Fluctuation-Induced Bistability of Fermionic Atoms Coupled to a Dissipative Cavity

Luisa Tolle,[1] Ameneh Sheikhan,[1] Thierry Giamarchi,[2] Corinna Kollath,[1] and Catalin-Mihai Halati[2]
[1]*Physikalisches Institut, University of Bonn, Nussallee 12, 53115 Bonn, Germany*
[2]*Department of Quantum Matter Physics, University of Geneva, Quai Ernest-Ansermet 24, 1211 Geneva, Switzerland*

(Received 24 September 2024; revised 23 December 2024; accepted 11 March 2025; published 31 March 2025)

We investigate the steady state phase diagram of fermionic atoms subjected to an optical lattice and coupled to a high finesse optical cavity with photon losses. The coupling between the atoms and the cavity field is induced by a transverse pump beam. Taking fluctuations around the mean-field solutions into account, we find that a transition to a self-organized phase takes place at a critical value of the pump strength. In the self-organized phase the cavity field takes a finite expectation value and the atoms show a modulation in the density. Surprisingly, at even larger pump strengths two self-organized stable solutions of the cavity field and the atoms occur, signaling the presence of a bistability. We show that the bistable behavior is induced by the atoms-cavity fluctuations and is not captured by the mean-field approach.

DOI: 10.1103/PhysRevLett.134.133602

The emergence of a bistability is a fascinating effect often occurring in nature. Such bistable systems can reach two distinct stable states depending on their history, i.e. the preparation procedure, or their previous dynamics. This concept is important in many scientific fields as for example in biology, chemistry, engineering, and physics. The presence of a bistability leads to profound implications, since the dynamics of the system can become highly sensitive to the initial conditions and external perturbations.

One very well known bistability in physics is the optical bistability [1]. This phenomenon occurs due to nonlinear effects, causing the existence of two stable states with different light intensities. A typical device is the Fabry-Perot cavity containing a nonlinear optical medium with a refractive index depending on the light intensity. Furthermore, the underlying nonlinearity can also emerge from the optomechanical interactions of a cold atomic gas coupled to the field of the optical cavity [2], e.g., as it has been recently investigated for the fermionic atoms in optical cavities throughout the BEC-BCS crossover [3].

Whether bistabilities can also occur in presence of dissipation is of course an important question. Bistable behaviors have been indeed identified around dissipative quantum phase transitions, either due to the spontaneous breaking of a weak-symmetry [2,4–14], or due to the first order character of the transition [15–25]. In the latter, a hysteresis behavior between the two competing phases can appear, depending on the nature of the initially prepared state. However, typically in these cases the bistability occurs within a mean-field approach and the competing states resolve to metastable states when quantum fluctuations around the mean-field solutions are taken into account [7,26–28]. In such cases the quantum dynamics exhibits a unique steady state, with the density matrix consisting of the admixture of the mean-field bistable states. This is in contrast to the presence of multiple stable steady states occurring from the presence of a strong symmetry [29–32].

Intriguingly, in equilibrium, for classical systems or systems described by Ginzburg-Landau type theories, such as magnets, superconductors, or liquid crystals, fluctuations have been shown to change the nature of a phase transition from second-order to a first-order character [33–37], with a coexistence of phases possible around the critical point [38].

Here we show a novel mechanism, which we refer to as fluctuation-induced bistability, in which bistability occurs in a *dissipative quantum* system, *only* if quantum fluctuations around a mean-field solution are taken into account. We identify this phenomenon for interacting spinful fermionic atoms confined to optical lattices and coupled to the field of a dissipative cavity, a paradigmatic example of an open quantum system. We include the fluctuations in the atoms-cavity coupling on top of the mean-field solution resulting from the adiabatic elimination of the cavity field. The fluctuations induce a thermalization process resulting in steady states with a self-consistently determined temperature. In the strong atoms-cavity coupling regime two (several) different self-organized states exist, which differ in their effective self-consistently determined temperatures. Whereas the dominant contribution is the same in both bistable solutions, the nature of the admixed excited states change. In particular, in the second steady solution, excited states with double occupied sites become crucial, with a cooling mechanism emerging due to resonant photon-assisted transitions between states with double occupancies and other atomic excitations [see sketch Fig. 1(c)]. Thus, we pinpoint the origin of the multistability in the interplay between the *short-range* atomic interactions and the *global-range* coupling via the cavity-induced self-consistent potential. We show the existence of the

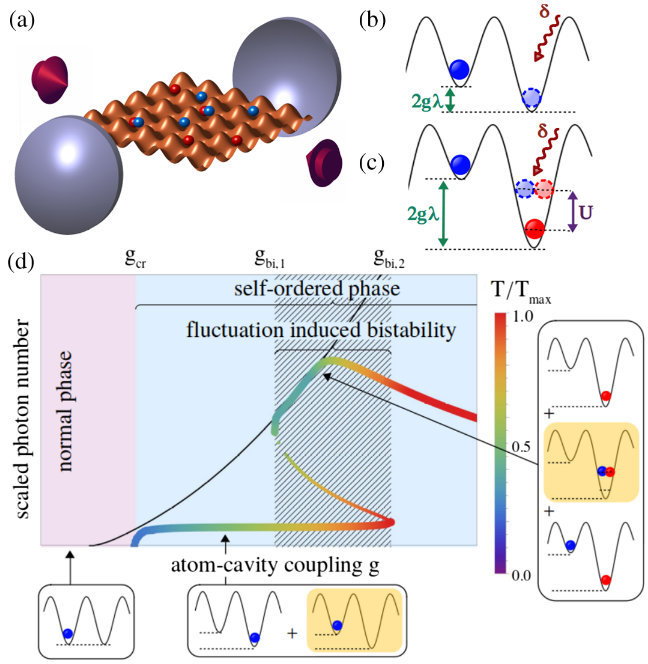

FIG. 1. (a) Fermi-Hubbard system coupled to a single-mode cavity. The two fermion species have an on-site interaction U and a tunneling amplitude J. They are coupled globally to the cavity mode, strength g, by pumping with a retroreflected transverse laser beam. Photons with a detuning δ leak through the mirrors at rate Γ. (b), (c) Sketches of important excited-state transitions in the cavity potential with effective energy difference $2g\lambda$, where the rescaled cavity field $\lambda = \langle \hat{a} + \hat{a}^\dagger \rangle / L^d$ is the order parameter. Resonances between the higher energy sites and doubly occupied sites play a crucial role in the physics (see text). (d) Scaled photon number $N_{\rm pho}$ as a function of atom-cavity coupling g. The color of the points represents the effective self-consistently determined temperature $T/T_{\rm max}$. The black line is the atom-cavity mean-field result ($T = 0$ MF). The sketches highlight the dominant occupations contributing to the different steady states. The yellow-colored contribution is the dominant excited state admixed by the self-consistent effective temperature.

fluctuation-induced bistability is independent of the dimensionality of the atomic gas and we expect that the underlying mechanism is present in a large class of atoms-cavity models with comparable atomic and photonic energy scales. The need to consider the fluctuations around the mean-field solution is even more remarkable, due to the general belief of the validity of mean-field methods for long-range couplings.

