Localization Transition for Interacting Quantum Particles in Colored-Noise Disorder

Giacomo Morpurgo,[1,2,3] Laurent Sanchez-Palencia,[4] and Thierry Giamarchi[1]
[1]*Department of Quantum Matter Physics, University of Geneva, 1205 Geneva, Switzerland*
[2]*International Solvay Institutes, 1050 Brussels, Belgium*
[3]*Center for Nonlinear Phenomena and Complex Systems, Université libre de Bruxelles, CP 231, Campus Plane, B-1050 Brussels, Belgium*
[4]*CPHT, CNRS, Ecole Polytechnique, IP Paris, F-91128 Palaiseau, France*

(Received 28 July 2025; revised 19 December 2025; accepted 9 February 2026; published 4 March 2026)

We investigate the localization transition of interacting particles in a one-dimensional system with colored-noise disorder, where backward scattering processes are suppressed beyond a cutoff. Employing two complementary renormalization group procedures, we derive the phase diagram and reveal a significant shift in the localization transition point, governed by correlations. Our numerical analysis further demonstrates that the scaling of the localization length with disorder strength deviates markedly from the conventional behavior observed in typical localized phases, highlighting the unique impact of correlated disorder on interacting quantum systems. Application to optical disorder used in cold atom experiments is discussed.

DOI: 10.1103/c81j-h58j

The discovery of Anderson localization (AL) [1] has unveiled the profound impact of disorder on the electronic properties of noninteracting materials, sparking extensive investigations into disorder effects within quantum systems. This impact is particularly pronounced in low-dimensional systems, where even an infinitesimal amount of disorder can fundamentally alter the nature of electronic wave functions, resulting in localization. A major challenge in this domain is elucidating the interplay between disorder and interactions [2–4]. The most striking manifestations of this interplay are observed in low-dimensional systems, where both interaction effects [5] and disorder are maximal. This confluence gives rise to a plethora of novel phenomena, including the emergence of Bose glass phases [6,7] and many-body localization [8–10].

To date, investigations have predominantly focused on white-noise disorder. However, experimental realizations typically exhibit finite spatial correlations, thus constituting so-called colored-noise disorder. Such correlations can significantly influence the behavior of diverse systems, ranging from crystallography to superconductivity and AL [11–13]. This has spurred a substantial amount of theoretical and experimental research within quantum simulation platforms, such as ultracold atom and cavity polariton systems, where disorder correlations are well characterized and can be engineered. For instance, extensive studies have been conducted on quasiperiodic models both with and without interactions [14–24]. In contrast, truly disordered systems with finite correlations have received less attention. A simple model that captures colored-noise correlations involves decreasing Fourier components with a cutoff. Owing to fundamental optical constraints, this approach is precisely what is achieved in speckle potentials within cold atomic systems [25–29]. In 1D systems, suppression of backscattering beyond the cutoff effectively suppresses single-particle AL, leading to the appearance of a pseudomobility edge where the localization length varies by orders of magnitude [30]. This phenomenon can be harnessed to control AL through correlation engineering in such potentials [31,32]. Additionally, studies have explored AL of collective excitations in weakly interacting 1D bosons within Bogoliubov formalism [33–36]. In contrast, the impact of colored-noise disorder correlations on the localization of quantum particles in the strongly interacting regime remains unexplored.

In this Letter, we show that an interacting 1D quantum system transitions from localization to delocalization under colored-noise disorder with a momentum cutoff. Using universal Tomonaga-Luttinger liquid (TLL) theory, we address both bosons and fermions within a unified framework. Our most significant finding reveals that, at the cutoff, the critical properties of the localization-delocalization transition are profoundly altered compared to those observed with standard white-noise disorder. Renormalization group (RG) analysis indicates a shift in the critical point from the Luttinger parameter $K^* = 3/2$ (white noise) to $K^* = 1$ (colored noise) in the weak-disorder and Gaussian-correlated regime. This result is corroborated through direct perturbative RG analysis of a microscopic interacting Fermi model. Additionally, we observe an unusual scaling of the localization length with disorder strength. Our results reveal the substantial impact of spatial disorder correlations on the critical properties of interacting quantum systems, with direct implications for speckle potentials and other forms of colored-noise disorder implementable via digital mirror devices (DMDs).

