\Omega^2\delta}{\delta^2 + \Gamma^2/4}\hat{\Delta}^2. \quad (5)$$

This represents an atom-only description of the complex hybrid system obtained by eliminating the cavity field in the limit $J, U, \hbar\Omega^2/\delta \ll \hbar\delta$ [49,50] and can include global-range dissipative processes [50,57,61,71,89]. $\hat{H}_{\text{atom-only}}$ includes directly the effective global-range nature induced by the coupling to the cavity in $\hat{H}_{\text{glo}}$. The simulations of the atom-only model, Eq. (5), make use of the matrix product state implementation of the time-dependent variational principal [75,90,91]. We show that, even for a different global-range coupling, the same crossover in the correlation spreading occurs as for the coupling to the cavity field.

In Figs. 1(b) and 1(d), showing the dynamics of $\mathcal{C}_{nn}(d,t)$ for several distances, we plot both the results for the atom-cavity Hamiltonian, Eq. (2) (continuous lines), and the results for the corresponding atom-only model, Eq. (5) (dashed lines). At small values of the long-range interactions, Fig. 1(b), we obtain a very good quantitative agreement in the dynamics of $\mathcal{C}_{nn}(d,t)$. In Fig. 1(d), we observe that the initial rise of the correlations $d = 1$ and $d = 2$ agrees in the two models, confirming that they are mostly influenced by the short-range terms. While in both cases we have a simultaneous increase of $\mathcal{C}_{nn}(d > 3, t)$, the rise of $\mathcal{C}_{nn}(d > 3, t)$ for the atom-only model occurs at slightly earlier times than for the atom-cavity model. In particular, the scaling we find is following more closely $\mathcal{C}_{nn}(d,t) \propto \Omega^4 t^6$, as shown in Figs. 2(c) and 2(d). We note that by decreasing the photonic timescale, i.e., increasing δ, while keeping Ω^2/δ constant, the difference between the long distance correlations of the two models decreases (not shown) and eventually they agree in the limit in which $\hbar\delta$ is the largest energy scale. In this regime also the atom-cavity model shows the scaling $\mathcal{C}_{nn}(d,t) \propto \Omega^4 t^6$, implying that the main effect of the fast photons is to introduce the effective global interaction. The qualitative agreement in the behavior of the correlations in the two models shows that the main features of the competition of the light-cone and the distance-independent propagation due to the global-range interactions are more generic than the case of hybrid atom-cavity systems.

Influence of a dissipative cavity field—A seldom explored question is how the presence of dissipation alters the spreading of correlations. Seminal works showed that for quasilocal Lindblad operators Lieb-Robinson-like bounds exist [92–96], observed these bounds numerically [97], and characterized entanglement measures [98,99]. The dissipation range can further alter the spatiotemporal behavior of correlations [100]. Thus, we aim to understand the effects of dissipation in the form of cavity losses, Eq. (1), which is a highly nonlocal dissipation for the atoms due to the global coupling.

For the atom-cavity system, Eqs. (1) and (2), photonic dissipation alters the position of the crossover between the local and the global spreading of correlations. By increasing the strength of dissipation, we go from the regime in which the supersonic propagation dominates to a regime with a mostly light-cone evolution [see Fig. 3(a) compared to Figs. 1(e) and 1(f)]. Thus, whereas the dissipation globally couples to the atoms, the photon losses rescale the effective global atom-atom interactions to a lower value [see the coefficient in $\hat{H}_{\text{glo}}$ Eq. (5)], making the light-cone propagation more prominent. However, for the same strength of $\hat{H}_{\text{glo}}$, stronger dissipation helps the supersonic propagation.