The considered platform of ultracold atoms coupled to optical cavities has established itself for the study of dissipative phenomena [2,39]. Experimentally, bosonic atoms have been placed in cavities both as three-dimensional Bose-Einstein condensates [4,40–45], or confined additionally to an optical lattice [46–49], more recently also the coupling between cavities and cold fermionic gases has been realized [3,50–53]. Generally, these type of systems exhibit a phase transition between a normal phase with an empty cavity and a self-organized phase exhibiting a finite cavity occupation [2]. The theoretical studies for fermionic systems have focused on the nature of the self-organization phase transition [54–61], employing the attractor dynamics to stabilize exotic states of matter [62–70], topological effects [71–76], pairing in superfluids [77], or aspects of the nonequilibrium dynamics [78,79].

The dissipative dynamics of the cold atomic gas in an optical cavity [as sketched in Fig. 1(a)], pumped with a standing-wave transverse laser beam far-detuned from the atomic resonance, can be described by a Lindblad master equation [2,39,80–82]

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \frac{\Gamma}{2}(2\hat{a}\,\hat{\rho}\,\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\,\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}). \quad (1)$$

Here $\hat{\rho}$ is the density matrix containing both the atomic and photonic degrees of freedom. The dissipator with amplitude Γ describes photon losses from the cavity mode by the photon annihilation jump operator $\hat{a}$. The Hamiltonian is given by $\hat{H} = \hat{H}_{\rm FH} + \hat{H}_{\rm cav} + \hat{H}_{\rm ac}$ [2,39]. The first term is the Hubbard model $\hat{H}_{\rm FH} = -J\sum_{\langle j,l\rangle,\sigma}(\hat{c}^\dagger_{j\sigma}\hat{c}_{l\sigma} + {\rm H.c.}) + U\sum_j \hat{n}_{j\uparrow}\hat{n}_{j\downarrow}$, where $\hat{c}_{j\sigma}$ and $\hat{c}^\dagger_{j\sigma}$ are fermionic operators for the atoms on site j and spin $\sigma \in \{\uparrow, \downarrow\}$, $\langle j, l\rangle$ denotes neighbouring sites, and the local density operator is $\hat{n}_{j,\sigma} = \hat{c}^\dagger_{j,\sigma}\hat{c}_{j,\sigma}$. L is the number of sites along each dimension of the lattice system d and N the total number of atoms. J is the tunneling amplitude and $U > 0$ the repulsive on-site interaction strength. The second term $\hat{H}_{\rm cav} = \hbar\delta\hat{a}^\dagger\hat{a}$, describes the cavity mode in the rotating frame of the pump beam, with δ the detuning between the cavity mode and the transverse pump beam. The laser-assisted dispersive atoms-cavity coupling, $\hat{H}_{\rm ac} = -(\hbar g/\sqrt{L^d})(\hat{a} + \hat{a}^\dagger)\hat{\Delta}$, is chosen such that the cavity mode is commensurable with twice the periodicity of the lattice spacing. This effectively creates a bipartite lattice with different sublattices A, B and a coupling of the cavity field to the imbalance between the occupation of the different sublattices $\hat{\Delta} = \sum_{j\in A,\sigma}\hat{n}_{j\sigma} - \sum_{l\in B,\sigma}\hat{n}_{l\sigma}$ with the effective pump strength g [82]. We note that we do not consider in $\hat{H}_{\rm ac}$ the dispersive coupling $\propto \hat{a}^\dagger\hat{a}\,\hat{\Delta}$ [2], which often leads to bistabilities due to optical nonlinearities.

A common method for dealing with cavity-atoms systems is to adiabatically eliminate the cavity mode and perform a mean-field approximation for the coupling term $\hat{H}_{\rm ac}$ [in the following called zero-temperature mean-field method ($T = 0$ MF)] [2,39]. After a fast decay, the photons are assumed to be in a coherent state with $\lambda \equiv \langle \hat{a} + \hat{a}^\dagger\rangle/\sqrt{L^d} = \{2g\delta/[\delta^2 + (\Gamma/2)^2]\}(\langle\hat{\Delta}\rangle/L^d)$ and the atoms in the self-consistently determined ground state of the effective Hamiltonian $\hat{H}_{\rm eff} = \hat{H}_{\rm FH} - \hbar g\lambda\hat{\Delta}$. We note that the MF treatment breaks the weak $\mathbb{Z}_2$ symmetry of the model, $(\hat{a}, \hat{\Delta}) \rightarrow (-\hat{a}, -\hat{\Delta})$, thus, in the following we restrict ourselves to the solution with $\langle\hat{a}\rangle \geq 0$, resulting in the A sublattice to have the lower energy.

However, the fluctuations in the coupling were shown to play an important role in the determination of the steady state in several setups [28,32,83–87]. Therefore, we employ a recently developed method, based on the many-body adiabatic elimination technique [88–93], to include perturbatively the fluctuations on top of the atoms-cavity mean-field [32,85]. Because of the complexity of the resulting equations, we perform a further approximation assuming the atomic steady state to be a thermal state, $\hat{\rho}_{\rm at} = e^{-\beta\hat{H}_{\rm eff}(\lambda)}/Z$. This is motivated by the level spacing distribution of $\hat{H}_{\rm eff}$ obeying Gaussian orthogonal ensemble (GOE) statistics in the presence of the staggered potential [94].