Using the bosonized representation [5,37], which describes well the low-energy properties of the system, the Hamiltonian reads

$$H = H_0 + H_W, \tag{1}$$

$$H_0 = \frac{1}{2\pi}\int dx\, u\left[K\big(\nabla\theta(x,\tau)\big)^2 + \frac{1}{K}\big(\nabla\phi(x,\tau)\big)^2\right], \tag{2}$$

$$H_W = \frac{1}{\pi\alpha}\int dx\, W(x)\cos\big(2\phi(x) - 2k_{\mathrm{F}}x\big), \tag{3}$$

where ϕ and θ are two bosonic fields with the commutation relation $[(1/\pi)\nabla\phi(x), \theta(x')] = -i\delta(x - x')$, u is the speed of sound, and K is the dimensionless (interaction-dependent) Luttinger parameter. This representation applies to fermionic, bosonic, or spin systems both in the continuum and in a lattice. The quantity $k_{\mathrm{F}} = \pi\rho_0$, with ρ_0 being the average density, is the Fermi wave vector, and α is an ultraviolet cutoff of the order of the lattice spacing for a lattice model. For fermionic systems, $K = 1$ corresponds to free fermions, while $K > 1$ ($K < 1$) corresponds to attractive (repulsive) interactions. For bosonic systems with contact-repulsive interactions, $K \in [1, \infty[$, with $K = 1$ corresponding to infinite repulsion (Tonks limit) and $K \to +\infty$ to free bosons [5,38].

The term H_W represents the backscattering of particles with momentum close to k_{F} (scattering with transfer momentum $2k_{\mathrm{F}}$) from a disorder potential $W(x)$, as shown in Fig. 1(a). In 1D, the forward and backward scatterings can be decoupled [6]. Since only the backscattering affects the current, we neglect the forward component of the disorder in this Letter (see the Supplemental Material [39] for more details on the forward scattering).

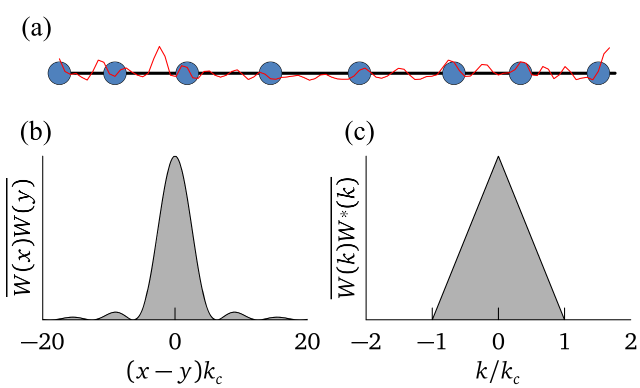

FIG. 1. Sketch of a colored-noise disorder as considered in this Letter. (a) Realization of a 1D speckle potential (red line) with quantum particles (blue disks). (b) Two-point correlation function of a speckle potential in real space. (c) Two-point correlation function of a speckle potential in momentum space. The latter has a triangular form with a high-momentum cutoff at $|k| = k_c$.