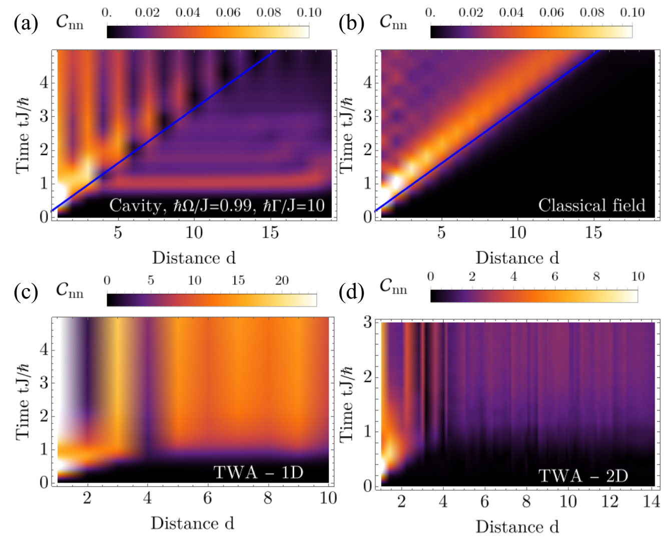

FIG. 3. (a) The propagation of $\mathcal{C}_{nn}(d,t)$, in the presence of dissipation, Eqs. (1) and (2), for $\hbar\Omega/J = 0.99$, $\hbar\Gamma/J = 10$, $N = 10$ particles, $L = 20$ sites, $U/J = 2$, $\hbar\delta/J = 2$. The blue line is a guide to the eye, approximating the front of the light-cone propagation for $\Omega = 0$ and $\Gamma = 0$. (b) The propagation of correlations when cavity field is replaced with a classical field (see Supplemental Material [75]), for the same parameters as in (a). Correlations $\mathcal{C}_{nn}(d,t)$ calculated from TWA in (c) 1D for a lattice with $L = 21$ and in (d) 2D for lattice with $L \times L = 21 \times 21$. For the TWA simulations we have used $U/J = 0.1$, $\hbar\delta/J = 2$, $\hbar\Gamma/J = 1$, $\sqrt{N}\hbar\Omega/J = 4$ in (c) and $\sqrt{N}\hbar\Omega/J = 8$ in (d). We initialize the atoms with alternating densities of $n = 10$ and $n = 11$ bosons corresponding to the total atom numbers (c) $N = 220$ and (d) $N = 4630$.

Key ingredients for the supersonic propagation—Our results show the presence of the supersonic propagation for global coupling to the cavity and atom-only global interactions. Thus, we aim to identify the underlying carrier of the correlations, using different approximations to highlight the required ingredients. In Fig. 3(b), we show results obtained employing a mean-field description of the cavity field, for which the cavity mode is modeled by a time-dependent classical field coupled to the mean value $\langle\hat{\Delta}\rangle$ [75]. In this mean-field approach, we can only recover the light-cone evolution while the distance-independent spreading of correlations is absent [see Fig. 3(b)]. This is due to the inability of the cavity in the mean-field description to transport and create fluctuations. Note that also in the minimal model, Eq. (4), the amplitude of the supersonic spreading would vanish in the mean-field approximation. Thus, we identify fluctuations, which can have different origins, as the carriers of the supersonic correlations. One source of fluctuations in the cavity field is the stochastic noise arising from photon losses. By including such a term in the mean-field description, which translates to a correlated noise in the atomic evolution, we recover a distance-independent propagation, albeit with a reduced contribution than in Fig. 3(a) (see Supplemental Material [75]).

To analyze the role of the fluctuations arising in the atom-cavity coupling, we employ the truncated Wigner approximation (TWA). In the TWA, the cavity field and each atomic site is described as a complex stochastic classical field [75,101] and it has been previously shown to describe light-cone spreading of correlations in Bose-Hubbard models [102]. For the 1D case at high filling, we show in Fig. 3(c) the TWA results, for an initial state with a staggered atomic occupation alternating between $n = 10$ and $n = 11$ [75]. We highlight that the qualitative behavior is the same as discussed in Fig. 1(e) for the tMPS results: at short times, light-cone spreading of the correlations occurs, followed by a distance-independent rise of correlations at large distances. Thus, the fluctuations of the global coupling through the cavity field determine the supersonic spreading.

Spreading of correlations in two dimensions—Establishing the TWA as a valid approach to simulate the quantum dynamics of correlations in this model gives us the opportunity to study this effect in higher dimensions. In Fig. 3(d), we show the results for a 2D atomic system coupled to the cavity field, where the atomic initial state is a checkerboard with alternating $n = 10$ and $n = 11$ occupations. Compared to the 1D simulations, the magnitude of $\mathcal{C}_{nn}$ is reduced, which we attribute to radial spread of the correlations in 2D. Nevertheless, the key features that emerge from the competing short- and long-range processes are well visible in 2D, the light-cone propagation of correlations on short times and the supersonic spreading on longer times.

Conclusions—We investigated the dynamics of density-density correlations emerging from the interplay of short- and global-range couplings, finding two key features, a light-cone propagation at short distances and times followed by a supersonic, distance-independent spreading. The crossover time between these dynamical features and their relative strength depend crucially on the ratio between the global coupling and the short-range tunneling. The general character of our results is highlighted by the robustness to the different quench situations, the dimensionality of the system, the presence of dissipation, or the form of the global coupling. We further checked the stability of the features under local atomic dephasing (see Supplemental Material [75]). We identify that the emerging dynamics relies on the presence of fluctuations in the long-range couplings, stemming either from the cavity field or the interaction itself. The role of fluctuations is essential for propagating the correlations outside of the light cone, which further underline the importance of capturing the fluctuations in the atom-cavity coupling in obtaining the correct quantum dynamics [67,72,73]. We emphasize that our results cannot be inferred from considerations based on general bounds on the propagation of correlations, which would be dominated by the contributions of the global-range couplings, such that numerical simulations of the quantum dynamics are needed. The studied effects could be realized and investigated in current state-of-the-art experiments of ultracold atoms coupled to optical cavities. However, due to its generality, our findings are applicable to a much wider class of experimental systems. Our results can guide the experimental investigations in controlling the spreading of correlations in platforms with competing interaction scales.

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Data availability—The data that support the findings of this article are openly available [103].

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