Importantly, the fluctuations in the atoms-cavity coupling determine the effective temperature at which the atomic system thermalizes and by this causes the admixture of excited states of the effective Hamiltonian in the density matrix of the system, in contrast to the $T = 0$ MF method. The effective inverse temperature $\beta = 1/k_B T$ and the cavity field strength λ are determined self-consistently from the mean-field relation and the steady state condition of the energy transfer $(\partial/\partial t)\langle\hat{H}_{\rm eff}\rangle_T = 0$ [85,95], given by

$$\frac{\partial}{\partial t}\langle\hat{H}_{\rm eff}\rangle_T \propto \int d\omega \frac{\hbar\omega\,{\rm Im}\,[\chi_T(\omega)]}{1-e^{-\beta\hbar\omega}} \frac{\Gamma/(2\pi)}{(\omega+\delta)^2+(\Gamma/2)^2}, \quad (2)$$

with self-consistently computed retarded susceptibility of the operator $\hat{\Delta}$, $\chi_T(\omega) = -(i/\hbar)\int_0^\infty dt e^{i\omega t}\langle[\hat{\Delta}(t),\hat{\Delta}(0)]\rangle_T$, which is evaluated for the thermal state $\hat{\rho}_{\rm at}$. As sketched in Fig. 1(d), this approach leads to different values of the temperature throughout the phase diagram and, as we discuss in the following, is one of the crucial ingredients for the occurrence of the bistability. To gain further insights regarding the self-consistent solutions, it is useful to go to the spectral representation of the energy transfer, namely,

$$\frac{\partial}{\partial t}\langle\hat{H}_{\rm eff}\rangle_T \propto \sum_{n,m}|\Delta_{nm}|^2 \frac{e^{-\beta E_m}(E_n-E_m)\Gamma}{(E_n-E_m+\hbar\delta)^2+(\hbar\Gamma/2)^2},$$

with E_n the energy of the eigenstate $|n\rangle$ of $\hat{H}_{\rm eff}$ and $\Delta_{nm} = \langle n|\hat{\Delta}|m\rangle$ [85,97].

In the following, we investigate the case of equal number of spin-up and spin-down particles. While we do not explicitly consider the total spin symmetry sectors, we checked that we obtain the same effective temperatures in typical spin sectors.

The nontrivial self-consistent solution(s) obtained might not be stable. Thus, we generalized the criterion of their stability [2,98,99] for finite temperature states [95]. We determine the stability under variation of the cavity field quadratures, considering the λ dependence of $E_n(\lambda), \beta(\lambda)$ and $\Delta_{nm}(\lambda)$, and obtain that for stable solutions the following holds

$$\frac{\delta^2+(\Gamma/2)^2}{2\delta g} > \sum_n \frac{e^{-\beta E_n}}{L^d Z}\left[\frac{\partial\Delta_{nn}}{\partial\lambda} - \Delta_{nn}\left(\frac{\partial\beta}{\partial\lambda}E_n + \beta\frac{\partial E_n}{\partial\lambda}\right)\right] + \sum_{n,m}\frac{e^{-\beta(E_n+E_m)}}{L^{2d}Z^2}\Delta_{nn}\left(\frac{\partial\beta}{\partial\lambda}E_m + \beta\frac{\partial E_m}{\partial\lambda}\right). \quad (3)$$

In order to obtain the steady state solutions we need to determine the eigenstates of $\hat{H}_{\rm eff}$, which we compute by two approaches. In the numerical approach, we employ the exact diagonalization (ED) method of $\hat{H}_{\rm eff}$ in the case of small one-dimensional systems at fixed particle number. Analytically, we perform a perturbation theory in the kinetic energy $J \ll U$, which allows us to go to the thermodynamic limit in any dimension. Here, the atom number is conserved only on average by introducing a chemical potential term in the Hamiltonian, $-\mu\sum_j \hat{n}_j$, and solving $\langle\hat{n}\rangle_T \equiv N/L^d$ for μ [95]. The energy transfer in this perturbative approach is given by

$$\frac{\partial}{\partial t}\langle\hat{H}_{\rm eff}\rangle_T \propto J^2\left[\frac{e^{\beta\mu}+e^{\beta(3\mu-U)}}{2g\lambda}\left(\frac{-e^{\beta\hbar g\lambda}}{(2g\lambda+\delta)^2+(\Gamma/2)^2} + \frac{e^{-\beta\hbar g\lambda}}{(2g\lambda-\delta)^2+(\Gamma/2)^2}\right) + \sum_{p=\pm1}\frac{e^{2\beta\mu}}{2g\lambda-pU/\hbar}\left(\frac{-e^{\beta(2\hbar g\lambda-pU)}}{(2g\lambda-pU/\hbar+\delta)^2+(\Gamma/2)^2} + \frac{1}{(2g\lambda-pU/\hbar-\delta)^2+(\Gamma/2)^2}\right)\right]. \quad (4)$$

Our results for an atomic filling of $n = 1/2$ show a very rich steady state diagram, see Fig. 1(d). At low pump power g the system is in the normal phase, with a vanishing photon number. Above a critical value $g_{\rm cr}$ the transition to the self-organized phase occurs, signaled by the emergence of a finite cavity field and a corresponding increase of the atomic sublattice density imbalance. Surprisingly, for even larger pump powers a novel type of bistability occurs within the self-organized phase, which we call *fluctuation-induced bistability.* We emphasize that the bistability is not present in the atoms-cavity mean-field approach [black line, Fig. 1(d)] and only occurs determining the effective temperature self-consistently. We show its absence for an externally fixed finite temperature, see Ref. [95].

The interplay of the cavity field and the interaction energy of the atoms is crucial for the presence of the bistability.

The first solution with the lower scaled photon number (marked by subscript 1) is mainly determined by the balance of the processes originating from transitions between eigenstates of $\hat{H}_{\mathrm{eff}}$, which are singly occupied on the lower or upper potential sublattice, i.e., $E_n - E_m \approx 2\hbar g\lambda_1$, contributing to Eq. (4) by the terms in the first line. The sign of the terms in Eq. (4) implies either a cooling or a heating nature of the processes in the self-consistent dynamics towards the steady state. Close to the self-organization threshold g_{cr}, the resonance between atomic excited states [sketched in Fig. 1(d)] and the photon energy leads to a decrease in the self-consistent temperature, see Ref. [95]. For large pump strength in the first solution, to balance the cooling (first term upper line) and heating (second term upper line) the temperature becomes relatively high $k_B T_1 > 8zJ$ [see color coding in Fig. 1(d)], with z the lattice coordination number. In contrast, for the second solution with higher scaled photon number (marked by subscript 2) the effective temperature is much lower. Transitions between states with either double occupancy, or two singly occupied sites [see Fig. 1(c)] captured by the lower line in Eq. (4) with $p = 1$ become important. The interaction energy is crucial, i.e., $E_n - E_m \approx 2\hbar g\lambda_2 - U$. In particular, the efficient transfer of energy from the atoms to the cavity mode due to the resonances between excited states and the photonic energy δ leads to a cooling mechanism, driving the atoms to a steady state with a much lower temperature in the bistable region. The two stable solutions are connected by a third unstable solution [smaller dots in Fig. 1(d)]. The occurrence of two stable solutions signals the fluctuation-induced bistability (blue hatched region).