We consider a colored-noise disorder that can potentially make the backscattering vanish. One example is the speckle disorder (SD) [40,41], which has been instrumental in observing single-particle AL in cold atomic gases [26,42,43]. It stems from the square of a random, Gaussian-correlated, complex field, so that it is non-Gaussian and nonsymmetric. Its two-point spatial correlation function is a squared sinc function; see Fig. 1(b). The speckle disorder has finite momentum support, such that the second moment of the correlations of the potential vanishes above a momentum cutoff k_c; see Fig. 1(c). It can be either repulsive (blue-detuned, BSD) or attractive (red-detuned, RSD). In order to disentangle some of the effects due to the colored noise from the existence of odd moments for BSD and RSD due to their non-Gaussian character, we also study a Gaussian, colored disorder (GCD) with the same correlation function. In both cases, the second moment reads

$$\overline{W_k W_{-k'}} = W_0^2(1 - |k|/k_c)\theta(k_c - |k|)\Omega\delta_{k,k'}, \tag{4}$$

where the overbar denotes the average over disorder realizations, W_0 is the disorder intensity, Ω is the volume of the system, and $\delta_{k,k'}$ is the Kronecker delta. Note that the factor Ω comes from the use of discrete values of k, and the spatial correlations do not depend on the volume of the system. More details can be found in the Supplemental Material [39].

To consider the combined effects of disorder and interactions, we treat the disorder term in Eq. (1) using a perturbative RG procedure. Working along the lines of Ref. [18], we integrate the short-distance properties by increasing the cutoff $\alpha(l) = \alpha e^l$. This is equivalent to integrating the momenta in a shell around $2k_{\mathrm{F}}$ with width $1/\alpha(l)$. It yields the RG equations [39]

$$\begin{aligned}\frac{\partial K}{\partial l} = -\frac{K^2 y^2}{2}\frac{1}{\Omega}\sum_k\left(1 - \frac{|k|}{k_c}\right)\theta(k_c - |k|)\\ \times\left[J_0\big((k + 2k_{\mathrm{F}})\alpha(l)\big) + J_0\big((k - 2k_{\mathrm{F}})\alpha(l)\big)\right],\end{aligned} \tag{5}$$

$$\frac{\partial y^2}{\partial l} = (4 - 2K)y^2, \tag{6}$$

where $y = (\alpha W_0/u)$, and J_0 is the Bessel function. The latter acts as a "window" filtering the modes far from the Fermi wave vector by a value of order $1/\alpha(l)$. At this order, the RG equations depend only on the second moment of the disorder correlations, and they are thus identical for SD and GCD. The appearance of the Bessel functions is due to our choice of a hard cutoff in real space and, similarly to Ref. [18], we replace them with windows $[J_0(q\alpha) \to \theta(1 - |q|\alpha)]$ centered at $2k_{\mathrm{F}}$ or $-2k_{\mathrm{F}}$. Intuitively, the RG procedure amounts to making these windows narrower and narrower, thus capturing only the physics which occurs there at low energy (i.e., at $\sim \pm 2k_{\mathrm{F}}$).

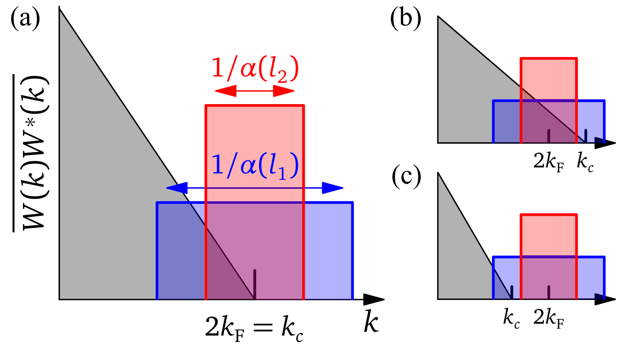

FIG. 2. Bosonized RG procedure: As $\alpha(l)$ increases ($l_1 < l_2$), the windows around $2k_\mathrm{F}$ shrink, capturing only the low-energy physics. (a) Case $k_c = 2k_\mathrm{F}$. The weight of the disorder in this window becomes vanishingly small as α increases. (b) Case $k_c > 2k_\mathrm{F}$. As α increases, there is always a finite weight of the disorder in the shrinking window. We recover the physics of an uncorrelated Gaussian disorder. (c) Case $k_c < 2k_\mathrm{F}$. There is a certain l^* after which there is not any weight of the disorder inside the shrinking window. We recover the physics of nondisordered systems.

Three cases of interest arise depending on the value of k_c compared to the Fermi level. For $k_c < 2k_\mathrm{F}$, there is a scale l^*, such that $\alpha(l > l^*) > 1/(2k_\mathrm{F} - k_c)$ and the window does not contain any disorder anymore, owing to the finite support of the latter; see Fig. 2(c). Hence, no backscattering occurs at this order in the RG, which implies the suppression of localization. Note that higher-order perturbation terms in the disorder may induce backscattering [30]. However, for the case of weak disorder that we consider here, such terms corresponding to a higher power of the disorder would be extremely small and lead potentially only to a huge localization length.

For $k_c > 2k_\mathrm{F}$, backscattering is always present at all scales. For $\alpha(l) \gg 1/(k_c - 2k_\mathrm{F})$, the second moment of the disorder is almost constant within the window—see Fig. 2(b)—and we recover the same RG equations as for an uncorrelated Gaussian disorder [6,18]. In that case, the momentum cutoff in the spectrum of the disorder is irrelevant, and we find a localization-delocalization transition (for weak disorder) at the usual critical point $K^* = 3/2$.

The most interesting case, and the central point of our Letter, corresponds to $k_c = 2k_\mathrm{F}$, for which backscattering is vanishingly allowed at all scales; see Fig. 2(a). In this case, as we progress in the RG, the window shrinking around $2k_\mathrm{F}$ always contains disorder, but with a smaller and smaller weight. The linear decrease of the spectral weight $\overline{W_k W_{-k'}}$ implies that the sum over k in Eq. (5) scales quadratically with $\alpha(l)$, which yields $\partial K/\partial l \propto -y^2/\alpha^2(l)$. Introducing $\tilde{y} = y/\alpha(l)$, we then find the RG equations

$$\frac{\partial K}{\partial l} = -\frac{K^2}{4\pi k_c}\tilde{y}^2, \tag{7}$$

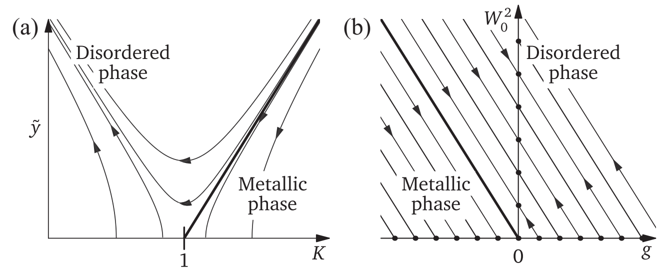

FIG. 3. Phase diagrams. (a) Sketch of the phase diagram versus the Luttinger parameter and renormalized disorder strength, with flow lines from the bosonized RG procedure for $k_c = 2k_\mathrm{F}$. The separatrix (bold line) separates the disordered (localized) and metallic (delocalized) phases, with the critical point at K^* in the weak-disorder limit. (b) Sketch of the phase diagram versus interaction and disorder strengths, with flow lines from the diagrammatic RG procedure around the noninteracting Fermi point. Here, the flow is made of straight lines, and we have a line of fixed points in the absence of interactions ($g = 0$).