To investigate this behavior in more detail, in Fig. 2 we show the results for the photon number, the sublattice imbalance, the atomic temperature, and the double occupancy of the atomic state versus the pump power for the two different methods. Let us focus first on the perturbative results for small J in the thermodynamic limit (red and purple symbols). The scaled photon number is zero below a critical pump strength, here approximately $\hbar g_{\mathrm{cr}}/J \sim 3.85$. Above g_{cr} we see a rapid increase of the cavity field creating an effective staggered potential and therefore a sublattice imbalance which signals the transition to the self-organized phase. Although such a transition to a self-organized staggered pattern has been known from the $T = 0$ MF approach [2,39], the critical pump strength found is considerably lower due to the absence of the fluctuations compared to the value from the finite temperature transition observed here. Additionally, a nonmonotonous behavior is observed in the sublattice imbalance for the fermionic atoms [Fig. 2(b)]. In the self-organized regime Δ has a maximum around $2g\lambda_1 = \delta$ (cyan vertical line) and

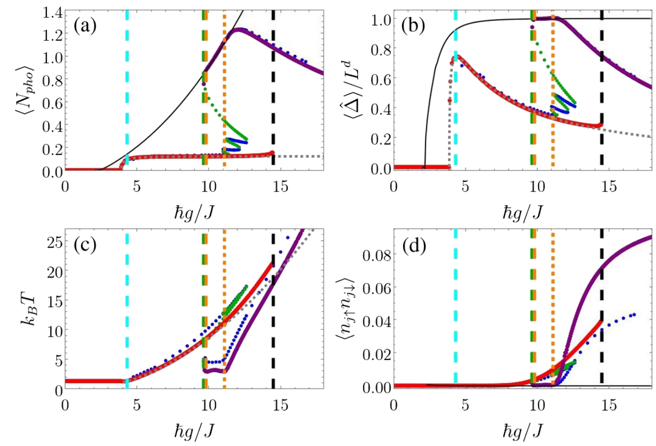

FIG. 2. (a) Scaled cavity photon number $\langle N_{\mathrm{pho}} \rangle = \langle \hat{a}^\dagger \hat{a} \rangle / L^d = \{[\delta^2 + (\Gamma/2)^2]/4\delta^2\}\lambda^2$, (b) atomic even-odd imbalance $\langle \hat{\Delta} \rangle / L^d$, (c) effective temperature and (d) average double occupancy as a function of the atoms-cavity coupling strength $\hbar g/J$. We show the following results: in the thermodynamic limit, $L \to \infty$ for $U \gg J$ stable solutions 1 (red) and 2 (purple), and for $U \to \infty$ (gray, dashed); for $L = 8$ the solution stable (blue), unstable (green); for $L = 8$, within the $T = 0$ MF approximation (black). The other parameters used are $U/J = 40, \hbar\delta/J = 5, \hbar\Gamma/J = 3$. Vertical lines denote the resonances, $2g\lambda_1 = \delta$ (cyan), $2g\lambda = U/\hbar$ (black, dashed), $2g\lambda_2 = U/\hbar \pm \delta$ (orange, short or long-dashed), $4g\lambda_2 = U/\hbar - \delta$ (green, dashed).

decreases for larger values of the pump strength. This decrease is mainly due to the quick rise of the finite temperature of the system [Fig. 2(c)] to a value of the order of the effective staggered potential $2\hbar g\lambda_1$ via the coupling to the cavity. At this temperature, reached at intermediate pump strength, $\hbar g/J \gtrsim 5$, atoms can be excited to the effectively higher lattice sites leading to the decrease of the sublattice imbalance compared to the atoms-cavity mean-field. This causes a relatively low number of photons per site that only weakly depends on the coupling strength g. As mentioned, around $\hbar g_{\mathrm{bi},1}/J = 9.65$ a second stable solution appears, with much larger number of photons, an almost maximal value of the atomic density imbalance and a low effective temperature [Fig. 2(c)].

We can understand the importance of the on-site interaction for the bistability by analyzing Eq. (4). We recover the first solution by neglecting all double occupancy, taking only the first two lines in Eq. (4), corresponding to $U \to \infty$ [gray dashed lines in Fig. 2]. However, the ending of the first solution and occurrence of the second one can only be obtained by considering terms corresponding to resonances $2g\lambda_2 = U/\hbar \pm \delta$ [95]. Thus, the bistability is related to processes stemming from the intricate interplay of the short-range atomic interactions and the global coupling to the self-organized photonic field.

For higher values of the pump strength, a drastic rise of the temperature can be found, determining an increase in the occupation of the excited states for the second solution, leading to the presence of double occupancies [Fig. 2(d)] and to a strong decrease of the scaled photon number and the atomic imbalance [Figs. 2(a) and 2(b)]. At even larger values of the pump strength $\hbar g_{\mathrm{bi},2}/J = 14.45$ the lower solution and the bistability region end. So far we focused on the description of the perturbative solution for small hopping J valid in the thermodynamic limit regardless of dimensionality. However, multistable solutions are observed also using ED to compute the energies and eigenstates of small one-dimensional systems $L \leq 8$ and solve fully Eq. (4) (blue, green symbols in Fig. 2). We associate this multistable region with the splitting of the resonances by finite size effects and find that the multistable region depends strongly on the system size. Nevertheless, the finite size systems capture qualitatively the same phenomenon of a multi-stable region caused by resonances, giving us confidence that it is not an artifact of the perturbative approach.

In Fig. 3 we show the dependence of the self-organization transition and the fluctuation-induced bistability region on U/J and $\hbar\delta/J$. The critical pump strength increases approximately linearly with the detuning $g_{\mathrm{cr}} \sim \delta$ at large detuning, similarly to the result obtained for bosonic atoms [85]. As the pump gets further detuned, the effective coupling to the cavity field decreases. Thus, to reach the same effective atom-cavity coupling, a larger amplitude of the pump field is required.

In contrast, the region of the bistability shows a very strong dependence on the interaction strength. We can obtain an approximate position of the lower onset [95]

$$g_{\mathrm{bi},1} = \sqrt{(U/\hbar - \delta)[(\Gamma/2)^2 + \delta^2]/(2\delta)}. \qquad (5)$$

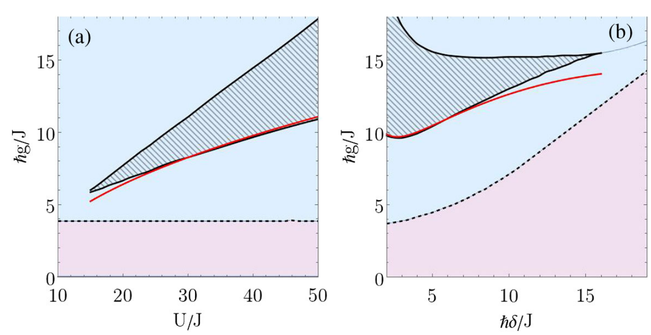

FIG. 3. Phase diagrams obtained from solving the equations of motion Eq. (4) together with the self-consistency and particle conservation as a function of (a) atoms-field coupling strength $\hbar g/J$ and atomic on-site interaction U/J for $\hbar\delta/J = 5$, $\hbar\Gamma/J = 3$ and (b) $\hbar g/J$ and cavity-pump detuning $\hbar\delta/J$ for $U/J = 40$, $\hbar\Gamma/J = 5$. With purple we mark the normal phase, with blue the self-organized phase, in the hatched region we show the fluctuation-induced bistability region within the self-organized phase. The red line represents the analytic approximation for $\hbar g_{\mathrm{bi},1}/J$, Eq. (5).

In particular, the relative strong dependence on U/J is not present in the onset of the self-organization phase for which g_{cr} is almost independent on U/J for the considered parameters. The upper boundary seems to evolve almost linearly with U/J, such that a wider bistable region is present at larger interactions. In contrast, increasing δ causes the bistability region to become smaller and to disappear beyond a certain large value. Whereas the lower boundary of the bistability region shows a similar rise with δ than the self-organization transition, the upper boundary becomes almost independent of the detuning above Γ.

To summarize, we have investigated fermionic atoms in optical lattices coupled to an optical cavity taking fluctuations beyond the mean-field decoupling into account. We find a transition between a normal and a self-organized phase and more surprisingly, the occurrence of a fluctuation-induced bistability region, which does not exist in the $T = 0$ MF solution. Within the bistable regime, an efficient cooling of the atomic ensemble takes place by a facilitated energy transfer between excited eigenstates of the atomic system to the photonic mode close to resonances. Many questions remain open, in particular on the dissipative dynamics of such systems, which might show nonthermal behavior in their approach towards the steady state, caused by the importance of these resonances.

We expect that the emergence of the fluctuation-induced bistability to be a generic feature of hybrid atoms-cavity systems in which one can control independently the coupling to the cavity and the atomic energies. As in our equations of motion we mainly use the spectrum of effective atomic Hamiltonian, the bistability should emerge for coupled systems where the resonances between the atomic levels and photonic excitations can be obtained.

Acknowledgments—We thank T. Donner, J.-P. Eckmann, S. B. Jäger, F. Mivehvar, H. Ritsch, and A. Rosch for fruitful discussions. We acknowledge support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Project No. 277625399-TRR 185 (B4) and Project No. 277146847-CRC 1238 (C05) and under Germany's Excellence Strategy—Cluster of Excellence Matter and Light for Quantum Computing (ML4Q) EXC 2004/1—390534769, and by the Swiss National Science Foundation under Division II Grant No. 200020-219400. This research was supported in part by the National Science Foundation under Grant No. NSF PHY-1748958 and No. PHY-2309135.

Data availability—The supporting data for this Letter are openly available at Zenodo [100].

[1] G. S. He, *Nonlinear Optics and Photonics* (Oxford University Press, New York, 2014).
[2] H. Ritsch, P. Domokos, F. Brennecke, and T. Esslinger, Cold atoms in cavity-generated dynamical optical potentials, Rev. Mod. Phys. 85, 553 (2013).

[3] V. Helson, T. Zwettler, K. Roux, H. Konishi, S. Uchino, and J.-P. Brantut, Optomechanical response of a strongly interacting Fermi gas, Phys. Rev. Res. **4**, 033199 (2022).

[4] K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger, Dicke quantum phase transition with a superfluid gas in an optical cavity, Nature (London) **464**, 1301 (2010).

[5] J. Keeling, M. J. Bhaseen, and B. D. Simons, Collective dynamics of Bose-Einstein condensates in optical cavities, Phys. Rev. Lett. **105**, 043001 (2010).

[6] A. Le Boité, G. Orso, and C. Ciuti, Steady-state phases and tunneling-induced instabilities in the driven dissipative Bose-Hubbard model, Phys. Rev. Lett. **110**, 233601 (2013).

[7] M. Benito, C. Sánchez Muñoz, and C. Navarrete-Benlloch, Degenerate parametric oscillation in quantum membrane optomechanics, Phys. Rev. A **93**, 023846 (2016).

[8] M.-J. Hwang, P. Rabl, and M. B. Plenio, Dissipative phase transition in the open quantum Rabi model, Phys. Rev. A **97**, 013825 (2018).

[9] H. Wilming, M. J. Kastoryano, A. H. Werner, and J. Eisert, Emergence of spontaneous symmetry breaking in dissipative lattice systems, J. Math. Phys. (N.Y.) **58**, 033302 (2017).

[10] M. Fitzpatrick, N. M. Sundaresan, A. C. Y. Li, J. Koch, and A. A. Houck, Observation of a dissipative phase transition in a one-dimensional circuit QED lattice, Phys. Rev. X **7**, 011016 (2017).

[11] J. S. Ferreira and P. Ribeiro, Lipkin-Meshkov-Glick model with Markovian dissipation: A description of a collective spin on a metallic surface, Phys. Rev. B **100**, 184422 (2019).

[12] K. C. Stitely, A. Giraldo, B. Krauskopf, and S. Parkins, Nonlinear semiclassical dynamics of the unbalanced, open Dicke model, Phys. Rev. Res. **2**, 033131 (2020).