$$\frac{\partial \tilde{y}}{\partial l} = (1 - K)\tilde{y}. \tag{8}$$

The corresponding RG flow is shown in Fig. 3(a). It shows that in this case, the critical point is at $K^* = 1$, instead of the value $K^* = 3/2$ for white-noise disorder. Hence, for $K < 1$, any arbitrary weak disorder implies localization (disordered phase). Instead, for $K > 1$, we find a localization transition where a finite amount of disorder is necessary to localize, while too-weak disorder implies delocalization (metallic phase). The shift of the critical point implies that a colored noise having backscattering vanishing linearly at $2k_\mathrm{F}$ can dominate only for significantly less attractive interactions for fermions (and therefore, more repulsive interactions for bosons) than for standard white-noise disorder. This remarkable shift of the transition point is consistent with the intuitive fact that the backscattering at exactly $2k_\mathrm{F}$ would be zero for such a disorder. Nevertheless, due to backscattering at finite momenta around $2k_\mathrm{F}$, interactions restore localization at $2k_\mathrm{F}$. Note that the value of K^* directly relies on the scaling of the disorder correlation functions at the cutoff k_c. For correlations behaving as $[1 - (|k|/k_c)]^\nu$, following the same steps, we obtain a transition point at $K^* = [(3 - \nu)/2]$. We thus recover the two limit cases: For $\nu = 0$ (white-noise disorder or colored-noise disorder with $2k_\mathrm{F} < k_c$), $K^* = 3/2$. For $\nu = 1$ (colored-noise disorder with $2k_\mathrm{F} = k_c$), $K^* = 1$. A colored-noise disorder with $0 < \nu < 1$ may be realized using a mask with varying transmission for SD [31,32] or directly engineered using DMDs in cold atom experiments. More details on the derivation of the RG equations and their generalization to $\nu \neq 1$ can be found in the Supplemental Material [39] (specificities linked to the colored disorder) and in Ref. [18] (core idea).

The linear vanishing of the spectral weight of the disorder for standard SD has the advantage to bring back the transition around the noninteracting point for fermions ($K = 1$). This allows us to perform a perturbative RG analysis directly on the microscopic model. Moreover, it avoids the necessity to carefully distinguish elastic and inelastic processes around the noninteracting point in the bosonization procedure [6], which is something highly nontrivial for the correlated disorder discussed here. To proceed, we consider the Fermi Hamiltonian

$$\begin{aligned} H = &\sum_r \sum_k v_{\mathrm{F}}(\varepsilon_r k - k_{\mathrm{F}}) c^\dagger_{r,k} c_{r,k} \\ &+ \frac{g}{2\Omega} \sum_r \sum_{k,k',q} c^\dagger_{r,k+q} c^\dagger_{-r,k'-q} c_{-r,k'} c_{r,k} \\ &+ \frac{1}{\Omega} \sum_{k,q\sim 0} \left[W_{q-2k_{\mathrm{F}}} c^\dagger_{R,k+q} c_{L,k} + W_{q+2k_{\mathrm{F}}} c^\dagger_{L,k+q} c_{R,k} \right], \end{aligned} \tag{9}$$

where the kinetic term is linearized around the Fermi momentum, v_{F} is the Fermi velocity, g is the strength of the interactions, $\varepsilon_r = \pm 1$ for respectively right and left movers, and $W_{q-2k_{\mathrm{F}}}$ is defined as before in Eq. (4). We impose an ultraviolet cutoff in momentum space $\Lambda \sim (1/\alpha)$, equivalent to the one used in bosonization. We expand up to second order in interactions g and disorder lines W_0^2, and we look for the second-order diagrams which renormalize the first-order interaction and backscattering diagrams. In both cases, there is a single gW_0^2 diagram to be computed. We find that both second-order diagrams diverge logarithmically with Λ. More details on the procedure [44,45] can be found in the Supplemental Material [39]. Upon varying the cutoff as $\Lambda(l) = \Lambda e^{-l}$, we find the RG equations

$$\frac{\partial g}{\partial l} = -\frac{g W_0^2}{2\pi k_c v_{\mathrm{F}}^2}, \tag{10}$$

$$\frac{\partial W_0^2}{\partial l} = \frac{g W_0^2}{2\pi v_{\mathrm{F}}}. \tag{11}$$

These equations describe a flow that follows straight lines in the g-W_0^2 parameter space; see Fig. 3(b). This flow confirms the predictions of the bosonization approach. For attractive interactions, $g < 0$, the flow reduces the disorder strength. There is a separatrix between the delocalized (metallic) phase, where the disorder fully vanishes, and the localized (disordered) phase, where it grows up to a finite value through RG. Remarkably, we find that the noninteracting line is a line of stable fixed points where the amplitude of the disorder remains constant under the RG flow.