[13] M. Soriente, T. L. Heugel, K. Omiya, R. Chitra, and O. Zilberberg, Distinctive class of dissipation-induced phase transitions and their universal characteristics, Phys. Rev. Res. **3**, 023100 (2021).

[14] F. Mivehvar, Conventional and unconventional Dicke models: Multistabilities and nonequilibrium dynamics, Phys. Rev. Lett. **132**, 073602 (2024).

[15] R. Labouvie, B. Santra, S. Heun, and H. Ott, Bistability in a driven-dissipative superfluid, Phys. Rev. Lett. **116**, 235302 (2016).

[16] M. Biondi, G. Blatter, H. E. Türeci, and S. Schmidt, Nonequilibrium gas-liquid transition in the driven-dissipative photonic lattice, Phys. Rev. A **96**, 043809 (2017).

[17] T. Fink, A. Schade, S. Höfling, C. Schneider, and A. Imamoglu, Signatures of a dissipative phase transition in photon correlation measurements, Nat. Phys. **14**, 365 (2017).

[18] M. Foss-Feig, P. Niroula, J. T. Young, M. Hafezi, A. V. Gorshkov, R. M. Wilson, and M. F. Maghrebi, Emergent equilibrium in many-body optical bistability, Phys. Rev. A **95**, 043826 (2017).

[19] J. Hannukainen and J. Larson, Dissipation-driven quantum phase transitions and symmetry breaking, Phys. Rev. A **98**, 042113 (2018).

[20] F. Minganti, A. Biella, N. Bartolo, and C. Ciuti, Spectral theory of Liouvillians for dissipative phase transitions, Phys. Rev. A **98**, 042118 (2018).

[21] A. Nava and M. Fabrizio, Lindblad dissipative dynamics in the presence of phase coexistence, Phys. Rev. B **100**, 125102 (2019).

[22] L. Garbe, P. Wade, F. Minganti, N. Shammah, S. Felicetti, and F. Nori, Dissipation-induced bistability in the two-photon Dicke model, Sci. Rep. **10**, 13408 (2020).

[23] F. Ferri, R. Rosa-Medina, F. Finger, N. Dogra, M. Soriente, O. Zilberberg, T. Donner, and T. Esslinger, Emerging dissipative phases in a superradiant quantum gas with tunable decay, Phys. Rev. X **11**, 041046 (2021).

[24] J. Benary, C. Baals, E. Bernhart, J. Jiang, M. Röhrle, and H. Ott, Experimental observation of a dissipative phase transition in a multi-mode many-body quantum system, New J. Phys. **24**, 103034 (2022).

[25] B. Gábor, D. Nagy, A. Dombi, T. W. Clark, F. I. B. Williams, K. V. Adwaith, A. Vukics, and P. Domokos, Ground-state bistability of cold atoms in a cavity, Phys. Rev. A **107**, 023713 (2023).

[26] J. J. Mendoza-Arenas, S. R. Clark, S. Felicetti, G. Romero, E. Solano, D. G. Angelakis, and D. Jaksch, Beyond mean-field bistability in driven-dissipative lattices: Bunching-antibunching transition and quantum simulation, Phys. Rev. A **93**, 023821 (2016).

[27] H. Landa, M. Schiró, and G. Misguich, Multistability of driven-dissipative quantum spins, Phys. Rev. Lett. **124**, 043601 (2020).

[28] C.-M. Halati, A. Sheikhan, H. Ritsch, and C. Kollath, Numerically exact treatment of many-body self-organization in a cavity, Phys. Rev. Lett. **125**, 093604 (2020).

[29] B. Buča and T. Prosen, A note on symmetry reductions of the Lindblad equation: Transport in constrained open spin chains, New J. Phys. **14**, 073007 (2012).

[30] V. V. Albert and L. Jiang, Symmetries and conserved quantities in Lindblad master equations, Phys. Rev. A **89**, 022118 (2014).

[31] D. Roberts and A. A. Clerk, Driven-dissipative quantum Kerr resonators: New exact solutions, photon blockade and quantum bistability, Phys. Rev. X **10**, 021022 (2020).

[32] C.-M. Halati, A. Sheikhan, and C. Kollath, Breaking strong symmetries in dissipative quantum systems: Bosonic atoms coupled to a cavity, Phys. Rev. Res. **4**, L012015 (2022).

[33] S. Brazovskiĭ, Phase transition of an isotropic system to a nonuniform state, Sov. J. Exp. Theor. Phys. **41**, 85 (1975), http://www.jetp.ras.ru/cgi-bin/e/index/e/41/1/p85?a=list.

[34] K. Binder, Theory of first-order phase transitions, Rep. Prog. Phys. **50**, 783 (1987).

[35] B. I. Halperin, T. C. Lubensky, and S.-k. Ma, First-order phase transitions in superconductors and smectic-*A* liquid crystals, Phys. Rev. Lett. **32**, 292 (1974).

[36] I. F. Herbut, A. Yethiraj, and J. Bechhoefer, Effect of order parameter fluctuations on the Halperin-Lubensky-Ma first-order transition in superconductors and liquid crystals, Europhys. Lett. **55**, 317 (2001).

[37] M. Janoschek, M. Garst, A. Bauer, P. Krautscheid, R. Georgii, P. Böni, and C. Pfleiderer, Fluctuation-induced

first-order phase transition in Dzyaloshinskii-Moriya helimagnets, Phys. Rev. B **87**, 134407 (2013).

[38] P. C. Hohenberg and J. B. Swift, Metastability in fluctuation-driven first-order transitions: Nucleation of lamellar phases, Phys. Rev. E **52**, 1828 (1995).

[39] F. Mivehvar, F. Piazza, T. Donner, and H. Ritsch, Cavity QED with quantum gases: New paradigms in many-body physics, Adv. Phys. **70**, 1 (2021).

[40] R. M. Kroeze, Y. Guo, V. D. Vaidya, J. Keeling, and B. L. Lev, Spinor self-ordering of a quantum gas in a cavity, Phys. Rev. Lett. **121**, 163601 (2018).

[41] V. D. Vaidya, Y. Guo, R. M. Kroeze, K. E. Ballantine, A. J. Kollár, J. Keeling, and B. L. Lev, Tunable-range, photon-mediated atomic interactions in multimode cavity QED, Phys. Rev. X **8**, 011002 (2018).

[42] H. Keßler, P. Kongkhambut, C. Georges, L. Mathey, J. G. Cosme, and A. Hemmerich, Observation of a dissipative time crystal, Phys. Rev. Lett. **127**, 043602 (2021).

[43] F. Ferri, R. Rosa-Medina, F. Finger, N. Dogra, M. Soriente, O. Zilberberg, T. Donner, and T. Esslinger, Emerging dissipative phases in a superradiant quantum gas with tunable decay, Phys. Rev. X **11**, 041046 (2021).

[44] D. Dreon, A. Baumgärtner, X. Li, S. Hertlein, T. Esslinger, and T. Donner, Self-oscillating pump in a topological dissipative atom–cavity system, Nature (London) **608**, 494 (2022).

[45] P. Kongkhambut, J. Skulte, L. Mathey, J. G. Cosme, A. Hemmerich, and H. Keßler, Observation of a continuous time crystal, Science **377**, 670 (2022).

[46] R. Landig, L. Hruby, N. Dogra, M. Landini, R. Mottl, T. Donner, and T. Esslinger, Quantum phases from competing short- and long-range interactions in an optical lattice, Nature (London) **532**, 476 (2016).

[47] L. Hruby, N. Dogra, M. Landini, T. Donner, and T. Esslinger, Metastability and avalanche dynamics in strongly correlated gases with long-range interactions, Proc. Natl. Acad. Sci. U.S.A. **115**, 3279 (2018).

[48] J. Klinder, H. Keßler, M. Wolke, L. Mathey, and A. Hemmerich, Dynamical phase transition in the open Dicke model, Proc. Natl. Acad. Sci. U.S.A. **112**, 3290 (2015).

[49] J. Klinder, H. Keßler, M. R. Bakhtiari, M. Thorwart, and A. Hemmerich, Observation of a superradiant Mott insulator in the Dicke-Hubbard model, Phys. Rev. Lett. **115**, 230403 (2015).

[50] V. Helson, T. Zwettler, F. Mivehvar, E. Colella, K. Roux, H. Konishi, H. Ritsch, and J.-P. Brantut, Density-wave ordering in a unitary Fermi gas with photon-mediated interactions, Nature (London) **618**, 716 (2023).

[51] T. Zwettler, G. del Pace, F. Marijanovic, S. Chattopadhyay, T. Bühler, C.-M. Halati, L. Skolc, L. Tolle, V. Helson, G. Bolognini, A. Fabre, S. Uchino, T. Giamarchi, E. Demler, and J.-P. Brantut, Non-equilibrium dynamics of long-range interacting Fermions, arXiv:2405.18204.

[52] X. Zhang, Y. Chen, Z. Wu, J. Wang, J. Fan, S. Deng, and H. Wu, Observation of a superradiant quantum phase transition in an intracavity degenerate Fermi gas, Science **373**, 1359 (2021).

[53] Z. Wu, J. Fan, X. Zhang, J. Qi, and H. Wu, Signatures of prethermalization in a quenched cavity-mediated long-range interacting Fermi gas, Phys. Rev. Lett. **131**, 243401 (2023).

[54] J. Larson, G. Morigi, and M. Lewenstein, Cold Fermi atomic gases in a pumped optical resonator, Phys. Rev. A **78**, 023815 (2008).

[55] Q. Sun, X.-H. Hu, A.-C. Ji, and W. M. Liu, Dynamics of a degenerate Fermi gas in a one-dimensional optical lattice coupled to a cavity, Phys. Rev. A **83**, 043606 (2011).

[56] F. Piazza and P. Strack, Umklapp superradiance with a collisionless quantum degenerate Fermi gas, Phys. Rev. Lett. **112**, 143003 (2014).

[57] F. Piazza and P. Strack, Quantum kinetics of ultracold fermions coupled to an optical resonator, Phys. Rev. A **90**, 043823 (2014).

[58] Y. Chen, Z. Yu, and H. Zhai, Superradiance of degenerate Fermi gases in a cavity, Phys. Rev. Lett. **112**, 143004 (2014).

[59] J. Keeling, M. J. Bhaseen, and B. D. Simons, Fermionic superradiance in a transversely pumped optical cavity, Phys. Rev. Lett. **112**, 143002 (2014).

[60] Y. Chen, H. Zhai, and Z. Yu, Superradiant phase transition of Fermi gases in a cavity across a Feshbach resonance, Phys. Rev. A **91**, 021602(R) (2015).

[61] J.-S. Pan, Superradiant phase transitions in one-dimensional correlated Fermi gases with cavity-induced umklapp scattering, Phys. Rev. A **105**, 013306 (2022).

[62] L. Dong, L. Zhou, B. Wu, B. Ramachandhran, and H. Pu, Cavity-assisted dynamical spin-orbit coupling in cold atoms, Phys. Rev. A **89**, 011602(R) (2014).

[63] J. Fan, X. Zhou, W. Zheng, W. Yi, G. Chen, and S. Jia, Magnetic order in a Fermi gas induced by cavity-field fluctuations, Phys. Rev. A **98**, 043613 (2018).

[64] E. Colella, R. Citro, M. Barsanti, D. Rossini, and M.-L. Chiofalo, Quantum phases of spinful Fermi gases in optical cavities, Phys. Rev. B **97**, 134502 (2018).

[65] E. Colella, F. Mivehvar, F. Piazza, and H. Ritsch, Hofstadter butterfly in a cavity-induced dynamic synthetic magnetic field, Phys. Rev. B **100**, 224306 (2019).

[66] F. Schlawin, A. Cavalleri, and D. Jaksch, Cavity-mediated electron-photon superconductivity, Phys. Rev. Lett. **122**, 133602 (2019).

[67] Z. Zheng and Z. D. Wang, Cavity-induced Fulde-Ferrell-Larkin-Ovchinnikov superfluids of ultracold Fermi gases, Phys. Rev. A **101**, 023612 (2020).

[68] X. Nie and W. Zheng, Nonequilibrium phases of a Fermi gas inside a cavity with imbalanced pumping, Phys. Rev. A **108**, 043312 (2023).

[69] E. Colella, S. Ostermann, W. Niedenzu, F. Mivehvar, and H. Ritsch, Antiferromagnetic self-ordering of a Fermi gas in a ring cavity, New J. Phys. **21**, 043019 (2019).

[70] F. Mivehvar, H. Ritsch, and F. Piazza, Superradiant topological peierls insulator inside an optical cavity, Phys. Rev. Lett. **118**, 073602 (2017).