Besides the phase diagrams shown in Fig. 3, computing physical properties, such as the nature of the metallic and disordered phases and transport properties, is challenging. For the phase where the disorder is relevant, higher-order disorder correlations must in principle be taken into account, which we leave for future studies. For the metallic phase, one recovers in principle TLL behavior, characterized by power-law decaying correlation functions and dominant superconducting or superfluid quasi-long-range order. However, this is justified for a disorder with strictly no Fourier component beyond $2k_{\mathrm{F}}$. Higher terms in the disorder—for instance, combining the backward scattering with one forward scattering may generate such Fourier components at order W_0^4, restoring a strict critical point at $K^* = 3/2$. Nevertheless, for $1 < K < 3/2$, the localization length would be extremely large and, far from the critical point, of the order of $\xi \sim (1/D)^{2/(3-2K)}$ with $D = \overline{W(x)W(x)}$. Below such a length, the system would be fully controlled by the colored part of the disorder with its own "localization"-delocalization transition at $K^* = 1$.

To further analyze the consequences of the presence or not of higher moments in the disorder, we may again take advantage of the fact that the noninteracting line is a fixed line by RG in the disordered phase, and consider noninteracting spinless fermions with the tight-binding Hamiltonian

$$H = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + \mathrm{H.c.} + \sum_i W_i c_i^\dagger c_i, \tag{12}$$

where t is the hopping amplitude, W_i is the disorder potential at site i, and $\Omega = Na$, with a being the lattice spacing, and N the number of sites. We solve the Hamiltonian in Eq. (12) by exact numerical diagonalization, which therefore contains all scattering orders and includes all forward and backward scattering processes. We use a system of 10000 sites for all strengths of disorder. To disentangle the roles of the non-Gaussian character of speckles, their nonsymmetric property, and the existence of a spectral cutoff, we consider BSD, RSD, and GCD. In order to compare to uncorrelated Gaussian disorder (white noise), it is convenient in this section to characterize the disorder strength by its zero-distance correlations $D = \overline{W(x)W(x)} = (W_0^2 k_c/2\pi)$. We set the disorder cutoff to be $k_c = (\pi/2a)$. To extract the localization length ξ of the different eigenstates, we compute the corresponding inverse participation ratio, $\mathrm{IPR} = \int dx |\psi(x)|^4$, and average it over 1000 realizations of the disorder. For weak disorder, the IPR is related to the inverse of the localization length via $\mathrm{IPR} = (1/2\xi)$. The results for $k_{\mathrm{F}} = k_c/2$ are shown in Fig. 4 for BSD, RSD, and GCD. More details on the generation of each type of disorder, the average over the disorder realizations, effects of different system sizes N, as well as numerical results for other values of k_c are given in the Supplemental Material [39]. In particular, for $k_c < 2k_{\mathrm{F}}$, we find ξ of the order of the system size, consistently with effective delocalization.

For $k_c = 2k_{\mathrm{F}}$, the three types of disorder exhibit clear differences showing the role of non-Gaussianity and the presence of higher moments for BSD and RSD.