[71] C. Kollath, A. Sheikhan, S. Wolff, and F. Brennecke, Ultracold fermions in a cavity-induced artificial magnetic field, Phys. Rev. Lett. **116**, 060401 (2016).

[72] A. Sheikhan, F. Brennecke, and C. Kollath, Cavity-induced generation of nontrivial topological states in a two-dimensional Fermi gas, Phys. Rev. A **94**, 061603(R) (2016).

[73] A. Sheikhan, F. Brennecke, and C. Kollath, Cavity-induced chiral states of fermionic quantum gases, Phys. Rev. A **93**, 043609 (2016).

[74] F. P. M. Mendez-Cordoba, J. J. Mendoza-Arenas, F. J. Gomez-Ruiz, F. J. Rodriguez, C. Tejedor, and L. Quiroga, Renyi entropy singularities as signatures of topological criticality in coupled photon-fermion systems, Phys. Rev. Res. **2**, 043264 (2020).

[75] J.-S. Pan, X.-J. Liu, W. Zhang, W. Yi, and G.-C. Guo, Topological superradiant states in a degenerate Fermi gas, Phys. Rev. Lett. **115**, 045303 (2015).

[76] T. Chanda, R. Kraus, G. Morigi, and J. Zakrzewski, Self-organized topological insulator due to cavity-mediated correlated tunneling, Quantum **5**, 501 (2021).

[77] F. Schlawin and D. Jaksch, Cavity-mediated unconventional pairing in ultracold fermionic atoms, Phys. Rev. Lett. **123**, 133601 (2019).

[78] S. Wolff, A. Sheikhan, and C. Kollath, Dissipative time evolution of a chiral state after a quantum quench, Phys. Rev. A **94**, 043609 (2016).

[79] F. Marijanović, S. Chattopadhyay, L. Skolc, T. Zwettler, C.-M. Halati, S. B. Jäger, T. Giamarchi, J.-P. Brantut, and E. Demler, Dynamical instabilities of strongly interacting ultracold fermions in an optical cavity, arXiv:2406.13548.

[80] H. Carmichael, *An Open Systems Approach to Quantum Optics* (Springer Verlag, Berlin, Heidelberg, 1991), 10.1007/978-3-540-47620-7.

[81] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002), 10.1093/acprof:oso/9780199213900.001.0001.

[82] C. Maschler, I. B. Mekhov, and H. Ritsch, Ultracold atoms in optical lattices generated by quantized light fields, Eur. Phys. J. D **46**, 545 (2008).

[83] F. Damanet, A. J. Daley, and J. Keeling, Atom-only descriptions of the driven-dissipative Dicke model, Phys. Rev. A **99**, 033845 (2019).

[84] C.-M. Halati, A. Sheikhan, and C. Kollath, Theoretical methods to treat a single dissipative bosonic mode coupled globally to an interacting many-body system, Phys. Rev. Res. **2**, 043255 (2020).

[85] A. V. Bezvershenko, C.-M. Halati, A. Sheikhan, C. Kollath, and A. Rosch, Dicke transition in open many-body systems determined by fluctuation effects, Phys. Rev. Lett. **127**, 173606 (2021).

[86] S. B. Jäger, T. Schmit, G. Morigi, M. J. Holland, and R. Betzholz, Lindblad master equations for quantum systems coupled to dissipative bosonic modes, Phys. Rev. Lett. **129**, 063601 (2022).

[87] V. Link, K. Müller, R. G. Lena, K. Luoma, F. Damanet, W. T. Strunz, and A. J. Daley, Non-Markovian quantum dynamics in strongly coupled multimode cavities conditioned on continuous measurement, PRX Quantum **3**, 020348 (2022).

[88] J. J. García-Ripoll, S. Dürr, N. Syassen, D. M. Bauer, M. Lettner, G. Rempe, and J. I. Cirac, Dissipation-induced hard-core boson gas in an optical lattice, New J. Phys. **11**, 013053 (2009).

[89] F. Reiter and A. S. Sørensen, Effective operator formalism for open quantum systems, Phys. Rev. A **85**, 032111 (2012).

[90] E. M. Kessler, Generalized Schrieffer-Wolff formalism for dissipative systems, Phys. Rev. A **86**, 012126 (2012).

[91] D. Poletti, P. Barmettler, A. Georges, and C. Kollath, Emergence of glasslike dynamics for dissipative and strongly interacting bosons, Phys. Rev. Lett. **111**, 195301 (2013).

[92] B. Sciolla, D. Poletti, and C. Kollath, Two-time correlations probing the dynamics of dissipative many-body quantum systems: Aging and fast relaxation, Phys. Rev. Lett. **114**, 170401 (2015).

[93] F. Lange, Z. Lenarčić, and A. Rosch, Time-dependent generalized Gibbs ensembles in open quantum systems, Phys. Rev. B **97**, 165138 (2018).

[94] J. De Marco, L. Tolle, C.-M. Halati, A. Sheikhan, A. M. Läuchli, and C. Kollath, Level statistics of the one-dimensional ionic Hubbard model, Phys. Rev. Res. **4**, 033119 (2022).

[95] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.134.133602 for details on the many body adiabatic elimination technique for atoms-cavity coupled systems, the mean-field results at fixed temperature, the perturbation theory in kinetic terms, the stability condition for thermal steady states, and regarding the analytical calculations for the onset of the bistable solutions. The Supplemental Material includes Ref. [96]

[96] G. A. Baker and P. Graves-Morris, *Padé Approximants*, 2nd ed., Encyclopedia of Mathematics and its Applications (Cambridge University Press, Cambridge, England, 1996), 10.1017/CBO9780511530074.

[97] C.-M. Halati, External control of many-body quantum systems, Ph.D. thesis, University of Bonn, 2021, https://hdl.handle.net/20.500.11811/9343.

[98] L. Tian, Cavity-assisted dynamical quantum phase transition at bifurcation points, Phys. Rev. A **93**, 043850 (2016).

[99] C.-M. Halati, A. Sheikhan, and C. Kollath, Cavity-induced artificial gauge field in a Bose-Hubbard ladder, Phys. Rev. A **96**, 063621 (2017).

[100] L. Tolle, A. Sheikhan, T. Giamarchi, C. Kollath, and C.-M. Halati, Figure data for article “Fluctuation-induced Bistability of Fermionic Atoms Coupled to a Dissipative Cavity” [Data set], Zenodo, 10.5281/zenodo.13805265 (2024).