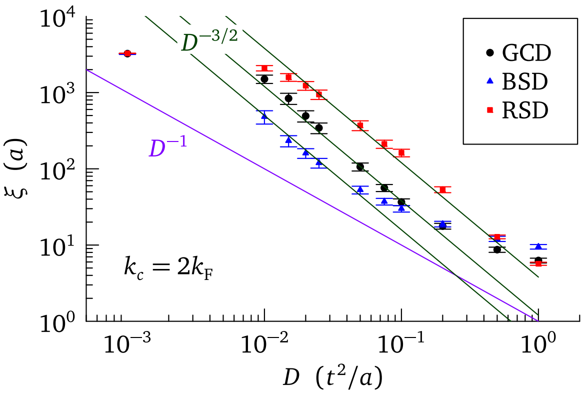

FIG. 4. Localization length ξ versus disorder strength D of the eigenstates of a noninteracting system at momentum $k_F = k_c/2$ for GCD (black circles), BSD (blue triangles), and RSD (red squares). The error bars indicate the standard deviation from disorder averaging (1000 realizations). Guides to the eye of $D^{-3/2}$ behavior for the three sets of data are shown as solid green lines, while the scaling D^{-1} expected for white-noise disorder is shown as a solid purple line for reference. The available range for ξ is limited on one side by the system size ($L/a = 10000$) and on the other side by the reaching of the strong-disorder regime, where usual scalings break down.

The difference between BSD and RSD may be attributed to odd-order disorder terms, which affect localization with opposite contributions [30,46], while GCD has no such terms. In all cases, clear deviations from the usual $1/D$ white-noise behavior are found, showing the role of the vanishing spectral weight of the disorder at $k_c = 2k_F$. As discussed above, higher-order terms in the disorder are expected to give a nonvanishing spectral weight of the disorder at $2k_F$, and thus, to a large but finite localization length. However, such terms are expected to give at least a $1/D^2$ behavior (note that third-order terms in SD have the same threshold at k_c as the second-order term), and none of the three disorders is apparently compatible with that scaling. For GCD and RSD, a scaling compatible with $D^{-3/2}$ is visible in the weak-disorder range and before the localization length is limited by finite-size effects. The precise dependence for BSD is more difficult to ascertain owing to the limited range of the data, but it also seems to show a crossover toward $D^{-3/2}$ at weak disorder. The origin of such a $D^{-3/2}$ scaling is not understood at the moment and is clearly an important target for more numerical investigations and further studies.

In summary, we have investigated the localization of interacting 1D quantum particles, both bosons and fermions, subjected to colored-noise disorder characterized by a vanishing Fourier spectrum. Such correlations are directly inspired by and generalize those inherent in speckle disorder. Our findings demonstrate that in the regime of weak disorder, they significantly modify the critical point for the localization-delocalization transition, a phenomenon governed by disorder correlations. Treating the non-Gaussianities of the speckle disorder by looking at higher-order processes would restore Anderson localization, as shown in Ref. [30], but with localization lengths larger by several orders of magnitude. We analyzed the behavior of the localization length for fermions along the RG fixed line (noninteracting fermions), revealing an unexpected scaling $1/D^{3/2}$ as a function of the disorder strength D. A deeper understanding and further exploration of the properties of such colored-noise disorder presents a significant challenge. On the theoretical front, additional analysis for the noninteracting case is clearly required. For the interacting case, despite the limitations to shorter systems, tensor network or Monte Carlo calculations should allow for a detailed study of the phase diagram and the transition-point shift. Experimentally, true speckle potentials, as well as those generated by digital mirror devices (DMDs), which could implement both GCD and SD, hold promise for observing disorder-induced localization for interacting quantum particles. Since state-of-the-art experimental setups can explore regimes with $k_c/2k_F$ ranging from 0.1 to 2.5, including the ratio $k_c/2k_F = 1$, they should enable accurate probing of the effects studied herein.

Acknowledgments—This work was supported by the Swiss National Science Foundation under Division II (Grant No. 200020-219400); the Agence Nationale de la Recherche under projects QuanTEdu-France (No. ANR-CMAQ-002 France 2030); and QUTISYM (No. ANR-23-PETQ-0002), GENCI-TGCC (Grant No. 2023-A0110510300), and the ERC Grant LATIS.

Data availability—The data that support the findings of this article are openly available [47].

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